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A

# COMPLETE

# ARITHMETIC,

UNITING

## MENTAL AND WRITTEN EXERCISES

IN A

## NATURAL SYSTEM OF INSTRUCTION.

By E. E. WHITE, M. A.

WILSON, HINKLE & CO.,
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# PREFACE.

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THIS work is called a COMPLETE ARITHMETIC, because it embraces all the subjects which properly belong to a school arithmetic, and because it treats these subjects both analytically and inductively. It is designed to be a complete text-book for pupils who have mastered the elements of numbers.

The work is characterized by the same features as the lower books of the series, viz.:

1. *It combines Mental and Written Arithmetic in a practical and philosophical manner.* This is done by making the mental exercises preparatory to the written; and thus these two classes of exercises, which have been so long and so unnaturally divorced, are united *as the essential complements of each other.*

2. *It faithfully embodies the inductive method of teaching.* The written methods are preceded by the analysis of mental problems, and both the written methods and the principles which they involve, are derived inductively from the analytic processes. The successive steps of each process are mastered by the pupil through the solution of problems, and he is required to deduce and state the rules before he is confronted with the author's generalization. All definitions which are deducible from the processes, and, with few exceptions, all principles and rules, are placed *after* the problems—a feature peculiar to this Series.

3. *It is specially adapted, both in matter and method, to the grade of pupils for which it is designed.* The greater portion of the work is devoted to a progressive and thorough treatment of subjects not embraced in the lower books—an arrangement which spe-

cially meets the wants of Graded Schools. But sixty pages cover the same ground as the Intermediate Arithmetic, and of these not more than twenty pages are in any sense a repetition. The repeated matter consists of definitions, principles, and rules, all the problems being new. The subjects before treated are not only concisely reviewed, but from *a higher stand-point.* Of the twenty-four pages devoted to the fundamental rules, eight present new abbreviated methods; and of twenty-eight pages devoted to Denominate Numbers, simple and compound, more than sixteen discuss new topics. A similar difference is observable in the treatment of Common Fractions and United States Money. Among the added articles worthy of special mention are those on Denominate Fractions, the Metric System, and Longitude and Time.

In the number of problems, the author has aimed to hit the golden mean between a paucity and an excess, and the greatest pains have been taken to make them sufficiently progressive, varied, and difficult, to afford the requisite drill and practice. Instead of rehashing old problems, with their incorrect data and obsolete terms, the author has gone to science and history for statistical information of practical value, and he has aimed to present the current values, terms, forms, and usages of American business. The mental problems will be found as difficult and comprehensive as those which constitute the latter half of the standard Mental Arithmetics, and are sufficiently numerous to afford thorough drills in analysis.

The explanations of the written processes are not designed to serve as models for the pupil to memorize and repeat. They are intended to supplement the analysis. In some cases, a formal analysis is given; in others, a principle is deduced or demonstrated; and in others, the process is described or its principles stated. Neither teacher nor pupil is denied the privilege of determining his own explanations.

Another characteristic feature of this work is the prominence

given to PRINCIPLES. A clear comprehension of the principles of arithmetic is essential to its thorough mastery, and their induction, proof, and illustration are mental exercises of great value. Until the pupil can step inductively from processes to principles, he has not a thorough knowledge of numbers. In this work the principles are concisely and formally stated in connection with the rules which are based upon them.

The author invites special attention to the sections treating of Percentage, Ratio and Proportion, and Involution and Evolution. Over eighty pages are devoted to Percentage and its applications, and it is believed that the treatment will be found not only full and thorough, but of great practical value. The student who masters these pages will certainly have a fair knowledge of the nature, laws, and usages of the business of the country. The introduction of *Formulas*, it is hoped, will prove a useful feature.

The thorough treatment of Ratio before Proportion, and of the latter before its application to the solution of problems, will make the mastery of this subject easy. The treatment of Involution and Evolution will not escape notice. The geometrical explanations of Square Root and Cube Root are the reverse of those usually given, and are believed to be new. They will be found both simple and natural.

All useless and obsolete subjects have been omitted; and those of interest only to more advanced students and teachers, are presented in an appendix. The typography and illustrations are beautiful and appropriate.

The COMPLETE ARITHMETIC is submitted to American teachers in the hope that it may not only be found new in its general plans and in many of its methods and details, but that it may prove eminently adapted to the present wants and condition of GRADED SYSTEMS OF INSTRUCTION.

COLUMBUS, OHIO, *July*, 1870.

## SUGGESTIONS TO TEACHERS.

1. The *Mental Problems* should be made a thorough drill in analysis; but, since the reasoning faculty is not trained by mere logical verbiage, the solution should be concise and simple. They should also be made introductory to the written processes of which they are often a complete elucidation. The corresponding problems in the two classes of exercises should be recited in connection, as well as separately.

2. All *Written Problems* should be solved by the pupils on slate or paper, and the solutions should be brought to the recitation for the teacher's inspection and criticism. From three to five minutes at the beginning of the recitation will suffice to compare the answers of the class, and ascertain the accuracy and neatness of each pupil's work. Time thus taken from the class-drill is more than made good by the increased interest, self-reliance, and accuracy, which the absence of answers secures. The explanations of the written processes, given by the pupil, should be both analytic and inductive.

3. The *Definitions* should be deduced and stated by the pupils under the guidance of the teacher, and this can usually be done in connection with the solution of the problems. See Int. Arith., p. 5, Sug. 3. When the definitions are placed before the problems, as in the applications of Percentage, they should be studied by the pupils, but their recitation may be deferred until the problems are solved, and the processes mastered.

4. The *Principles* should be taught inductively, when this is possible, and each should be proven or illustrated, or both, by the pupil. A thorough mastery of every principle should be made an essential condition of the pupil's progress. The recitation should secure a constant application of known principles, and a clear comprehension of all new ones.

5. The *Rules* should also be deduced and stated by the pupils. The true order is this: 1. A mastery of the process. 2. Recognition of the successive steps in order, and a statement of each. 3. Combination of these several statements into a general statement. 4. Comparison of this generalization with the author's rule. 5. Memorizing of the rule approved. See Int. Arith., p. 6.

6. When two or more methods or solutions are given, *the one preferred should be thoroughly taught.* It is well for pupils to understand different processes and explanations, but they should be made familiar with *one* of them.

7. Before a subject is left, the pupils should be required to make a topical analysis of the definitions, principles, and rules, and the same should be recited with accuracy and dispatch. Their knowledge of the subject should finally be tested by a series of questions and problems. See General Review, pp. 268–284.

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# COMPLETE ARITHMETIC.

## SECTION I.

### PRELIMINARY DEFINITIONS.

**Art. 1.** ***Arithmetic*** is the science of numbers, and the art of numerical computation.

As a *science*, Arithmetic treats of the relations, properties, and principles of numbers; and, as an *art*, it applies the science of numbers to their computation.

**2.** A ***Unit*** is one thing of any kind.

**3.** A ***Number*** is a unit or a collection of units.

**4.** An ***Integer*** is a number composed of whole or integral units; as, 5, 12, 20. It is also called a *Whole Number*.

**5.** Numbers are either *Concrete* or *Abstract*.

A ***Concrete Number*** is applied to a particular thing or quantity; as, 4 stars, 6 hours.

When a concrete number expresses the denominate units of currency, weight, or measure, it is called a *Denominate Number*. (Art. 174.)

An ***Abstract Number*** is not applied to a particular thing or quantity; as, 4, 6, 20.

A concrete number is composed of concrete units, and an abstract number of abstract units.

6. A ***Problem*** is a question proposed for solution.

7. An ***Example*** is a problem used to illustrate a process or a principle.

8. A ***Rule*** is a general description of a process.

9. An ***Arithmetical Sign*** is a character denoting an operation to be performed upon numbers, or a relation between them.

10. In the ***Mental Solution*** of a problem, the successive steps are determined mentally, and the results are held in the mind.

In the ***Written Solution*** of a problem, the results are written on a slate, paper, or other substance.

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## SECTION II.

### NOTATION AND NUMERATION.

#### MENTAL EXERCISES.

1. How many hundreds, tens, and units in 368? 427? 549? 608? 724? 806? 870?

2. How many hundred-thousands, ten-thousands, and thousands in 456048? 607803? 680435? 700450?

3. Read the thousands' period in 3045; 40607; 150482; 405360; 920400; 600060.

4. Read first the thousands' period and then the units' period in 65671; 120408; 400750; 650400; 80008.

5. Read 45037406; 520600480; 138405050.

6. Read 50008140; 600650508; 805000030.

7. Read 5308008450; 35006060600; 120500408080.

8. Read 7008360004; 302000860060; 500080800008.

### WRITTEN EXERCISES.

9. Express in figures the number composed of 5 thousands, 7 tens, and 3 units; 4 ten-thousands, 6 hundreds, and 5 units.

10. Express in figures 50 thousands and 40 units; 406 thousands and 30 units; 700 thousands and 7 units.

Express the following numbers in figures:

11. Five million five thousand five hundred.

12. Sixty million sixty thousand and sixty.

13. Seven hundred million seven hundred thousand seven hundred.

14. Five hundred and sixty million sixty-eight thousand.

15. Four billion fourteen million forty-five thousand.

16. Sixty-five billion six thousand and fifty.

17. Three hundred and fifty billion forty-nine million.

18. Seventeen trillion seventy billion seven hundred thousand four hundred.

19. Fifty-six trillion sixteen million and ninety.

20. Seven quadrillion eighty-five billion two hundred and four.

### DEFINITIONS AND PRINCIPLES.

**11.** There are three methods of expressing numbers:

1. By *words;* as, five, fifty, etc.
2. By *letters,* called the *Roman* method.
3. By *figures,* called the *Arabic* method.

**12.** ***Notation*** is the art of expressing numbers by figures or letters.

**13.** ***Numeration*** is the art of reading numbers expressed by figures or letters.

NOTE.—Notation may be defined to be the art of *writing* numbers, and Numeration, the art of *reading* numbers. In Arithmetic, the term notation is used to denote the Arabic method.

**14.** In the Roman Notation, numbers are expressed by means of seven *capital letters,* viz: I, V, X, L, C, D, M.

I stands for one; V for five; X for ten; L for fifty; C for one hundred; D for five hundred; M for one thousand, All other numbers are expressed by repeating or combining these letters.

**15.** In the Arabic Notation, numbers are expressed by means of ten characters, called figures, viz: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

The first of these characters, 0, is called *Naught*, or *Cipher*. It denotes nothing, or the absence of number.

The other nine characters are called *Significant Figures*, or *Numeral Figures*. They each express one or more units. They are also called *Digits*.

**16.** The successive figures which express a number, denote successive *Orders of Units*. A figure in units' place denotes *units of the first order*; in tens' place, *units of the second order*; in hundreds' place, *units of the third order*, and so on—the term *units* being used to express *ones* of any order.

**17.** Figures have two values, called *Simple* and *Local*.

The ***Simple Value*** of a figure is its value when standing in unit's place. It is also called its *Absolute* value.

The ***Local Value*** of a figure is its value arising from the order in which it stands. The local value of a figure is tenfold greater in hundreds' order than in tens' order.

**18.** The local value of each of the successive figures which express a number, is called a *Term*. The terms of 325 are 3 *hundreds*, 2 *tens*, and 5 *units*.

**19.** The figures denoting the successive orders of units, are divided into groups of three figures each, called *Periods*. The first or right-hand period is called *Units;* the second, *Thousands;* the third, *Millions;* the fourth, *Billions;* the fifth, *Trillions;* the sixth, *Quadrillions;* the seventh, *Quintillions;* the eighth, *Sextillions;* the ninth, *Septillions;* the tenth, *Octillions;* the eleventh, *Nonillions;* the twelfth, *Decillions*, etc.

NOTE.—The division of orders into periods of three figures each is the French method. In the English method, the period contains *six*

orders, the name of the first period being *Units*, the second *Millions*; the third *Billions*, etc.

**20.** The three orders of any period, counting from the right, denote respectively *Units*, *Tens*, and *Hundreds* of that period. They may be briefly read by calling the first order by the name of the period, and uniting the words *ten* and *hundred* in each period after the first with the period's name.

Thus, the orders of thousands' period may be read *thousands*, *ten-thousands*, *hundred-thousands*; the orders of millions' period, *millions*, *ten-millions*, *hundred-millions*, etc.

**21.** PRINCIPLES.—1. Ten units of the first order make one unit of the second order, ten units of the second order make one unit of the third, and, generally, *ten units of any order make one unit of the next higher order.* Hence,

2. *The value of the successive orders of figures increases tenfold from right to left.*

3. *The value of a figure is increased tenfold by each removal of it one order to the left, and is decreased tenfold by each removal of it one order to the right.*

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## SECTION III.

# ADDITION.

### MENTAL PROBLEMS.

1. Add by 6's from 1 to 73, thus: 1, 7, 13, 19, 25, etc.
2. Add by 7's from 3 to 73; from 6 to 90.
3. Add by 8's from 5 to 77; from 7 to 95.
4. Add by 9's from 4 to 76; from 8 to 98.
5. The ages of five boys are respectively 12, 10, 9, 8, and 7 years: what is the sum of their ages?
6. A rode 45 miles the first day, 42 miles the second day, and 38 miles the third day: how far did he ride in all?

SUGGESTION.—Add the tens and then the units of each couplet, thus: $45 + 40 = 85$, $85 + 2 = 87$; $87 + 30 = 117$, $117 + 8 = 125$. Or name only results, thus: 45, 85, 87; 117, 125. (Art. 22.)

7. A drover bought 37 sheep of one farmer, 44 sheep of another, 48 sheep of another, and 27 sheep of another: how many sheep did he buy?

8. The Senior class of a college contains 27 students, the Junior class 34, the Sophomore class 38, and the Freshman class 46: how many students in the college?

9. A grocer sold 18 sacks of flour on Monday, 23 on Tuesday, 27 on Wednesday, 24 on Thursday, 35 on Friday, and 37 on Saturday: how many sacks did he sell during the week?

10. A lady paid \$36 for a carpet, \$34 for a bureau, \$16 for a washstand, \$28 for a bedstead, and \$42 for chairs: how much did she pay for all?

11. A man paid \$85 for a horse, and \$17 for his keeping; and then sold him so as to gain \$15: for how much did he sell the horse?

12. Two men start from the same point, and travel in opposite directions, the one at the rate of 54 miles a day, and the other at the rate of 48 miles a day: how far will they be apart at the close of the second day?

## WRITTEN PROBLEMS.

13. Add 347, 4086, 7080, 29408, and 67736.

14. $667 + 3804 + 45608 + 304867 + 87609 =$ what?

15. Add four thousand and fifty-six; sixty-three thousand seven hundred; seven million nine thousand and ninety-nine; and fifty-six million nine hundred and seventy-eight.

16. Add eight million eighty thousand eight hundred; seven hundred thousand and seventy; five million eighty-six thousand seven hundred and eight; and sixty million six hundred thousand and seventy.

17. A grain dealer bought wheat as follows: Monday, 2480 bushels; Tuesday, 788 bushels; Wednesday, 1565 bushels; Thursday, 2684 bushels; Friday, 985 bushels; Saturday, 3867 bushels. How many bushels did he buy during the week?

18. Ohio contains 39964 square miles; Indiana, 33809; Illinois, 55409; Michigan, 56243; Wisconsin, 53924; Minnesota, 83531; Iowa, 55045; and Missouri, 65350. What is the total area of these eight States?

19. The population of these States in 1860 was as follows: Ohio, 2339511; Indiana, 1350428; Illinois, 1711951; Michigan, 756890; Wisconsin, 778714; Minnesota, 189923; Iowa, 674913; Missouri, 1182012. What was their total population?

20. The territory of the United States has been acquired as follows:

| | Square miles. |
|---|---|
| Territory ceded by England, 1783, | 815615 |
| Louisiana, as acquired from France, 1803, | 930928 |
| Florida, as acquired from Spain, 1821, | 59268 |
| Texas, as admitted to the Union, 1845, | 237504 |
| Oregon, as settled by treaty, 1846, | 280425 |
| California, etc., as conquered from Mexico, 1847, | 649762 |
| Arizona, as acquired from Mexico by treaty, 1854, | 27500 |
| Alaska, as acquired from Russia by purchase, 1867, | 577390 |

What is the total area of the United States?

## ADDITION OF TWO COLUMNS.

**22.** There is a practical advantage in adding two columns at one operation. Some accountants add three or more columns in this manner.

21. Add 67, 58, 43, 36, and 54.

PROCESS.

$$\begin{array}{r} 67 \\ 58 \\ 43 \\ 36 \\ \underline{54} \\ 258 \end{array}$$

Add thus: $54 + 30 = 84$, $+ 6 = 90$; $90 + 40 = 130$, $+ 3 = 133$; $133 + 50 = 183$, $+ 8 = 191$; $191 + 60 = 251$, $+ 7 = 258$.

Or thus, naming only results: 54, 84, 90; 130, 133, 183; 191; 251, 258.

NOTE.—The process consists in first adding the tens of each couplet, and then the units. If preferred, the units may first be added, and then the tens. Sufficient practice will enable the accountant to add two columns without separating the numbers into tens and units.

22. Add 37, 40, 63, 84, 67, 22, and 70.
23. Add 95, 46, 77, 66, 88, 63, 33, and 44.
24. Add 67, 76, 45, 54, 38, 83, 27, and 72.
25. Add 68, 86, 97, 79, 86, 68, 78, and 87.
26. Add 45, 60, 57, 86, 83, 76, 49, 58, and 84.
27. Add 56, 75, 83, 96, 69, 73, 37, 38, and 205.
28. Add 27, 72, 33, 38, 69, 96, 75, 57, and 336.
29. Add 235, 88, 77, 66, 55, 44, 33, 22, and 11.
30. Add 405, 56, 43, 47, 74, 36, 63, 75, and 66.
31. Add 46, 67, 72, 38, 99, 87, 65, 74, and 88.
32. Add 73, 86, 47, 56, 69, 65, 58, 33, 52, and 94.

## DEFINITIONS AND PRINCIPLES.

**23.** ***Addition*** is the process of finding the sum of two or more numbers.

The ***Sum*** of two or more numbers is a number containing as many units as all of them, taken together. It is also called the *Amount*.

**24.** The ***Sign of Addition*** is a short vertical line bisecting an equal horizontal line, thus: $+$. It is called *plus*.

**25.** The ***Sign of Equality*** is two short horizontal parallel lines, thus: $=$. It is read *equals* or *is equal to*. Thus, $7 + 8 = 15$ is read 7 *plus* 8 *equals* 15.

**26.** ***Like Numbers*** are composed of units of the same kind. Thus, 4 balls and 8 balls, or 4 dimes and 8 dimes, or 4 and 8, are like numbers.

**27.** PRINCIPLES.—1. *Only like numbers can be added.*

2. *Only like orders of figures can be added.*

3. *The sum is of the same kind or order as the numbers added.*

4. *The sum is the same whatever be the order in which the numbers are added.*

NOTE.—See appendix for method of proof by "casting out the 9's."

# SECTION IV.

# SUBTRACTION.

### MENTAL PROBLEMS.

1. Count by 4's from 61 back to 1, thus: 61, 57, 53, etc.
2. Count by 6's from 53 back to 5; from 74 back to 2.
3. Count by 7's from 66 back to 3; from 85 back to 1.
4. Count by 8's from 75 back to 3; from 94 back to 6.
5. Count by 9's from 73 back to 1; from 96 back to 6.
6. A grocer having a certain number of sacks of flour, bought 48 sacks, and sold 33 sacks, and then had 34 sacks on hand: how many sacks had he at first?
7. A man sold a horse for $95, which was $28 more than the horse cost him: what was the cost of the horse?
8. Two men start at once from the same point, and travel in the same direction, one traveling 52 miles a day, and the other but 39 miles: how far will they be apart at the close of the second day?
9. A man earns $85 a month, and pays $18 for house rent, and $35 for other expenses: how much does he save each month?
10. A gentleman being asked his age said, that if he should live 27 years longer, he should then be three score and ten: what was his age?
11. From a piece of carpeting containing 68 yards, a merchant sold 27 yards to one man and 18 yards to another: how many yards of the piece were left?
12. A man bought a carriage for $135, paid $21 for repairing it, and then sold it for $170: how much did he gain?
13. A boy earned 65 cents, and his father gave him 33 cents; he paid 45 cents for an arithmetic and 18 cents for a slate: how much money had he left?
14. There are 85 sheep in three fields; there are 36 sheep

in the first field, and 28 sheep in the second: how many sheep in the third field?

15. John had 33 chestnuts, and Charles 25; John gave Charles 14 chestnuts, and Charles gave his sister as many as he then had more than John: how many chestnuts did the sister receive?

### WRITTEN PROBLEMS.

16. A builder contracted to build a school-house for $25460, and the job cost him $21385: what were his profits?

17. The earth's mean distance from the sun (old value) is 95274000 miles, and that of Mars is 145168136: how much farther is Mars from the sun than the earth?

18. The population of Illinois in 1860 was 1711951, and in 1865 its population was 2141510: what was the increase in five years?

19. The population of Massachusetts in 1860 was 1231066, and in 1865 it was 1267031: what was the increase in five years?

20. The area of the Chinese Empire is 4695334 square miles, and the area of the United States is 3578392 square miles: how much greater is the Chinese Empire than the United States?

21. The area of Europe is 3781280 square miles: how much greater is Europe than the United States? The Chinese Empire than Europe?

22. In 1866, Ohio produced 99766822 bushels of corn, and Illinois 155844350 bushels: how many bushels did Illinois produce more than Ohio?

23. A man bought a farm for $5867, and built upon it a house at a cost of $1850, and then sold the farm for $7250: how much did he lose?

24. An estate of $13450 was divided between a widow and two children; the widow's share was $6340, the son's $1560 less than the widow's, and the rest fell to the daughter: what was the daughter's share?

25. A man deposited in a bank at one time $850, at an-

other, $367, and at another, $670; he then drew out $480, and $375: how much money had he still in bank?

26. A man bought a farm for $6450, giving in exchange a house worth $4500, a note for $1150, and paying the difference in money: how much money did he pay?

27. A grain dealer bought 15640 bushels of wheat, and sold at one time 3465 bushels, at another, 4205, and at another, 1080: how many bushels remained?

28. A has 320 acres of land; B has 65 acres more than A; C has 124 acres less than both A and B; and D has as many acres as both A and C less the number of acres owned by B. How many acres have B, C, and D respectively? How many have all?

29. From the sum of 45003 and 13478, take their difference.

### DEFINITIONS AND PRINCIPLES.

**28.** ***Subtraction*** is the process of finding the difference between two numbers.

The ***Difference*** is the number found by taking one number from another. It is also called the *Remainder*.

The ***Minuend*** is the number diminished.

The ***Subtrahend*** is the number subtracted.

**29.** The ***Sign of Subtraction*** is a short horizontal line, made thus —. It is called *minus*. Thus 12 — 5 is read 12 *minus* 5.

**30.** PRINCIPLES.—1. *The minuend, subtrahend, and difference are like numbers.*

2. *The minuend is the sum of the subtrahend and difference.*

3. *If the minuend and subtrahend be equally increased, the difference will not be changed.*

4. *The adding of* 10 *to a term of the minuend and* 1 *to the next higher term of the subtrahend, increases the minuend and subtrahend equally.*

# SECTION V.

# MULTIPLICATION.

### MENTAL PROBLEMS.

1. There are 24 hours in a day: how many hours in 7 days? In 9 days? 11 days?

2. There are 60 minutes in an hour: how many minutes in 8 hours? In 12 hours? 15 hours?

3. If a man earn $63 a month, and spend $48, how much will he save in 12 months?

4. If 12 men can do a piece of work in 15 days, how long will it take one man to do it?

5. If 35 bushels of oats will feed 8 horses 25 days, how long will they feed one horse?

6. Two men start from the same place and travel in opposite directions, one at the rate of 28 miles a day, and the other at the rate of 32 miles a day: how far will they be apart at the end of five days?

7. Two men are 450 miles apart: if they approach each other, one traveling 30 miles a day and the other 35 miles a day, how far apart will they be at the end of 6 days?

8. A cask has two pipes, one discharging into it 90 gallons of water an hour, and the other drawing from it 75 gallons an hour: how many gallons of water will there be in the cask at the end of 12 hours?

9. A had $24, B four times as much as A less $16, and C twice as much as A and B together plus $17: how much money had B and C?

10. A farmer sold to a grocer 15 pounds of butter, at 30 cents a pound, and bought 8 pounds of sugar, at 15 cents a pound, and 9 pounds of coffee, at 20 cents a pound: how much was still due him?

### WRITTEN PROBLEMS.

11. Multiply 624 by 45; by 405; by 4005.
12. Multiply 38400 by 27; by 607; by 6007.
13. Multiply 7863 by 69; by 6900; by 64000.
14. Multiply 48000 by 760; by 7600000.
15. There are 5280 feet in a mile: how many feet in 608 miles? In 3300 miles?
16. The earth moves 1092 miles in a minute: how far does it move in 1440 minutes, or one day?
17. A square mile contains 640 acres, and the state of Ohio contains, in round numbers, 40000 square miles: how many acres in the state?
18. If a garrison of 380 soldiers consume 56 barrels of flour in 75 days, how many soldiers will the same amount of flour supply one day?
19. A man bought a farm, containing 472 acres, at $24 an acre, and after investing $3450 in buildings, he sold the farm, at $33 an acre: did he gain or lose, and how much?

### DEFINITIONS AND PRINCIPLES.

**31.** ***Multiplication*** is the process of taking one number as many times as there are units in another.

The ***Multiplicand*** is the number taken or multiplied.

The ***Multiplier*** is the number denoting how many times the multiplicand is taken.

The ***Product*** is the number obtained by multiplying.

The multiplicand and multiplier are *Factors* of the product, and the product is a *Multiple* of each of its factors.

**32.** The ***Sign of Multiplication*** is an oblique cross, made thus: ×. It is read *multipled by*, or *times*.

When placed between two numbers, it shows that they are to be multiplied together; and, since the order of the factors does not affect the product, either number may be made the multiplier. The multiplier is usually written after the sign.

**33.** The product may be obtained by adding the multiplicand to itself as many times less one as there are units in the multiplier. Hence, *Multiplication is a short method of finding the sum of several equal numbers.*

**34.** Principles.—1. *The Multiplicand may be either concrete or abstract.*

2. *The multiplier must always be regarded as abstract.*

3. *The product and multiplicand are like numbers.*

4. *The product is not affected by changing the order of the factors.*

5. *The multiplicand equals the product divided by the multiplier.*

6. *The multiplier equals the product divided by the multiplicand.*

7. *The division of either the multiplicand or the multiplier by any number divides the product by that number.*

## ABBREVIATED PROCESSES.

### Case I.

#### The Multiplier 10, 100, 1000, etc.

1. There are 7 days in a week: how many days in 10 weeks? In 100 weeks?

2. There are 24 hours in a day: how many hours in 10 days? 100 days?

3. If a railway train run 30 miles an hour, how far will it run in 10 hours? 1000 hours?

4. If a freight car will carry 18 head of cattle, how many cattle will 10 cars carry? 100 cars? 1000 cars?

5. There are 12 months in a year: how many months in 100 years? 1000 years?

#### WRITTEN PROBLEMS.

6. Multiply 648 by 100.

Process: $648 \times 100 = 64800$. The annexing of a cipher to a number removes the significant figures one place to the left, and hence increases their value 10 times; the annexing of two ciphers removes

the significant figures two places to the left, and increases their value 100 times. Hence, the annexing of two ciphers to 648 multiplies it by 100.

7. Multiply 456 by 10; by 10000.
8. Multiply 3050 by 100; 100000.
9. Multiply 347000 by 1000; by 1000000.
10. Multiply 889000 by 10000; by 100.

**35.** PRINCIPLE.—*The removal of a figure one order to the left increases its value tenfold.*

**36.** RULE.—To multiply by 10, 100, 1000, etc., *Annex to the multiplicand as many ciphers as there are ciphers in the multiplier.*

## Case II.

### The Multiplier a convenient part of 10, 100, 1000, etc.

NOTE.—If the class is not sufficiently familiar with the subject of fractions, this case may be omitted.

11. There are 24 sheets of paper in a quire: how many sheets in $2\frac{1}{2}$ quires? In $3\frac{1}{3}$ quires?

12. There are 60 minutes in an hour: how many minutes in $3\frac{1}{3}$ hours? In $12\frac{1}{2}$ hours?

13. If a workman earn $40 a month, how much will he earn in $2\frac{1}{2}$ months? In $12\frac{1}{2}$ months?

14. At 36 cents a yard, what will 25 yards of cloth cost? $33\frac{1}{3}$ yards?

15. At 24 cents a dozen, what will $12\frac{1}{2}$ dozens of eggs cost? $16\frac{2}{3}$ dozens?

### WRITTEN PROBLEMS.

16. Multiply 459 by $33\frac{1}{3}$.

PROCESS.

3 ) 45900
15300 *Prod.*

Since $33\frac{1}{3}$ is $\frac{1}{3}$ of 100, $33\frac{1}{3}$ times $459 = \frac{1}{3}$ of 100 times $459 = \frac{1}{3}$ of 45900. Or, multiply the multiplicand by 100, and divide the product by 3.

17. Multiply 486 by $3\frac{1}{3}$; by $33\frac{1}{3}$.
18. Multiply 1688 by $12\frac{1}{2}$; by 25; by 50.

19. Multiply 40648 by $16\frac{2}{3}$; by $33\frac{1}{3}$; by $333\frac{1}{3}$.
20. Multiply 3468 by 25; by 125; by 250.
21. Multiply 4086 by $16\frac{2}{3}$; by $166\frac{2}{3}$; by $333\frac{1}{3}$.
22. Multiply 10366 by 50; by $33\frac{1}{3}$; by $66\frac{2}{3}$.

**37.** PRINCIPLE.—*If the multiplier be multiplied by a given number, and the resulting product be divided by the same number, the quotient will be the true product.*

**38.** RULE.—To multiply by a convenient part of 10, 100, 1000, etc., *Multiply by* 10, 100, 1000, *etc., and divide the product by the number of times the multiplier has been increased.*

## Case III.

### The Multiplier a little less than 10, 100, 1000, etc.

23. Multiply 467 by 98.

PROCESS.

46700
934
45766, *Prod.*

Since $98 = 100 - 2$, the product of 467 by $98 = 467 \times 100 - 467 \times 2$, or $46700 - 934$. In multiplying by 100 the multiplicand is taken two times more than it should be.

24. Multiply 5672 by 99; by 999.
25. Multiply 40863 by 97; by 997.
26. Multiply 8679 by 998; by 9998.
27. Multiply 618734 by 95; by 99995.

**39.** RULE.—To multiply by a number a little less than 10, 100, 1000, etc., *Multiply by* 10, 100, 1000, *etc., and subtract from the product the multiplicand multiplied by the difference between the multiplier and* 10, 100, 1000, *etc., as the case may be.*

## Case IV.

### The Multiplier 14, 15, 16, etc., or 31, 51, 61, etc.

28. Multiply 7856 by 14; by 41.

| 1ST PROCESS. | 2D PROCESS. |
|---|---|
| $7856 \times 14$ | $7856 \times 41$ |
| 31424 | 31424 |
| 109984, *Product.* | 322096, *Product.* |

NOTE.—An inspection of each process will suggest its explanation. The second partial product need not be written, as the successive terms can be added mentally to the proper terms of the first partial product.

29. Multiply 38407 by 13; by 15; by 17.
30. Multiply 4960 by 16; by 18; by 19.
31. Multiply 360978 by 31; by 51; by 71.
32. Multiply 48706 by 61; by 81; by 91.
33. Multiply 34087 by 17; by 71; by 18.

**40.** RULES.—1. To multiply by 13, 14, 15, etc., *Multiply by the units' term, and add the successive products after the first, which is units, to the successive terms of the multiplicand.*

2. To multiply by 31, 41, 51, etc., *Multiply by the tens' term, and add the successive products to the successive terms of the multiplicand beginning with tens.*

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# SECTION VI.

# DIVISION.

### MENTAL PROBLEMS.

1. There are 7 days in a week: how many weeks in 63 days? 98 days? 126 days?

2. There are eight quarts in a peck: how many pecks in 72 quarts? 120 quarts? 144 quarts?

3. There are 60 minutes in an hour: how many hours in 480 minutes? 720 minutes? 1440 minutes?

4. A man paid $3600 for a farm, paying at the rate of $40 an acre: how many acres in the farm?

5. A grocer bought 12 barrels of flour for $90, and sold them so as to gain $18: how much did he receive per barrel?

6. Two men are 120 miles apart, and are traveling toward each other, one at the rate of 7 miles an hour, and the other at the rate of 8 miles an hour: in how many hours will they meet?

7. If a man can build a wall in 84 days, how long will it take 7 men to build it?

8. If 8 men can do a piece of work in 15 days, how long will it take 12 men to do it?

9. If a quantity of provisions will supply a ship's crew of 20 men 15 weeks, how large a crew will it supply 25 weeks?

10. If a man can do a piece of work in 40 days, by working 8 hours a day, how long would it take him if he should work 10 hours a day?

11. A man earns \$16 while a boy earns \$9: how many dollars will the man earn while the boy is earning \$72?

12. The fore wheels of a carriage are each 9 feet in circumference, and the hind wheels are each 12 feet: if the fore wheels each rotate 400 times in going a certain distance, how many rotations will each hind wheel make?

13. Five times Harry's age plus 4 times his age, minus 6 times his age, plus 7 times his age, minus 5 times his age, equals 60 years: how old is Harry?

14. A number multiplied by 6, divided by 3, multiplied by 8, and divided by 4, equals 96: what is the number?

## WRITTEN PROBLEMS.

15. Divide 486 by 6; by 8; by 9.

16. Divide 8408 by 12; by 24; by 36.

17. Divide 84600 by 900; by 12000.

18. Divide 412304 by 3600; by 303000.

19. The dividend is 1059984 and the divisor is 306: what is the quotient?

20. The dividend is 2185750 and the quotient is 250: what is the divisor?

21. The product is 1123482 and the multiplier is 246: what is the multiplicand?

22. How many passenger cars, costing \$2450 each, can be bought for \$100450?

23. There are 5280 feet in a mile, and the height of Mount Everest, in Asia, is 29100 feet: what is its height in miles?

24. There are 3600 seconds in an hour: how many hours in 738000 seconds?

25. Divide the product of 480 and 256 by their difference.

## DEFINITIONS AND PRINCIPLES.

**41.** ***Division*** is the process of finding how many times one number is contained in another; or, it is the process of finding one of the equal parts of a number.

The ***Dividend*** is the number divided.

The ***Divisor*** is the number by which the dividend is divided.

The ***Quotient*** is the number of times the divisor is contained in the dividend.

The ***Remainder*** is the part of the dividend which is left undivided.

**42.** The ***Sign of Division*** is a short horizontal line between two dots, thus: $\div$. It is read *divided by*. Thus, $16 \div 4$ is read 16 *divided by* 4.

Division is also expressed by writing the dividend above and the divisor below a short horizontal line. Thus, $\frac{18}{3}$ is read 18 *divided by* 3.

**43.** There are two methods of division, called *Short Division* and *Long Division.*

In ***Short Division,*** the partial products and partial dividends are not written, but are formed mentally.

In ***Long Division,*** the partial products and partial dividends are written.

**44.**—1. One number is contained in another as many times as it must be taken to produce it. Hence, *Division is the reverse of multiplication.*

2. One number is contained in another as many times as it can be taken from it. Hence, *Division is a brief method of finding how many times one number can be subtracted from another.*

**45**. PRINCIPLES.—1. *The divisor and quotient are factors of the dividend.*

2. When division finds how many times one number is contained in another, *the divisor and dividend are* LIKE NUMBERS, *and the quotient is an abstract number.*

3. When division finds one of the equal parts of a number, *the divisor is an abstract number, and the dividend and quotient are* LIKE NUMBERS.

4. *The multiplying of both divisor and dividend by the same number does not change the value of the quotient.*

5. *The dividing of both dividend and divisor by the same number does not change the value of the quotient.*

## ABBREVIATED PROCESSES.

### Case I.

#### The Divisor 10, 100, 1000, etc.

1. There are 10 cents in a dime: how many dimes in 80 cents? 120 cents? 240 cents?

2. There are 10 dimes in a dollar: how many dollars in 70 dimes? 250 dimes? 2500 dimes?

3. There are 100 cents in a dollar: how many dollars in 800 cents? 2400 cents? 7500 cents?

4. At $10 a barrel, how many barrels of flour can be bought for $90? For $150?

5. At $100 apiece, how many horses can be bought for $1200? For $2500? For $45000?

#### WRITTEN PROBLEMS.

6. Divide 450 by 10.

PROCESS.

45|0
45, Quotient.

7. Divide 3852 by 100.

PROCESS.

38|52
38, *Quotient.*
52, *Remainder.*

The explanation of these processes is obvious. The cutting off of the right-hand figure removes all the other figures one place to the right, and thus decreases their value *ten* times. The cutting off of two figures removes the other figures two places to the right, and de-

creases their value *one hundred* times. The figures cut off denote the remainder.

8. Divide 356000 by 100; by 1000.
9. Divide 46035 by 100; by 1000.
10. Divide 384602; by 1000; by 10000.
11. Divide 95000000 by 10000; by 1000000.

**46.** PRINCIPLE.—*The removal of a figure one order to the right decreases its value tenfold.*

**47.** RULE.—To divide by 10, 100, 1000, etc., *Cut off as many figures from the right of the dividend as there are ciphers in the divisor. The figures cut off denote the remainder.*

## Case II.

### The Divisor ending with one or more Ciphers.

12. There are 20 quires of paper in a ream: how many reams in 80 quires? 160 quires?
13. There are fifty cents in a half-dollar: how many half-dollars in 150 cents? 350 cents?
14. There are 60 minutes in an hour: how many hours in 240 minutes? 720 minutes?
15. A barrel of beef contains 200 pounds: how many barrels will 1200 pounds make? 3600 pounds?

### WRITTEN PROBLEMS.

16. Divide 71400 by 3400.

PROCESS.

```
34|00 ) 714|00 ( 21
        68
        --
         34
         34
         --
```

First divide both divisor and dividend by 100, which is done by cutting off the two right-hand figures. Then divide 714, the new dividend, by 34, the new divisor.

17. Divide 58864 by 4500.

PROCESS.

```
45|00 ) 588|64 ( 13
        45
        ---
        138
        135
        ---
          3
Remainder, 364
```

First divide both dividend and divisor by 100, which, in the case of the dividend, leaves a remainder of 64. Next divide 588 by 45, leaving a remainder of 3, which is 3 *hundreds* since the dividend (588) is hundreds. The first remainder is 64 *units* which, annexed to the 3 hundreds, give 364, the true remainder.

18. Divide 63200 by 7900; by 7000.
19. Divide 116000 by 2500; by 4800.
20. Divide 172800 by 14400; by 18000.
21. Divide 129600 by 4800; by 64000.

**48.** PRINCIPLE.—*The dividing of both divisor and dividend by the same number does not change the value of the quotient.*

**49.** RULE.—To divide by a number ending in one or more ciphers, 1. *Cut off the ciphers from the right of the divisor, and an equal number of figures from the right of the dividend.*

2. *Divide the new dividend thus formed by the new divisor, and the result will be the quotient.*

3. *Annex the figures cut off from the dividend to the remainder, if there be one, and the result will be the true remainder.*

## Case III.

### The Divisor a convenient part of 10, 100, etc.

22. At $3\frac{1}{3}$ cents apiece, how many lemons can be bought for 90 cents? For 240 cents?

SUGGESTION.—Since 10 is 3 times $3\frac{1}{3}$, multiply the dividend by 3 and divide the product by 10.

23. At $12\frac{1}{2}$ cents a yard, how many yards of cloth can be bought for 75 cents? For 225 cents?

24. At $16\frac{2}{3}$ cents a bushel, how many bushels of coal can be bought for 150 cents? For 550 cents?

25. At $\$33\frac{1}{3}$ a head, how many cows can be bought for $200? For $1200?

### WRITTEN EXERCISES.

26. Divide 4375 by 125.

PROCESS.

$$\begin{array}{r} 4375 \\ 8 \\ \hline 35|000 \end{array}$$

35, *Quotient.*

27. Divide 13600 by $333\frac{1}{3}$.

PROCESS.

$$\begin{array}{r} 13600 \\ 3 \\ \hline 40|800 \end{array}$$

40, *Quotient.*

$800 \div 3 = 266\frac{2}{3}$ *Rem.*

28. Divide 6250 by $33\frac{1}{3}$; by 50.

29. Divide 4365 by 250; by $166\frac{2}{3}$.
30. Divide 15300 by $16\frac{2}{3}$; by $333\frac{1}{3}$.

**50.** Principle.—*The multiplying of both divisor and dividend by the same number does not change the value of the quotient.*

**51.** Rule.—To divide by a convenient part of 10, 100, 1000, etc., *Multiply the dividend by the number denoting how many times the divisor is contained in* 10, *or* 100, *or* 1000, *etc., and divide the product by* 10, *or* 100, *or* 1000, *etc.*

## Case IV.

### The Divisor a Composite Number.

31. Divide 18315 by 45.

Process.

$45 = 5 \times 9$

$$\begin{array}{r|l} 5 & \underline{18315} \\ 9 & \underline{3663} \\ & 407, \textit{ Quotient.} \end{array}$$

Illustrative Process.

$$\begin{array}{r|l} 5 & \underline{18315 \div 45 = 3663 \div 9} \\ 9 & \underline{3663 \div 9 = 407 \div 1} \\ & 407 \div 1 = 407 \end{array}$$

Since $45 = 5 \times 9$, the quotient obtained by dividing 18315 by 5, is 9 *times too large*, and hence this quotient (3663) divided by 9, is the true quotient.

The process of dividing by the factors of the divisor successively is the same in principle as the division of both dividend and divisor by these factors successively, which (Art. 48) does not change the value of the quotient. See "Illustrative Process."

32. Divide 58636 by 28; by 77.
33. Divide 13328 by 49; by 56; by 70.
34. Divide 31360 by 64; by 70; by 81.
35. Divide 3687 by 64.

Process.

$64 = 2 \times 8 \times 4$

$$\begin{array}{r|l} 2 & \underline{3687} \\ 8 & \underline{1843} \ldots\ldots 1 \text{ (1st Rem.)} = \ldots\ldots\ldots\ldots 1 \\ 4 & \underline{230} \ldots\ldots 3 \text{ (2d " )} = 3 \times 2 = \ldots\ldots 6 \\ & 57 \ldots\ldots 2 \text{ (3d " )} = 2 \times 8 \times 2 = \underline{32} \\ & \qquad\qquad \text{True Remainder} = 39 \end{array}$$

A unit of the first quotient equals 2 units of the dividend, and hence the second remainder (3) equals $3 \times 2$ units of the dividend.

A unit of the second quotient equals 8 units of the first quotient, and hence the third remainder (2) equals $2 \times 8$ units of the second quotient $= 2 \times 8 \times 2$ units of the dividend. Hence the first remainder is 1; the second 6; the third 32; and the total, or true remainder, 39.

NOTE.—The teacher can illustrate this process by considering the dividend (3687) *pints*. The first quotient will be quarts, the second pecks, and third bushels, and the first remainders will be 1 pt., the second, 3 qt., and the third, 2 pk. 1 pt. + 3 qt. + 2 pk. = 39 pt.

36. Divide 34567 by 63; by 72.
37. Divide 120473 by 56; by 81.
38. Divide 400671 by 64; by 77.
39. Divide 346000 by 55; by 96.
40. Divide 47633 by 90; by 110.

**52.** PRINCIPLE.—*The division of both divisor and dividend by the same number does not change the value of the quotient.*

**53.** RULE.—To divide by a composite number, 1. *Resolve the divisor into convenient factors; divide the dividend by one of these factors, the quotient thus obtained by another, and so on until all the factors are used as divisors. The last quotient will be the true quotient.*

2. *Multiply each remainder, except the first, by all the divisors preceding its own. The sum of these products and the first remainder will be the true remainder.*

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# SECTION VII.

## PROPERTIES OF NUMBERS.

### PRIME AND COMPOSITE NUMBERS AND FACTORS.

NOTE.—The terms *number*, *divisor*, and *factor*, used in this section, denote *integral* numbers.

1. What two numbers besides itself and 1 will exactly divide 10? 21? 35? 63? 77?

2. What numbers besides itself and 1 will exactly divide 7? 11? 17? 23? 37? 41?

3. What numbers will exactly divide 15? 13? 28? 29? 42? 43?

NOTE.—Since every integer is exactly divisible by itself and 1, these divisors need not be given.

4. What numbers will exactly divide 30? 31? 45? 53? 56? 67? 65?

5. Name all the prime numbers between 0 and 20; 30 and 50.

6. Name all the composite numbers between 20 and 30; 50 and 70.

7. What are the prime divisors of 6? 15? 18? 21? 30? 45? 50? 54?

8. What are the prime factors of 12? 24? 35? 39? 42?

9. What are the prime factors of 27? 36? 49? 56? 63? 66? 72? 84?

10. Of what numbers are 2 and 5 prime factors? 2, 3, and 5? 2, 5, and 7? 3, 5, and 7?

11. Of what numbers are 2, 2, and 3 prime factors? 2, 3, 3, and 5? 2, 3, 5, and 7?

12. What prime factor is common to 9 and 12? 15 and 25? 18 and 30? 21 and 28?

13. What prime factor is common to 24 and 27? 35 and 42? 44 and 77? 35 and 50? 63 and 70?

### WRITTEN EXERCISES.

14. What are the prime factors of 126?

PROCESS.

$$\begin{array}{r|l} 2 & 126 \\ \hline 3 & 63 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

Divide 126 by 2, a prime divisor; next divide the quotient 63 by 3, a prime divisor, and then divide the quotient 21 by 3, a prime divisor. The prime factors are 2, 3, 3, and 7.

$126 = 2 \times 3 \times 3 \times 7.$

What are the prime factors of

| | | | |
|---|---|---|---|
| 15. 160? | 18. 325? | 21. 462? | 24. 748? |
| 16. 175? | 19. 330? | 22. 490? | 25. 693? |
| 17. 256? | 20. 420? | 23. 594? | 26. 1155? |

What prime factors are common to

27. 45 and 63?
28. 50 and 80?
29. 96 and 256?
30. 200 and 250?
31. 175 and 325?
32. 144 and 180?

## DEFINITIONS, PRINCIPLES, AND RULES.

**54.** The ***Divisor*** of a number is any number that will exactly divide it.

**55.** Numbers are either *Prime* or *Composite*.

A ***Prime Number*** has no divisor except itself and one.

A ***Composite Number*** has other divisors besides itself and one.

Every composite number is the product of two or more numbers, called *factors*.

**56.** Two or more numbers are *prime to each other*, or *relatively prime*, when they have no common divisor except 1. Thus, 9 and 16 are prime to each other.

All prime numbers are prime to each other. Composite numbers may be relatively prime, as 9 and 10; 16 and 25.

**57.** A ***Factor*** of a number is its divisor.

A ***Prime Factor*** of a number is its prime divisor.

The terms divisor and factor differ only in their use, the former implying *division* and the latter *multiplication*. A divisor or factor of a number is also called its *measure*.

**58.** When a number is a factor of each of two or more numbers, it is called their ***Common Factor.*** Thus, 5 is a common factor of 15 and 20.

**59.** Whole numbers are either *Even* or *Odd*.

An ***Even Number*** is exactly divisible by 2; as, 2, 4, 6, 8, 10, 12, etc.

An ***Odd Number*** is not exactly divisible by 2; as, 1, 3, 5, 7, 9, 11, 13, etc.

All the even numbers except 2 are composite. Some of the odd numbers are composite and others are prime.

**60.** Principles.—1. *A factor of a number is a factor of any number of times that number.*

2. *A common factor of two or more numbers is a factor of their sum.*

3. *A composite number is the product of all its prime factors.*

4. *If a composite number composed of two factors be divided by one factor, the quotient will be the other factor.*

5. *If any composite number be divided by a factor, or by the product of any number of its factors, the quotient will be the product of the remaining factors.*

**61.** Rules.—1. To resolve a composite number into its prime factors, *Divide it by any prime divisor, and the quotient by any prime divisor, and so continue until a quotient is obtained which is a prime number. The several divisors and the last quotient are the prime factors.*

2. To find the common factors of two or more numbers, *Resolve the given numbers into their prime factors and select the factors which are found in all the numbers.*

## CANCELLATION.

33. Divide the product of 4, 7, 9, and 12 by the product of 4, 7, and 9.

Process.

$$\begin{array}{l} \text{Dividend,} \\ \text{Divisor,} \end{array} \quad \frac{\cancel{4} \times \cancel{7} \times \cancel{9} \times 12}{\cancel{4} \times \cancel{7} \times \cancel{9}} = 12.$$

Instead of forming the products, indicate the multiplication by the proper sign, and write the divisor underneath the dividend. Since dividing both dividend and divisor by the same number does not affect the value of the quotient (Art. 48), divide each by 4, 7, and 9. This may be done by canceling, as indicated in the process. The quotient is 12.

34. Multiply $4 \times 7$ by 12, and divide the product by 4 times 12.

35. Divide $6 \times 8 \times 20$ by $4 \times 20$.

36. Divide $5 \times 7 \times 11 \times 13\frac{1}{2}$ by $7 \times 13\frac{1}{2}$.

37. Divide $12 \times 16 \times 28$ by $9 \times 24 \times 21$.

PROCESS.

$$\frac{\cancel{12} \times \overset{8}{\cancel{16}} \times \overset{4}{\cancel{28}}}{9 \times \underset{\cancel{2}}{\cancel{24}} \times \underset{3}{\cancel{21}}} = \frac{8 \times 4}{9 \times 3} = \frac{32}{27} = 1\tfrac{5}{27}$$

Since dividing the factor of a number divides the number, cancel 12 in the dividend and divide 24 in the divisor by 12, giving 2. Cancel the 2 and divide 16 in the dividend by 2, giving 8. Divide the 28 in the dividend and 21 in the divisor, each by 7, giving 4 and 3. The uncanceled factors of the divisor are 8 and 4, and those of the dividend are 9 and 3. The quotient is $32 \div 27 = 1\frac{5}{27}$.

38. Divide $24 \times 27 \times 12\frac{1}{2}$ by $18 \times 54 \times 50$.

39. Divide $28 \times 30 \times 100$ by $21 \times 15 \times 33\frac{1}{3}$.

40. $40 \times 22 \times 35 \times 16\frac{2}{3} \div 20 \times 44 \times 50 \times 49 =$ what?

41. A farmer exchanged 12 barrels of apples, each containing 3 bushels, at 75 cts. a bushel, for 25 sacks of potatoes, each containing 2 bushels: how much did the potatoes cost a bushel?

42. If 9 men can do a piece of work in 16 days, working 10 hours a day, how many men can do it in 20 days, working 8 hours a day?

## DEFINITIONS, PRINCIPLES, AND RULE.

**62.** ***Cancellation*** is the omission of one or more of the factors of a number. It is used when both dividend and divisor contain one or more equal factors, to abbreviate the process of division.

**63.** PRINCIPLES.—1. *The canceling of one of the factors of a number divides the number by the factor canceled.*

2. *Canceling equal factors of both dividend and divisor divides them by the same number, and hence does not change the value of the quotient.*

3. *Dividing one of the composite factors of a product divides the product.*

**64.** RULE.—*Indicate the multiplications by the proper sign, and write the divisor underneath the dividend. Cancel the fac-*

*tors common to both dividend and divisor, and divide the product of the factors left in the dividend by the product of those left in the divisor.*

**Note.**—When all the expressed factors of either dividend or divisor are canceled, 1 remains as a factor.

## GREATEST COMMON DIVISOR.

1. What are the divisors of 15? 28? 45? 53? 75? 90? 91? 108?

2. What is a common divisor of 15 and 35? 42 and 56? 63 and 72? 64 and 80?

3. What is a common divisor of 27 and 36? 18, 30, and 42? 36, 54, and 72?

4. What is the greatest number that will exactly divide 32 and 48? 45 and 90? 60 and 96?

5. What is the greatest common divisor of 36 and 60? 45, 60, and 75? 18, 54, and 90?

6. What is the greatest common divisor of 24, 48, and 72? 16, 48, and 80? 20, 31, and 45?

7. Show that every common divisor of 12 and 16 is a divisor of 28, their *sum*.

8. Show that a common divisor of any two numbers is a divisor of their sum.

9. Show that every common divisor of 16 and 28 is a divisor of 12, their *difference*.

10. Show that a common divisor of any two numbers is a divisor of their difference.

### WRITTEN EXERCISES.

11. What is the greatest common divisor of 126 and 210?

Process by Factoring.

$$126 = \cancel{2} \times \cancel{3} \times 3 \times \cancel{7}$$
$$210 = \cancel{2} \times \cancel{3} \times 5 \times \cancel{7}$$
$$2 \times 3 \times 7 = 42, \ G.C.D.$$

Resolve 126 and 210 into their prime factors. Since every divisor of a number is a prime factor, or the product of two or more prime factors, the product of all the prime factors common to 126 and 210 will be their greatest common divisor.

What is the greatest common divisor of

| | |
|---|---|
| 12. 60 and 84? | 15. 112, 140, and 168? |
| 13. 63 and 126? | 16. 84, 126, and 210? |
| 14. 144 and 192? | 17. 128, 256, and 1280? |

18. What is the greatest common divisor of 288 and 528?

PROCESS BY DIVIDING.

```
288 ) 528 ( 1
      288
      ---
      240 ) 288 ( 1
            240
            ---
             48 ) 240 ( 5
                  240
                  ---
48 = G. C. D. of 288 and 528.
```

Divide 528 by 288, and 288 by the first remainder 240, and 240 by the second remainder 48; and, there being no remainder, 48 is the greatest common divisor of 288 and 528.

Since 48, which is the greatest divisor of itself, is a divisor of 240, it is the *greatest* common divisor of 48 and 240. Since 48 is a divisor of both 48 and 240, it is a divisor of 288, their *sum*, and since the greatest common divisor of two numbers is a divisor of their *difference*, 48 is the greatest common divisor of 240 and 288. Since 48 is the greatest common divisor of 240 and 288, it is a divisor of 528, their *sum*; and since the greatest common divisor of two numbers is a divisor of their difference, 48 is the greatest common divisor of 288 and 528.

NOTE.—Let the pupil show, in like manner, that the last divisor, in the solution of each of the following problems, is the greatest common divisor required.

What is the greatest common divisor of

| | |
|---|---|
| 19. 196 and 1728? | 27. $260 and $416? |
| 20. 336 and 576? | 28. $1815 and $3465? |
| 21. 407 and 888? | 29. 2145 lb. and 3471 lb.? |
| 22. 326 and 807? | 30. 175, 225, and 275? |
| 23. 756 and 1764? | 31. 240, 360, and 480? |
| 24. 1064 and 1274? | 32. 144, 216, and 648? |
| 25. 768 and 5184? | 33. 140, 308, and 819? |
| 26. 741 and 1938? | 34. 240, 336, and 1768? |

35. What is the greatest common divisor of 1065, 1730, and 2845?

36. What is the greatest common divisor of 156, 585, 442, and 1287?

37. What is the greatest common divisor of 2731 and 3120?

## DEFINITIONS, PRINCIPLES, AND RULES.

**65.** A ***Divisor*** of a number is a number that will exactly divide it.

A ***Common Divisor*** of two or more numbers is a number that will exactly divide each of them.

The ***Greatest Common Divisor*** of two or more numbers is the greatest number that will exactly divide each of them.

**66.** PRINCIPLES.—1. *Every prime factor, and every product of any two or more prime factors of a number, is a divisor of that number.* Conversely,

2. *Every divisor of a number is a prime factor, or the product of two or more of its prime factors.*

3. *The product of all the prime factors common to two or more numbers is their greatest common divisor.*

4. *The divisor of a number is a divisor of any number of times that number.*

5. *A common divisor of two numbers is a divisor of their sum, or of their difference.*

6. *Any common divisor of either of two numbers and their difference is a common divisor of the two numbers.*

**67.** RULES.—1. To find the greatest common divisor of two or more numbers by factoring, *Resolve the given numbers into their prime factors, and select the factors which are common. The product of the common factors will be the greatest common divisor.*

2. To find the greatest common divisor of two numbers by division, *Divide the greater number by the less, and the divisor by the remainder, and the second divisor by the second remainder, and so on, until there is no remainder. The last divisor will be the greatest common divisor.*

NOTE.—When there are three or more numbers, first find the greatest common divisor of two of them, and then the greatest common divisor of this G. C. D. and a third number, and so on.

## LEAST COMMON MULTIPLE.

1. What number will 16 exactly divide? 25? 30? 45?

NOTE.—A number will exactly divide its multiple.

2. What number is a multiple of 15? 24? 32? 54? 75? 100? 120? 150? 200?

3. How many multiples has every number?

4. What number will 8 and 10 both exactly divide? 9 and 12? 20 and 25?

5. What number is a common multiple of 5 and 12? 15 and 30? 25 and 50?

6. How many common multiples have two or more numbers?

7. What is the least number that 7 and 8 will both exactly divide? 9 and 12? 20 and 30? 25 and 75?

8. What number is the least common multiple of 7 and 10? 12 and 18? 8, 12, and 16?

9. How many least common multiples have two or more numbers?

10. Show that all the prime factors of a number are factors of its multiple, and, conversely, that a number containing all the prime factors of another number is its multiple.

### WRITTEN EXERCISES.

11. What is the least common multiple of 12, 18, and 30?

PROCESS BY FACTORING.

$$12 = \cancel{2} \times \cancel{2} \times 3$$
$$18 = 2 \times \cancel{3} \times \cancel{3}$$
$$30 = 2 \times 3 \times \cancel{5}$$

$$2 \times 2 \times 3 \times 3 \times 5 = 180, \textit{ L. C. M.}$$

Resolve the numbers into their prime factors, and select all the different factors, repeating each as many times as it is found in any number. The factor 2 is found twice in 12; the factor 3, twice in 18; and the factor 5, once in 30. The product of $2 \times 2 \times 3 \times 3 \times 5$ is the least common multiple required, since it is the least number which contains all the prime factors of 12, 18, and 30.

What is the least common multiple of

12. 8, 12, 20?
13. 9, 21, 42?
14. 32, 48, 80?
15. 27, 54, 108?
16. 18, 24, 72, 48?
17. 15, 35, 70, 105?
18. 25, 75, 100, 150?
19. $16, $40, $60, $72?

20. What is the least common multiple of 12, 15, 42, 70?

PROCESS BY DIVISION.

| | | | | |
|---|---|---|---|---|
| 2) | 12 | 15 | 42 | 70 |
| 3) | 6 | 15 | 21 | 35 |
| 5) | 2 | 5 | 7 | 35 |
| 7) | 2 | 1 | 7 | 7 |
| | 2 | 1 | 1 | 1 |

$2 \times 3 \times 5 \times 7 \times 2 = 420$, *L. C. M.*

Find all the prime factors by dividing the given numbers by any prime number that will exactly divide two or more of them, thus: Dividing by 2, it is found to be a prime factor of 12, 42, and 70. Write the quotients with the 15 underneath. Dividing by 3, it is found to be a prime factor of 6, 15, and 21, and hence it is a prime factor of 12, 15, and 42. Dividing by 5, it is found to be a prime factor of 5 and 35, and hence of 15 and 70. Dividing by 7, it is found to be a prime factor of 7 and 7, and hence of 42 and 70. The remaining quotient 2 is a prime factor of 12.

Hence, all the prime factors of 12, 15, 42, and 70 are 2, 3, 5, 7, and 2, and since the product of these several prime factors ($2 \times 3 \times 5 \times 7 \times 2 = 420$) is the least number that contains each of them, it is the least common multiple of 12, 15, 42, and 70.

What is the least common multiple of

21. 12, 18, 30?
22. 8, 28, 70?
23. 9, 20, 15, 36?
24. 15, 24, 25, 30?
25. 18, 21, 27, 36?
26. 30, 45, 48, 80, 120?
27. 16, 30, 40, 50, 75?
28. 15, 27, 35, 42, 70?
29. 8, 28, 20, 24, 32, 48?
30. 2, 3, 4, 5, 6, 7, 8, 9?

## DEFINITIONS, PRINCIPLES, AND RULES.

**68.** A ***Multiple*** of a number is any number which it will exactly divide.

NOTE.—Every number is an exact divisor of its product by an integer.

A ***Common Multiple*** of two or more numbers is any number which each of them will exactly divide.

The ***Least Common Multiple*** of two or more numbers is the least number which each of them will exactly divide.

NOTE.—The following definitions may be preferred by some teachers:
A Multiple of a number is the product arising from taking it two or more times.
A Common Multiple of two or more numbers is a number which is a multiple of each of them.
The Least Common Multiple of two or more numbers is the least number which is a multiple of each of them.

**69.** PRINCIPLES.—1. *Every multiple of a number contains all its prime factors.*

2. *A common multiple of two or more numbers contains all their prime factors.*

3. *The least common multiple of two or more numbers contains all their prime factors, and no other factors.*

4. *The least common multiple of two or more numbers contains each of their prime factors the greatest number of times it occurs in either number.*

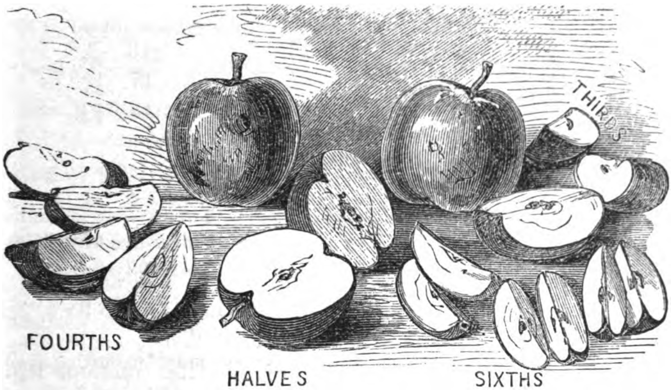
**70.** RULES.—1. To find the least common multiple of two or more numbers by factoring, *Resolve each of the numbers into its prime factors, and then select all the different factors, taking each the greatest number of times it is found in any number. The product of the different factors, thus selected, will be the least common multiple.*

2. To find the least common multiple of two or more numbers by division, *Write the numbers in a line, and divide by any prime divisor of two or more of them, writing the quotients and the undivided numbers underneath. Divide these resulting numbers by any prime divisor of two or more of them, and so proceed until no two of the resulting numbers have a common prime divisor. The product of the divisors and the last resulting numbers will be the least common multiple required.*

NOTE.—If no two of the given numbers have a common divisor, their product will be the least common multiple.

# SECTION VIII.

# FRACTIONS.




## NUMERATION AND NOTATION.

1. If a single thing be divided into two equal pieces, what part of the whole will one piece be?

2. If a single thing be divided into four equal pieces, what part of the whole will one piece be? Two pieces? Three pieces?

3. How many halves in a single thing or unit? How many fourths?

4. Which is the greater, one half or one fourth of a unit? How many fourths in one half?

5. What is meant by one third of a unit? Two thirds? One sixth? Three sixths? Two fifths? Four fifths?

**71.** Such parts of a unit as one half, two thirds, three fourths, etc., are called *Fractions*. A fraction may be expressed in figures by writing the figure denoting the number of equal parts, into which the unit is divided, below a short

horizontal line ($_6$), and the figure denoting the number of equal parts taken, above the same line ($^5$). Thus, $\frac{5}{6}$ expresses five sixths of a unit.

6. What does $\frac{5}{7}$ express? What does the figure 7, below the line, denote? The figure 5, above the line?

Read the following fractions, and tell, in each case, what each figure denotes:

| | | | |
|---|---|---|---|
| 7. $\frac{3}{5}$ | 10. $\frac{5}{8}$ | 13. $\frac{7}{10}$ | 16. $\frac{9}{20}$ |
| 8. $\frac{5}{6}$ | 11. $\frac{6}{9}$ | 14. $\frac{5}{12}$ | 17. $\frac{15}{24}$ |
| 9. $\frac{6}{7}$ | 12. $\frac{7}{6}$ | 15. $\frac{8}{15}$ | 18. $\frac{27}{40}$ |

Write the following fractions in figures:

| (19) | (20) | (21) |
|---|---|---|
| Two fifths; | Seven twelfths; | Twenty-four fortieths; |
| Seven ninths; | Ten thirteenths; | Thirty-five fiftieths; |
| Ten ninths. | Twenty seventeenths. | Forty fifty-fifths. |

22. Is the fraction $\frac{3}{4}$ greater or less than 1? Why?

23. Is $\frac{5}{4}$ greater or less than a unit? Why?

Compare the value of each of the following fractions with a unit or 1:

| | | | |
|---|---|---|---|
| 24. $\frac{5}{7}$ | 26. $\frac{10}{9}$ | 28. $\frac{9}{9}$ | 30. $\frac{41}{45}$ |
| 25. $\frac{8}{5}$ | 27. $\frac{7}{12}$ | 29. $\frac{12}{12}$ | 31. $\frac{33}{20}$ |

32. Deduce from the above examples a general statement of the value of fractions as compared with a unit or 1.

## DEFINITIONS AND PRINCIPLES.

**72.** A ***Fraction*** is one or more of the equal parts of a unit.

The unit divided is called the *Unit of the Fraction;* and one of the equal parts, into which it is divided, is called a *Fractional Unit.* An integer is composed of integral units, and a fraction of fractional units.

**73.** A ***Common Fraction*** is expressed in figures by two numbers, one written over the other, with a line between them.

NOTE.—Decimal fractions are a variety of common fractions. (Art. 112.)

The number above the line is called the *Numerator;* and the one below the line, the *Denominator.*

The ***Denominator*** of a fraction denotes the number of equal parts into which the unit is divided.

The ***Numerator*** of a fraction denotes the number of equal parts taken.

The numerator and denominator are called the ***Terms*** of the fraction.

**74.** PRINCIPLE.—*The value of a fraction is less than* 1 *when its numerator is less than its denominator; equal to* 1 *when its numerator equals its denominator; and more than* 1 *when its numerator is greater than its denominator.*

**75.** Common Fractions are *Proper* or *Improper.*

A ***Proper Fraction*** is one whose numerator is less than its denominator; as, $\frac{5}{6}$, $\frac{3}{8}$.

An ***Improper Fraction*** is one whose numerator is equal to or greater than its denominator.

The value of a proper fraction is *less* than one; and the value of an improper fraction is *equal to* or *greater* than one, and hence it is regarded as *not properly* the fraction of a unit.

**76.** Fractions are *Simple, Compound,* or *Complex.*

A ***Simple Fraction*** is a fraction not united with another, and both of whose terms are integral; as, $\frac{5}{6}$.

A ***Compound Fraction*** is a fraction of a fraction; as, $\frac{2}{3}$ of $\frac{3}{5}$; $\frac{2}{5}$ of $3\frac{1}{3}$.

A ***Complex Fraction*** is one having a fraction in one or both of its terms; as, $\frac{\frac{2}{3}}{4}$, $\frac{5}{\frac{3}{5}}$, $\frac{\frac{5}{6}}{\frac{3}{8}}$, $\frac{5\frac{1}{2}}{3\frac{1}{3}}$.

A ***Mixed Number*** is an integer and a fraction united; as, $5\frac{1}{2}$, $16\frac{1}{2}$.

**77.** The fraction $\frac{3}{5}$ may be considered as expressing 3 fifths of 1 unit, or 1 fifth of 3 units; and hence the numer-

ator of a fraction may denote the number of units to be divided, and the denominator the number of parts into which the numerator is to be divided. Thus, $\frac{5}{6}$ may be read 5 sixths, or 1 sixth of 5, or 5 divided by 6. Hence,

*A fraction may express an* UNEXECUTED *division, the numerator denoting the dividend, the denominator the divisor, and the fraction itself the quotient.*

## REDUCTION OF FRACTIONS.

### Case I.

#### Whole or Mixed Numbers reduced to Improper Fractions.

1. How many thirds in an apple? In 4 apples? 7 apples? 10 apples? 20 apples?

2. How many fifths in 3 melons? In five melons? 8 melons? 12 melons? 15 melons?

3. How many sixths in 1? 5? 8? 12? 20?

4. How many fourths of an inch in 2 and 1 fourth inches? In $3\frac{1}{4}$ inches? $6\frac{3}{4}$ inches? $30\frac{1}{4}$ inches?

5. How many fifths in $3\frac{1}{5}$? $4\frac{2}{5}$? $12\frac{3}{5}$? $16\frac{4}{5}$?

6. How many tenths in $5\frac{1}{10}$? $8\frac{3}{10}$? $12\frac{7}{10}$? $15\frac{9}{10}$?

#### WRITTEN PROBLEMS.

7. Reduce 225 to sevenths. $225\frac{5}{7}$ to sevenths.

| PROCESS. | PROCESS. |
|---|---|
| $\begin{array}{r} 225 \\ 7 \\ \hline \frac{1575}{7} \end{array}$, *Ans.* | $\begin{array}{r} 225\frac{5}{7} \\ 7 \\ \hline \frac{1580}{7} \end{array}$, *Ans.* |

8. Reduce 324 to ninths. $324\frac{7}{9}$ to ninths.
9. Reduce $48\frac{11}{15}$ to 15ths. $65\frac{13}{15}$ to 15ths.
10. Reduce $54\frac{7}{20}$ to 20ths. $135\frac{11}{30}$ to 30ths.
11. Reduce $63\frac{7}{12}$ to an improper fraction.
12. Reduce $74\frac{11}{15}$ to an improper fraction.
13. Reduce $206\frac{7}{30}$ to an improper fraction.

14. Reduce $145\frac{3}{40}$ to an improper fraction.

Reduce to an improper fraction,

| | | |
|---|---|---|
| 15. $137\frac{8}{25}$ | 17. $600\frac{13}{41}$ | 19. $208\frac{11}{35}$ |
| 16. $408\frac{9}{50}$ | 18. $365\frac{7}{24}$ | 20. $607\frac{5}{12}$ |

**78.** RULES.—1. To reduce an integer to a fraction, *Multiply the integer by the given denominator, and write the denominator under the product.*

2. To reduce a mixed number to a fraction, *Multiply the integer by the denominator of the fraction, to the product add the numerator, and write the denominator under the result.*

## Case II.

### Improper Fractions reduced to Whole or Mixed Numbers.

21. How many dollars in 8 half-dollars? 16 half-dollars? 30 half-dollars?

22. How many pints in 9 thirds of a pint? 15 thirds of a pint? 33 thirds of a pint?

23. How many days in 20 fifths of a day? 35 fifths of a day? 42 fifths of a day?

24. How many units in 36 ninths? 63 ninths? 75 ninths?

25. How many units in $\frac{28}{7}$? $\frac{56}{7}$? $\frac{65}{7}$? $\frac{72}{7}$?

26. How many units in $\frac{93}{10}$? $\frac{87}{12}$? $\frac{104}{25}$? $\frac{208}{50}$?

### WRITTEN PROBLEMS.

27. Reduce $\frac{256}{16}$ to a whole number.

PROCESS: $\frac{256}{16} = 256 \div 16 = 16$, *Ans.*

28. Reduce $\frac{263}{16}$ to a mixed number.

Reduce to a whole or a mixed number,

| | | | |
|---|---|---|---|
| 29. $\frac{324}{12}$ | 32. $\frac{636}{32}$ | 35. $\frac{1340}{35}$ | 38. $\frac{4000}{333}$ |
| 30. $\frac{265}{15}$ | 33. $\frac{720}{80}$ | 36. $\frac{2304}{50}$ | 39. $\frac{2388}{45}$ |
| 31. $\frac{705}{18}$ | 34. $\frac{1128}{64}$ | 37. $\frac{4260}{39}$ | 40. $\frac{42351}{21}$ |

**79.** RULE.—To reduce an improper fraction to an integer or mixed number, *Divide the numerator of the fraction by the denominator.*

## Case III.

### Simple Fractions reduced to Lowest Terms.

41. How many fourths of an inch in 2 eighths of an inch? In 4 eighths? 6 eighths?

42. How many sixths in 2 twelfths? In 4 twelfths? 8 twelfths? 10 twelfths?

43. How many sevenths in 4 fourteenths? In 6 fourteenths? 8 fourteenths? 12 fourteenths?

44. How many eighths in $\frac{6}{16}$? $\frac{10}{16}$? $\frac{14}{16}$? $\frac{20}{16}$?

45. How many tenths in $\frac{9}{30}$? $\frac{15}{30}$? $\frac{24}{30}$? $\frac{33}{30}$?

46. Reduce $\frac{9}{15}$, $\frac{12}{20}$, $\frac{18}{30}$ each to fifths.

47. Reduce $\frac{20}{30}$, $\frac{28}{24}$, $\frac{35}{42}$, and $\frac{30}{60}$ each to sixths.

48. Divide both terms of $\frac{9}{12}$ by 3, and show that the value of the fraction is not changed.

49. Show that the division of both terms of any fraction by the same number does not change its value.

#### WRITTEN PROBLEMS.

50. Reduce $\frac{105}{140}$ to its lowest terms.

PROCESS.

$$\frac{105 \div 5}{140 \div 5} = \frac{21}{28} \quad \frac{21 \div 7}{28 \div 7} = \frac{3}{4}$$

Or: $\dfrac{105 \div 35}{140 \div 35} = \dfrac{3}{4}$, *Ans.*

Divide both terms of $\frac{105}{140}$ by 5, reducing it to $\frac{21}{28}$; then divide both terms of $\frac{21}{28}$ by 7, reducing it to $\frac{3}{4}$. Since $\frac{3}{4}$ can not be reduced to smaller or lower terms, it is in its *lowest* terms. Or, divide both terms of $\frac{105}{140}$ by 35, their greatest common divisor, reducing the fraction to $\frac{3}{4}$.

Reduce to lowest terms,

| | | | |
|---|---|---|---|
| 51. $\frac{81}{189}$ | 54. $\frac{264}{480}$ | 57. $\frac{693}{792}$ | 60. $\frac{480}{1248}$ |
| 52. $\frac{105}{210}$ | 55. $\frac{360}{576}$ | 58. $\frac{720}{1125}$ | 61. $\frac{891}{1485}$ |
| 53. $\frac{216}{414}$ | 56. $\frac{567}{783}$ | 59. $\frac{288}{1728}$ | 62. $\frac{2700}{3780}$ |

63. Express the quotient of 195 divided by 105 in its simplest form. *Ans.* $\frac{13}{7}$.

64. Express the quotient of 462 divided by 441 in its simplest form. $128 \div 256$. $360 \div 288$.

65. Express the quotient of $576 \div 432$ in its simplest form. $216 \div 324$. $828 \div 506$.

### DEFINITIONS, PRINCIPLE, AND RULES.

**80.** A fraction is reduced to *lower terms* when it is changed to an equivalent fraction with smaller terms.

A fraction is in its *lowest terms* when its terms are prime to each other.

**81.** PRINCIPLE.—*The division of both terms of a fraction by the same number does not change its value.*

**82.** RULES.—To reduce a fraction to its lowest terms, 1. *Divide both terms of the fraction by any common divisor; then divide both terms of the resulting fraction by any common divisor; and so on, until the terms of the resulting fraction have no common divisor except* 1. Or,

2. *Divide both terms of the fraction by their greatest common divisor.*

## Case IV.

### Fractions reduced to Higher Terms, and to a Common Denominator.

66. How many eighths of a foot in 1 fourth of a foot? In 2 fourths? 3 fourths?

67. How many twelfths in 3 sixths? 4 sixths? 5 sixths?

68. How many fifteenths in $\frac{2}{3}$? $\frac{2}{5}$? $\frac{4}{5}$?

69. Change $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ each to twelfths.

70. Change $\frac{3}{4}$, $\frac{3}{5}$, and $\frac{7}{10}$ to fortieths.

71. Change $\frac{5}{6}$, $\frac{3}{10}$, and $\frac{7}{15}$ to sixtieths.

72. Change $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{7}{10}$ to thirtieths.

73. Change $\frac{4}{5}$, $\frac{5}{8}$, and $\frac{11}{20}$ to fortieths.

74. Multiply both terms of $\frac{2}{3}$ by 4, and show that the value of the fraction is not changed.

75. Show that the multiplication of both terms of any fraction by the same number does not change its value.

### WRITTEN PROBLEMS.

76. Reduce $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$ to equivalent fractions having a common denominator.

PROCESS.

$\frac{5}{6}$ $\frac{7}{8}$ $\frac{11}{12}$

$\frac{20}{24}$ $\frac{21}{24}$ $\frac{22}{24}$

Reduce the fractions to *twenty-fourths*, thus: $\frac{5}{6} = \frac{20}{24}$; $\frac{7}{8} = \frac{21}{24}$; $\frac{11}{12} = \frac{22}{24}$.

Reduce to a common denominator,

77. $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{6}$
78. $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{12}$
79. $\frac{2}{7}$, $\frac{5}{14}$, $\frac{1}{2}$
80. $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{16}$
81. $\frac{1}{3}$, $\frac{5}{6}$, $\frac{7}{9}$
82. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$
83. $\frac{3}{5}$, $\frac{5}{6}$, $\frac{7}{15}$, $\frac{11}{30}$
84. $\frac{4}{5}$, $\frac{9}{10}$, $\frac{13}{20}$, $\frac{11}{40}$
85. $\frac{2}{3}$, $\frac{3}{7}$, $\frac{5}{6}$, $\frac{4}{21}$

86. Reduce $\frac{5}{8}$, $\frac{7}{16}$, $\frac{11}{24}$, and $\frac{21}{32}$ to equivalent fractions having the least common denominator.

PROCESS.

$\frac{5}{8}$ $\frac{7}{16}$ $\frac{11}{24}$ $\frac{21}{32}$

$\frac{60}{96}$ $\frac{42}{96}$ $\frac{44}{96}$ $\frac{63}{96}$

The least common multiple of 8, 16, 24, and 32 is 96, and hence 96 is the least common denominator. Change the fractions to 96ths. $\frac{5}{8} = \frac{60}{96}$; $\frac{7}{16} = \frac{42}{96}$; $\frac{11}{24} = \frac{44}{96}$; $\frac{21}{32} = \frac{63}{96}$.

Reduce to the least common denominator,

87. $\frac{4}{9}$, $\frac{11}{12}$, $\frac{17}{36}$, $\frac{23}{24}$
88. $\frac{2}{3}$, $\frac{3}{5}$, $\frac{5}{6}$, $\frac{7}{8}$
89. $\frac{7}{10}$, $\frac{8}{15}$, $\frac{11}{20}$, $\frac{13}{30}$
90. $\frac{7}{12}$, $\frac{8}{21}$, $\frac{11}{28}$, $\frac{17}{42}$
91. $\frac{2}{5}$, $\frac{11}{12}$, $\frac{11}{20}$, $\frac{29}{30}$, $\frac{31}{60}$
92. $\frac{5}{9}$, $\frac{14}{35}$, $\frac{22}{45}$, $\frac{12}{63}$, $\frac{101}{115}$

### DEFINITIONS, PRINCIPLE, AND RULES.

**83.** A fraction is reduced to *higher terms* when it is changed to an equivalent fraction with greater terms.

**84.** Several fractions are reduced to a *Common Denominator* when they are changed to equivalent fractions, with the same denominator.

When the common denominator of several fractions is the smallest denominator which they can have in common, it is called their *Least Common Denominator*.

**85.** PRINCIPLE.—*The multiplication of both terms of a fraction by the same number does not change its value.*

**86.** Rules.—1. To reduce a fraction to higher terms, *Divide the given denominator by the denominator of the fraction, and multiply both terms by the quotient.*

2. To reduce fractions to the least common denominator, *Divide the least common multiple of the denominators by the denominator of each fraction, and multiply both terms by the quotient.*

### Case V.

**Compound Fractions reduced to Simple Fractions.**

93. How much is 1 half of 1 third of a pear? 1 half of 1 fourth of a pear?

94. A father divided $\frac{1}{2}$ of a pine-apple equally between 3 boys: what part of the pine-apple did each boy receive?

95. What is $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{5}$? $\frac{1}{3}$ of $\frac{1}{8}$?

96. What is $\frac{1}{2}$ of $\frac{1}{8}$? $\frac{1}{2}$ of $\frac{3}{8}$?

97. What is $\frac{1}{5}$ of $\frac{1}{3}$? $\frac{1}{5}$ of $\frac{2}{3}$? $\frac{2}{5}$ of $\frac{2}{3}$?

98. What is $\frac{1}{7}$ of $\frac{1}{9}$? $\frac{1}{7}$ of $\frac{5}{9}$? $\frac{3}{7}$ of $\frac{5}{9}$?

99. What is $\frac{2}{3}$ of $\frac{3}{4}$? $\frac{2}{3}$ of $\frac{5}{6}$? $\frac{2}{3}$ of $\frac{7}{8}$?

100. What is $\frac{3}{5}$ of $\frac{4}{5}$? $\frac{3}{7}$ of $\frac{5}{8}$? $\frac{5}{6}$ of $\frac{7}{12}$?

101. What is $\frac{1}{3}$ of 12? $\frac{1}{3}$ of $12\frac{1}{2}$? $\frac{1}{3}$ of $13\frac{1}{2}$?

Solution.—$\frac{1}{3}$ of $13\frac{1}{2} = \frac{1}{3}$ of 12, which is 4, $+ \frac{1}{3}$ of $1\frac{1}{2}$ or $\frac{3}{2}$, which is $\frac{3}{6}$ or $\frac{1}{2}$. $4 + \frac{1}{2} = 4\frac{1}{2}$. Hence, $\frac{1}{3}$ of $13\frac{1}{2}$ is $4\frac{1}{2}$.

102. What is $\frac{1}{4}$ of $17\frac{1}{2}$? $\frac{1}{4}$ of $21\frac{2}{3}$? $\frac{1}{4}$ of $33\frac{1}{3}$?

103. What is $\frac{2}{5}$ of 12? $\frac{2}{5}$ of $12\frac{1}{2}$? $\frac{3}{5}$ of $16\frac{1}{2}$?

104. What is $\frac{3}{7}$ of $22\frac{1}{3}$? $\frac{5}{8}$ of $25\frac{1}{3}$? $\frac{7}{9}$ of $37\frac{1}{2}$?

105. What is $\frac{5}{6}$ of $33\frac{1}{3}$? $\frac{3}{8}$ of $42\frac{1}{3}$? $\frac{2}{12}$ of $62\frac{1}{2}$?

**WRITTEN PROBLEMS.**

106. Reduce $\frac{5}{9}$ of $\frac{3}{15}$ of $7\frac{1}{2}$ to a simple fraction.

Process.

$$\tfrac{5}{9} \text{ of } \tfrac{3}{15} \text{ of } 7\tfrac{1}{2} = \frac{5 \times 3 \times 15}{9 \times 15 \times 2} = \frac{225}{270} = \frac{5}{6}, \textit{ Ans.}$$

$$\text{Or: } \tfrac{5}{9} \text{ of } \tfrac{3}{15} \text{ of } 7\tfrac{1}{2} = \frac{5 \times \cancel{3} \times \cancel{15}}{\underset{3}{\cancel{9}} \times \cancel{15} \times 2} = \frac{5}{6}$$

Reduce to a simple fraction,

107. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$
108. $\frac{2}{5}$ of $\frac{3}{7}$ of $2\frac{1}{2}$
109. $\frac{5}{7}$ of $\frac{14}{15}$ of $1\frac{7}{8}$
110. $\frac{4}{5}$ of $\frac{6}{7}$ of $2\frac{1}{3}$
111. $\frac{3}{5}$ of $\frac{5}{9}$ of $\frac{6}{7}$ of $3\frac{1}{3}$
112. $\frac{3}{4}$ of $\frac{4}{15}$ of $\frac{6}{7}$ of $4\frac{2}{3}$
113. $\frac{2}{5}$ of $\frac{4}{7}$ of $2\frac{5}{8}$ of $2\frac{1}{2}$
114. $\frac{2}{3}$ of $\frac{9}{10}$ of $\frac{7}{12}$ of $3\frac{1}{2}$

**87.** RULES.—To reduce a compound fraction to a simple fraction, 1. *Multiply the numerators together for a numerator, and the denominators together for a denominator.* Or,

2. *Indicate the continued multiplication of the numerators, and also of the denominators, and reduce the resulting fraction to its lowest terms by cancellation.*

### REVIEW PROBLEMS.

115. Reduce 16 to a fraction having 8 for a denominator.

116. Change $35 \div 21$ to a fraction in its lowest terms.

117. How many 15ths of a gallon in $33\frac{1}{3}$ gallons?

118. Reduce $\frac{304}{24}$ to a mixed number with the fraction in its lowest terms.

119. Reduce $\$\frac{90}{15}$, $\$\frac{137}{25}$, $\$\frac{485}{60}$, and $\$\frac{700}{11}$ each to whole or mixed numbers.

120. Reduce $\frac{2}{5}$, $\frac{7}{10}$, and $\frac{11}{15}$ each to 30ths.

121. Reduce $12\frac{1}{2}$, $18\frac{3}{4}$, and $33\frac{1}{3}$ each to 12ths.

122. Reduce $\frac{2}{15}$ of $\frac{1}{12}$ of $2\frac{1}{13}$ of 24 to a simple fraction.

123. Reduce $\frac{5}{8}$, $\frac{5}{6}$, and $\frac{7}{12}$ to a common denominator; to the least common denominator.

124. Reduce $\frac{5}{6}$, $5\frac{1}{2}$, and $\frac{3}{5}$ of $\frac{5}{9}$ to their least common denominator.

SUGGESTION.—First reduce $5\frac{1}{2}$ and $\frac{3}{5}$ of $\frac{5}{9}$ to simple fractions.

125. Reduce $\frac{4}{9}$, $\frac{2}{3}$ of $\frac{6}{7}$, and $\frac{4}{5}$ of $6\frac{2}{3}$ to their least common denominator.

126. Reduce $\frac{1}{2}$ of $2\frac{1}{2}$, $\frac{5}{6}$ of 3, and $\frac{3}{7}$ of $\frac{7}{9}$ of $6\frac{1}{4}$ to their least common denominator.

127. Reduce $\frac{3}{4}$ of $\frac{7}{9}$, $\frac{5}{6}$ of $2\frac{2}{5}$, and $\frac{3}{7}$ of $13\frac{1}{3}$ to their least common denominator.

128. Reduce $\frac{7}{8}$, $\frac{5}{7}$ of $5\frac{1}{4}$, $2\frac{1}{2}$ of $3\frac{1}{3}$, and $2\frac{3}{10}$ to their least common denominator.

# ADDITION OF FRACTIONS.

1. A clerk spends $\frac{4}{9}$ of his salary for board, $\frac{2}{9}$ of it for clothing, and $\frac{1}{9}$ for other expenses: what part of his salary does he spend?

2. How many ninths in $\frac{2}{9}$, $\frac{5}{9}$, and $\frac{1}{9}$?

3. A man traveled $\frac{1}{3}$ of his journey the first day, and $\frac{1}{4}$ of it the second day: what part of the journey did he travel in the two days?

4. How many twelfths in $\frac{1}{3}$ and $\frac{1}{4}$? $\frac{1}{4}$ and $\frac{1}{6}$?

5. A owns $\frac{2}{5}$ of a vessel, and B $\frac{3}{8}$ of it: what part of the vessel do both own?

6. What is the sum of $\frac{2}{5}$ and $\frac{3}{8}$? $\frac{3}{5}$ and $\frac{5}{8}$?

7. $\frac{3}{8}$ and $\frac{5}{12}$? $\frac{5}{6}$ and $\frac{7}{12}$? $\frac{2}{3}$ and $\frac{7}{9}$? $\frac{5}{12}$ and $\frac{7}{8}$?

8. $\frac{4}{5}$ and $\frac{4}{7}$? $\frac{3}{7}$ and $\frac{4}{9}$? $\frac{5}{7}$ and $\frac{9}{14}$? $\frac{5}{6}$ and $\frac{7}{9}$?

9. $\frac{1}{3}$ and $5\frac{1}{2}$? $2\frac{1}{3}$ and $6\frac{1}{2}$? $5\frac{1}{2}$ and $6\frac{1}{4}$? $8\frac{1}{3}$ and $9\frac{2}{5}$?

SUGGESTION.—First add the fractions and then the integers.

10. Show that fractions having a common denominator, express like fractional units, and that only like fractional units can be added.

## WRITTEN PROBLEMS.

11. What is the sum of $\frac{9}{23}$, $\frac{16}{23}$, and $\frac{10}{23}$?

PROCESS: $\frac{9}{23} + \frac{16}{23} + \frac{10}{23} = \frac{9+16+10}{23} = \frac{35}{23} = 1\frac{12}{23}$, *Ans.*

12. What is the sum of $\frac{11}{18}$, $\frac{13}{18}$, $\frac{17}{18}$, and $\frac{7}{18}$?

13. What is the sum of $\frac{13}{45}$, $\frac{23}{45}$, $\frac{31}{45}$, and $\frac{37}{45}$?

14. What is the sum of $\frac{141}{160}$, $\frac{97}{160}$, and $\frac{153}{160}$?

15. What is the sum of $\frac{5}{8}$, $\frac{7}{12}$, and $\frac{11}{16}$?

PROCESS.

$\frac{5}{8} + \frac{7}{12} + \frac{11}{16} =$
$\frac{30}{48} + \frac{28}{48} + \frac{33}{48} =$
$\frac{91}{48} = 1\frac{43}{48}$, *Ans.*

Since unlike fractional units can not be added, reduce the fractions $\frac{5}{8}$, $\frac{7}{12}$, and $\frac{11}{16}$ to a common denominator, and then add the resulting fractions.

16. Add $\frac{4}{5}$ and $\frac{1}{14}$.
17. $\frac{3}{8}$, $\frac{7}{12}$, and $\frac{9}{16}$.
18. $\frac{5}{6}$, $\frac{11}{12}$, and $\frac{13}{18}$.
19. $\frac{3}{5}$, $\frac{7}{10}$, and $\frac{13}{15}$.
20. $\frac{9}{14}$, $\frac{20}{21}$, and $\frac{31}{42}$.
21. $\frac{9}{10}$, $\frac{9}{20}$, $\frac{23}{30}$, and $\frac{41}{60}$.
22. $\frac{5}{6}$, $\frac{5}{9}$, $\frac{11}{15}$, and $\frac{17}{30}$.
23. $\frac{13}{18}$, $\frac{15}{27}$, $\frac{50}{81}$, and $\frac{2}{3}$.
24. $\frac{7}{8}$, $\frac{7}{12}$, $\frac{7}{18}$, and $\frac{7}{24}$.
25. $\frac{1}{12}$, $\frac{1}{15}$, $\frac{1}{20}$, and $\frac{1}{30}$.

26. Add $\frac{3}{4}$, $\frac{2}{3}$ of $\frac{3}{5}$, and $\frac{3}{7}$ of $\frac{5}{8}$ of $2\frac{1}{3}$.

PROCESS.

$\frac{2}{3}$ of $\frac{3}{5} = \frac{2}{5}$   $\frac{3}{7}$ of $\frac{5}{8}$ of $\frac{7}{3} = \frac{5}{8}$

$\frac{3}{4} + \frac{2}{5} + \frac{5}{8} = \frac{30}{40} + \frac{16}{40} + \frac{25}{40} = 1\frac{31}{40}$

Since $\frac{2}{3}$ of $\frac{3}{5} = \frac{2}{5}$, and $\frac{3}{7}$ of $\frac{5}{8}$ of $2\frac{1}{3} = \frac{5}{8}$, the sum of $\frac{3}{4} + \frac{2}{3}$ of $\frac{3}{5} + \frac{3}{7}$ of $\frac{5}{8}$ of $2\frac{1}{3} = \frac{3}{4} + \frac{2}{5} + \frac{5}{8}$.

27. Add $\frac{2}{3}$ of $\frac{3}{8}$, $\frac{3}{5}$ of $\frac{5}{13}$ of $2\frac{1}{6}$, and $\frac{7}{8}$.
28. Add $\frac{4}{5}$ of $2\frac{1}{2}$, $\frac{3}{4}$ of $\frac{4}{9}$, and $\frac{3}{8}$ of $\frac{2}{3}$ of 6.
29. Add $\frac{3}{14}$, $\frac{1}{7}$ of 5, and $\frac{6}{7}$ of $\frac{8}{9}$ of $\frac{5}{8}$.
30. Add $\frac{3}{15}$ of 2, $\frac{2}{3}$ of $\frac{3}{5}$, $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{5}{12}$, and $3\frac{1}{3}$.
31. Add $33\frac{1}{3}$, $37\frac{1}{2}$, $55\frac{3}{4}$, and $66\frac{2}{3}$.

PROCESS.

| | |
|---|---|
| $33\frac{1}{3}$ | $\frac{4}{12}$ |
| $37\frac{1}{2}$ | $\frac{6}{12}$ |
| $55\frac{3}{4}$ | $\frac{9}{12}$ |
| $66\frac{2}{3}$ | $\frac{8}{12}$ |
| $193\frac{1}{4}$, *Ans.* | |

The sum of $33\frac{1}{3}$, $37\frac{1}{2}$, $55\frac{3}{4}$, and $66\frac{2}{3}$, equals the sum of $\frac{1}{3} + \frac{1}{2} + \frac{3}{4} + \frac{2}{3}$ added to the sum of $33 + 37 + 55 + 66$. $\frac{1}{3} + \frac{1}{2} + \frac{3}{4} + \frac{2}{3} = 2\frac{3}{12}$ or $2\frac{1}{4}$. Write the $\frac{1}{4}$ under the fractions and add the 2 with the integers. The sum is $193\frac{1}{4}$.

32. Add $39\frac{1}{3}$, $56\frac{3}{4}$, $88\frac{1}{6}$, and $104\frac{5}{12}$.
33. Add 45, $87\frac{3}{4}$, $66\frac{2}{3}$, and $75\frac{1}{2}$.
34. Add $12\frac{1}{2}$, $16\frac{2}{3}$, $18\frac{3}{4}$, $30\frac{1}{4}$, $33\frac{1}{3}$, and $62\frac{1}{2}$.
35. Add $\frac{3}{5}$, $\frac{2}{3}$ of $\frac{3}{4}$, $16\frac{2}{3}$, and $48\frac{1}{6}$.
36. Add \$5.12$\frac{1}{2}$, \$3.18$\frac{3}{4}$, \$8.25, and \$3.81$\frac{1}{4}$.
37. Add $\frac{5}{7}$, $\frac{9}{14}$, $\frac{3}{7}$ of $5\frac{1}{4}$, and $65\frac{5}{21}$.
38. Add $\frac{3}{4}$, $\frac{3}{5}$, $\frac{3}{8}$, $\frac{7}{10}$, and $\frac{5}{12}$ of $2\frac{2}{5}$.

## PRINCIPLES AND RULES.

**83.** PRINCIPLES.—1. *Only like fractional units can be added.* Hence,

2. *Fractions must have a common denominator before they can be added.*

**89.** RULES.—1. To add fractions, *Reduce the fractions to a common denominator, add the numerators of the new fractions, and under the sum write the common denominator.*

2. To add mixed numbers, *Add the fractions and the integers separately, and combine the results.*

NOTES.—1. Compound fractions must be reduced to simple fractions before they can be added.

2. When mixed numbers are small they may be reduced to improper fractions and then added.

## SUBTRACTION OF FRACTIONS.

1. A boy spent $\frac{3}{8}$ of his money for a slate: what part of his money has he left?

2. How much is $\frac{8}{8}$ less $\frac{3}{8}$? $\frac{8}{8}$ less $\frac{5}{8}$? $\frac{8}{8}$ less $\frac{7}{8}$?

3. How much is $\frac{11}{12}$ less $\frac{3}{12}$? $\frac{11}{12}$ less $\frac{5}{12}$? $\frac{11}{12}$ less $\frac{9}{12}$?

4. A bought $\frac{3}{4}$ of a bushel of clover seed and sold $\frac{1}{3}$ of a bushel to B: what part of a bushel has A left?

SUGGESTION.—Change $\frac{3}{4}$ and $\frac{1}{3}$ to twelfths.

5. How much is $\frac{3}{4}$ less $\frac{1}{3}$? $\frac{3}{4}$ less $\frac{2}{3}$? $\frac{2}{3}$ less $\frac{1}{4}$?

6. $\frac{5}{6}$ less $\frac{3}{4}$? $\frac{3}{4}$ less $\frac{2}{5}$? $\frac{4}{5}$ less $\frac{3}{4}$? $\frac{2}{5}$ less $\frac{1}{3}$?

7. $\frac{4}{7}$ less $\frac{3}{5}$? $\frac{2}{3}$ less $\frac{2}{7}$? $\frac{6}{7}$ less $\frac{3}{4}$? $\frac{2}{5}$ less $\frac{1}{7}$?

8. $\frac{5}{8}$ less $\frac{1}{2}$? $\frac{7}{8}$ less $\frac{3}{5}$? $\frac{4}{7}$ less $\frac{3}{8}$? $\frac{5}{7}$ less $\frac{1}{2}$?

9. $\frac{7}{12}$ less $\frac{1}{3}$? $\frac{11}{12}$ less $\frac{3}{4}$? $\frac{5}{8}$ less $\frac{1}{12}$? $\frac{5}{6}$ less $\frac{5}{8}$?

10. $5\frac{1}{2}$ less $3\frac{1}{4}$? $6\frac{2}{3}$ less $4\frac{1}{2}$? $9\frac{3}{4}$ less $7\frac{1}{3}$? $12\frac{1}{2}$ less $6\frac{1}{4}$?

11. Why can not $\frac{2}{3}$ be subtracted from $\frac{3}{4}$ without first reducing the fractions to a common denominator?

### WRITTEN PROBLEMS.

12. Subtract $\frac{19}{35}$ from $\frac{27}{35}$.

PROCESS: $\frac{27}{35} - \frac{19}{35} = \frac{27-19}{35} = \frac{8}{35}$, *Ans.*

13. Subtract $\frac{37}{90}$ from $\frac{56}{90}$; $\frac{37}{85}$ from $\frac{75}{85}$; $\frac{49}{88}$ from $\frac{73}{88}$.

14. Subtract $\frac{7}{15}$ from $\frac{17}{18}$.

PROCESS: $\frac{17}{18} - \frac{7}{15} = \frac{85}{90} - \frac{42}{90} = \frac{43}{90}$, *Ans.*

How much is

| | | |
|---|---|---|
| 15. $\frac{14}{15} - \frac{5}{9}$? | 18. $\frac{33}{50} - \frac{11}{20}$? | 21. $\frac{10}{33} - \frac{9}{44}$? |
| 16. $\frac{13}{18} - \frac{7}{12}$? | 19. $\frac{19}{21} - \frac{11}{14}$? | 22. $\frac{23}{30} - \frac{17}{54}$? |
| 17. $\frac{16}{27} - \frac{5}{18}$? | 20. $\frac{7}{12} - \frac{4}{15}$? | 23. $\frac{29}{60} - \frac{13}{45}$? |

24. From $\frac{1}{2}$ of $\frac{4}{5}$ take $\frac{2}{7}$ of $\frac{7}{8}$ of $\frac{2}{3}$.

PROCESS: $\frac{1}{2}$ of $\frac{4}{5} = \frac{2}{5}$  $\frac{2}{7}$ of $\frac{7}{8}$ of $\frac{2}{3} = \frac{1}{6}$  $\frac{2}{5} - \frac{1}{6} = \frac{7}{30}$, *Ans.*

25. From $\frac{3}{4}$ of $\frac{5}{9}$ take $\frac{3}{5}$ of $\frac{2}{3}$ of $\frac{5}{9}$.
26. From $\frac{4}{5}$ of 7 take $\frac{1}{2}$ of $\frac{3}{4}$ of 7.
27. From $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{3}{4}$ take $\frac{5}{6}$ of $\frac{2}{3}$ of $\frac{5}{8}$.
28. From $\frac{3}{5}$ of $\frac{2}{7}$ of $2\frac{1}{2}$ take $\frac{3}{11}$.
29. From $340\frac{2}{5}$ take $247\frac{3}{4}$.

PROCESS.

| | |
|---|---|
| $340\frac{2}{5}$ | $\frac{8}{20}$ |
| $247\frac{3}{4}$ | $\frac{15}{20}$ |
| $92\frac{13}{20}$, *Ans.* | |

First subtract the fractions and then the integers. Since $\frac{15}{20}$ is greater than $\frac{8}{20}$, add $\frac{20}{20}$ to $\frac{8}{20}$, making $\frac{28}{20}$, and then subtract $\frac{15}{20}$ from $\frac{28}{20}$, writing the difference, $\frac{13}{20}$, under the fractions, and adding 1 ($\frac{20}{20}$) to the 7 units before subtracting the integers.

| | |
|---|---|
| 30. $93\frac{3}{4} - 46\frac{7}{8} = ?$ | 33. $241\frac{7}{8} - 153\frac{5}{12} = ?$ |
| 31. $56\frac{3}{5} - 37\frac{1}{2} = ?$ | 34. $\$2.33\frac{1}{3} - \$1.62\frac{1}{2} = ?$ |
| 32. $108\frac{2}{7} - 90\frac{2}{3} = ?$ | 35. $\$3.12\frac{1}{2} - \$2.48\frac{3}{4} = ?$ |

36. What fraction added to $\frac{3}{8}$ will make $\frac{11}{12}$?
37. What number added to $6\frac{3}{4}$ will make $16\frac{2}{3}$?
38. From the sum of $\frac{5}{6}$ and $\frac{3}{4}$ take their difference.
39. From $\frac{5}{8} + \frac{3}{5}$ take $\frac{5}{6} - \frac{3}{5}$.
40. From $\frac{3}{5} + \frac{2}{3} + \frac{7}{10}$ take $\frac{3}{7}$ of $1\frac{1}{6}$.
41. From $\frac{7}{9} + \frac{3}{4}$ take $\frac{7}{12} - \frac{2}{3}$ of $\frac{5}{8}$.
42. From a cask containing $45\frac{1}{2}$ gallons of sirup, a grocer sold one customer $16\frac{3}{4}$ gallons and another $21\frac{3}{8}$ gallons: how many gallons remained unsold?
43. A man bequeathed $\frac{3}{10}$ of his property to his wife, $\frac{5}{12}$ of it to his children, and the remainder to a college for its better endowment. What part of his property did the college receive?
44. A man owning $\frac{2}{3}$ of a factory, sold $\frac{2}{5}$ of his share: what part of the factory did he still own?

45. Two ninths of a pole is in the mud, $\frac{2}{7}$ of it in the water, and the rest of it in the air: what part of the pole is in the air?

46. The part of a pole broken off by the wind was $\frac{5}{8}$ of the whole pole, and $\frac{3}{5}$ of the part still standing was above the ground: what part of the pole was in the ground?

### PRINCIPLES AND RULES.

**90.** PRINCIPLES.—1. *The minuend and subtrahend must denote like fractional units.* Hence,

2. *Fractions must have a common denominator before their difference can be found.*

**91.** RULES.—1. To subtract fractions, *Reduce the fractions to a common denominator, subtract the numerator of the subtrahend from the numerator of the minuend, and under the difference write the common denominator.*

2. To subtract mixed numbers, *Subtract first the fractions, and then the integers, and unite the results.*

NOTES.—1. Compound fractions must be reduced to simple fractions before they can be subtracted.

2. When mixed numbers are small they may be reduced to improper fractions, and then subtracted.

## MULTIPLICATION OF FRACTIONS.

### Case I.

#### Fractions multiplied by Integers.

1. How much is twice 2 ninths of an inch? 4 times 2 ninths of an inch?

2. If a basket hold $\frac{3}{4}$ of a bushel, how many bushels will 8 baskets hold? 10 baskets?

3. How much is 8 times $\frac{3}{4}$? 10 times $\frac{3}{4}$? 20 times $\frac{3}{4}$?

4. 6 times $\frac{5}{7}$? 8 times $\frac{7}{9}$? 9 times $\frac{7}{11}$? 12 times $\frac{7}{8}$?

5. 7 times $\frac{1}{12}$? 9 times $\frac{4}{15}$? 8 times $\frac{11}{20}$? 11 times $\frac{8}{9}$?

6. 6 times $5\frac{1}{2}$? 9 times $6\frac{2}{3}$? 7 times $12\frac{1}{2}$? 10 times $7\frac{3}{5}$?

7. 8 times $12\frac{1}{2}$? 6 times $16\frac{2}{3}$? 5 times $33\frac{1}{3}$? 7 times $30\frac{1}{4}$?

8. Why does multiplying the numerator of $\frac{4}{15}$ by 3 multiply the fraction by 3?

9. Why does dividing the denominator of $\frac{4}{15}$ by 3 multiply the fraction by 3?

10. In how many ways may a fraction be multiplied by an integer?

### WRITTEN PROBLEMS.

Multiply

| | | |
|---|---|---|
| 11. $\frac{8}{15}$ by 9. | 15. $\frac{25}{144}$ by 12. | 19. $62\frac{1}{2}$ by 36. |
| 12. $\frac{13}{24}$ by 10. | 16. $\frac{47}{256}$ by 16. | 20. $45\frac{3}{5}$ by 80. |
| 13. $\frac{5}{21}$ by 24. | 17. $\frac{31}{360}$ by 60. | 21. $\$5.18\frac{3}{4}$ by 32. |
| 14. $\frac{11}{63}$ by 45. | 18. $\frac{111}{625}$ by 25. | 22. $\$66\frac{2}{3}$ by 52. |

### PRINCIPLE AND RULES.

**92.** PRINCIPLE.—*A fraction is multiplied by multiplying its numerator or dividing its denominator.*

**93.** RULES.—1. To multiply a fraction by an integer, *Multiply the numerator or divide the denominator.*

2. To multiply a mixed number by an integer, *Multiply the fraction and the integer separately, and add the products.*

## Case II.

### Integers multiplied by Fractions.

23. If a ton of hay cost \$16, what will $\frac{1}{4}$ of a ton cost? $\frac{3}{4}$ of a ton?

24. If an acre of land is worth \$50, what is $\frac{1}{5}$ an acre worth? $\frac{4}{5}$ of an acre?

25. What is $\frac{1}{6}$ of 42? $\frac{5}{6}$ of 42? $\frac{7}{6}$ of 42?

26. What is $\frac{3}{7}$ of 56? $\frac{5}{7}$ of 56? $\frac{7}{8}$ of 56?

27. $\frac{5}{9}$ of 63? $\frac{7}{12}$ of 84? $\frac{7}{11}$ of 99? $\frac{7}{9}$ of 56?

SOLUTION.—$\frac{1}{9}$ of $56 = 6\frac{2}{9}$, and $\frac{7}{9}$ of $56 = 7$ times $6\frac{2}{9} = 43\frac{5}{9}$.

28. $\frac{5}{8}$ of 66? $\frac{4}{9}$ of 66? $\frac{5}{12}$ of 66? $\frac{6}{7}$ of 74?

29. What is $16 \times \frac{3}{4}$? $50 \times \frac{4}{5}$? $42 \times \frac{5}{6}$?

SOLUTION.—Since $\frac{3}{4} = \frac{3}{4}$ of 1, $16 \times \frac{3}{4} = \frac{3}{4}$ of $16 \times 1 = \frac{3}{4}$ of $16 = 12$.

30. $57 \times \frac{5}{8}$? $75 \times \frac{5}{9}$? $87 \times \frac{7}{12}$? $95 \times \frac{7}{10}$? $76 \times \frac{6}{7}$?
31. $47 \times \frac{5}{9}$? $68 \times \frac{7}{8}$? $75 \times \frac{5}{6}$? $83 \times \frac{9}{10}$? $100 \times \frac{5}{12}$?
32. Show that the product of an integer by a fraction equals the fraction of the integer.

### WRITTEN PROBLEMS.

33. Multiply 654 by $\frac{7}{12}$.

PROCESS.

$$\begin{array}{r} 12\,)\,\underline{654} \\ 54\frac{1}{2} \\ \underline{7} \\ 381\frac{1}{2},\ \textit{Ans.} \end{array} \qquad \text{Or:} \qquad \begin{array}{r} 654 \\ \underline{7} \\ 12\,)\,\underline{4578} \\ 381\frac{1}{2} \end{array}$$

Since $\frac{7}{12} = 7$ times $\frac{1}{12}$, or $\frac{1}{12}$ of 7, the product of $654 \times \frac{7}{12} = 7$ times $\frac{1}{12}$ of 654, or $\frac{1}{12}$ of 7 times 654.

34. 66 by $\frac{7}{9}$.
35. 58 by $\frac{13}{15}$.
36. 92 by $\frac{7}{22}$.
37. 784 by $\frac{13}{40}$.
38. 648 by $\frac{25}{26}$.
39. 564 by $\frac{33}{40}$.
40. 757 by $\frac{2}{3}$ of $\frac{6}{7}$.
41. 908 by $\frac{3}{4}$ of $2\frac{1}{2}$.
42. 588 by $\frac{4}{15}$ of $3\frac{1}{3}$.

43. Multiply 256 by $27\frac{5}{8}$. 406 by $33\frac{5}{6}$.

SUGGESTION.—Since $256 \times 27\frac{5}{8} = 256 \times 27 + 256 \times \frac{5}{8}$, first multiply by the integer and then by the fraction, and add the products.

44. 66 by $8\frac{2}{3}$.
45. 72 by $9\frac{5}{8}$.
46. 96 by $8\frac{5}{12}$.
47. 645 by $12\frac{3}{4}$.
48. 465 by $18\frac{3}{5}$.
49. 406 by $33\frac{1}{3}$.
50. 745 by $60\frac{2}{3}$.
51. 385 by $45\frac{4}{15}$.
52. 708 by $60\frac{3}{8}$.

### PRINCIPLE AND RULES.

**94.** PRINCIPLE.—*The product of an integer by a fraction equals the fraction of the integer.*

**95.** RULES.—1. To multiply an integer by a fraction, (1) *Divide the integer by the denominator, and multiply the quotient by the numerator.* Or, (2) *Multiply the integer by the numerator, and divide the product by the denominator.*

2. To multiply an integer by a mixed number, *Multiply by the integer and the fraction separately, and add the products.*

## Case III.

### Fractions multiplied by Fractions.

53. What is $\frac{1}{3}$ of $\frac{1}{4}$? $\frac{1}{3}$ of $\frac{3}{4}$? $\frac{2}{3}$ of $\frac{3}{4}$?
54. $\frac{3}{4}$ of $\frac{5}{6}$? $\frac{2}{3}$ of $\frac{5}{8}$? $\frac{5}{6}$ of $\frac{7}{8}$? $\frac{3}{8}$ of $\frac{5}{9}$?
55. What is $\frac{3}{4} \times \frac{5}{6}$? $\frac{5}{8} \times \frac{3}{7}$? $\frac{7}{9} \times \frac{5}{7}$? $\frac{7}{10} \times \frac{3}{14}$?

SUGGESTION.—$\frac{3}{4} \times \frac{5}{6} = \frac{3}{4}$ of $\frac{5}{6}$; $\frac{5}{8} \times \frac{3}{7} = \frac{5}{8}$ of $\frac{3}{7}$, etc.

56. $\frac{5}{9} \times \frac{9}{11}$? $\frac{6}{7} \times \frac{14}{15}$? $\frac{7}{12} \times \frac{5}{7}$? $\frac{4}{5} \times \frac{10}{11}$? $\frac{7}{9} \times \frac{9}{14}$?
57. What is $\frac{1}{3}$ of $12\frac{1}{2}$? $\frac{1}{3}$ of $13\frac{1}{4}$? $\frac{2}{3}$ of $13\frac{1}{4}$.

SOLUTION.—$\frac{1}{3}$ of $13\frac{1}{4} = \frac{1}{3}$ of $12 + \frac{1}{3}$ of $1\frac{1}{4} = 4 + \frac{5}{12} = 4\frac{5}{12}$; and $\frac{2}{3}$ of $13\frac{1}{4} = 2$ times $4\frac{5}{12} = 8\frac{5}{6}$.

58. $\frac{3}{4}$ of $16\frac{2}{3}$? $\frac{4}{5}$ of $22\frac{1}{2}$? $\frac{5}{8}$ of $42\frac{1}{3}$? $\frac{7}{10}$ of $62\frac{1}{3}$?
59. $\frac{7}{6}$ of $37\frac{1}{6}$? $\frac{5}{6}$ of $42\frac{2}{3}$? $\frac{7}{9}$ of $65\frac{1}{4}$? $\frac{5}{12}$ of $100\frac{1}{3}$?
60. Show that $\frac{3}{4} \times \frac{5}{6} = \frac{5}{6}$ of $\frac{3}{4}$.

### WRITTEN PROBLEMS.

61. Multiply $\frac{13}{15}$ by $\frac{3}{4}$.

PROCESS.

$$\frac{13}{15} \times \frac{3}{4} = \frac{13 \times 3}{15 \times 4} = \frac{13}{20}, \textit{ Ans.}$$

Since $\frac{3}{4} = \frac{1}{4}$ of 3, the product of $\frac{13}{15} \times \frac{3}{4} = \frac{1}{4}$ of 3 times $\frac{13}{15} = \frac{1}{4}$ of $\frac{13 \times 3}{15} = \frac{13 \times 3}{15 \times 4} = \frac{13}{20}$.

62. $\frac{13}{16}$ by $\frac{11}{13}$.
63. $\frac{10}{11}$ by $\frac{22}{27}$.
64. $\frac{19}{25}$ by $\frac{15}{38}$.
65. $\frac{10}{13}$ by $\frac{39}{40}$.
66. $\frac{3}{7}$ by $\frac{4}{5}$ of $\frac{5}{6}$.
67. $\frac{7}{8}$ of $\frac{4}{21}$ by $\frac{3}{10}$.
68. $\frac{3}{4}$ by $\frac{5}{6}$ by $\frac{7}{8}$.
69. $2\frac{1}{3}$ by $\frac{3}{10}$ of $\frac{15}{21}$.
70. $2\frac{1}{2}$ by $3\frac{1}{3}$.
71. $4\frac{1}{2}$ by $5\frac{1}{2}$.
72. $6\frac{1}{4}$ by $3\frac{2}{3}$.
73. $10\frac{1}{2}$ by $2\frac{2}{3}$.

74. What will $\frac{7}{8}$ of a yard of cloth cost at $\$\frac{5}{6}$ a yard? At $\$\frac{7}{12}$ a yard?

75. What will $5\frac{1}{2}$ pounds of flour cost at $4\frac{1}{2}$ cents a pound? At $6\frac{1}{4}$ cents a pound?

76. What will $2\frac{1}{2}$ pounds of tea cost at $\$1\frac{3}{4}$ a pound? At $\$1\frac{1}{3}$ a pound?

77. What is the cost of 35 barrels of flour at $\$6\frac{1}{3}$ a barrel? At $\$7\frac{1}{4}$ a barrel?

78. A man owned $\frac{7}{12}$ of a ship which was sold for \$13250: what was his share of the money?

79. What is the product of $\frac{7}{10}$, $\frac{2}{3}$ of $2\frac{1}{2}$, $\frac{3}{4}$ of $\frac{7}{10}$ of $\frac{20}{21}$, and $2\frac{1}{3}$?

80. What will $12\frac{1}{2}$ pounds of butter cost at $18\frac{3}{4}$ cents a pound? At $22\frac{1}{2}$ cents a pound?

### PRINCIPLE AND RULES.

**96.** PRINCIPLE.—*The product of a fraction by a fraction equals the fraction of the fraction.*

**97.** RULES.—1. To multiply a fraction by a fraction, *Multiply the numerators together, and also the denominators.*

2. To multiply a mixed number by a mixed number, *Reduce the mixed numbers to improper fractions, and proceed as above.*

NOTES.—1. Mixed numbers may be multiplied together by *first multiplying the integers; next multiplying each integer by the fraction united with the other integer; next multiplying the two fractions; and then adding the four products.* Thus, $18\frac{3}{4} \times 12\frac{1}{2} = 18 \times 12 + 18 \times \frac{1}{2} + 12 \times \frac{3}{4} + \frac{3}{4} \times \frac{1}{2}$. But in most cases it is shorter to reduce the mixed numbers to improper fractions.

2. Cases I and II may be included in Case III, by changing the integer to the form of a fraction. Thus, $\frac{3}{8} \times 5 = \frac{3}{8} \times \frac{5}{1}$, and $8 \times \frac{3}{5} = \frac{8}{1} \times \frac{3}{5}$.

3. The process of multiplying fractions may be shortened by cancellation. Compound fractions need not be reduced to simple fractions, since $\frac{2}{3} \times \frac{4}{5}$ of $\frac{10}{11} = \frac{2}{3} \times \frac{4}{5} \times \frac{10}{11}$.

## DIVISION OF FRACTIONS.

### Case I.

#### Fractions divided by Integers.

1. If a man can do $\frac{6}{7}$ of a piece of work in 3 days, how much can he do in 1 day?

2. A man divided $\frac{8}{9}$ of a farm equally between 4 sons: what part of the farm did each receive?

3. If 5 yards of muslin cost $\frac{4}{5}$ of a dollar, what will 1 yard cost?

4. If 10 oranges cost $\frac{5}{8}$ of a dollar, what will 1 orange cost?

5. If 8 bushels of oats cost $\$2\frac{2}{3}$, what will 1 bushel cost?

6. If $\frac{10}{11}$ of a melon be divided into 5 equal parts, what will each part be?

7. Why does dividing the numerator of $\frac{8}{9}$ by 4 divide the fraction by 4?

8. Why does multiplying the denominator of $\frac{8}{9}$ by 4 divide the fraction by 4?

9. In how many ways may a fraction be divided by an integer?

## WRITTEN PROBLEMS.

10. Divide $\frac{12}{25}$ by 6.

PROCESS.

$\frac{12}{25} \div 6 = \dfrac{12 \div 6}{25} = \frac{2}{25}$, *Ans.*

Or: $\frac{12}{25} \div 6 = \dfrac{12}{25 \times 6} = \frac{2}{25}$.

Since $\frac{12}{25} \div 1 = \frac{12}{25}$, $\frac{12}{25} \div 6 = \frac{1}{6}$ of $\frac{12}{25} = \dfrac{12 \div 6}{25}$, or $\dfrac{12}{25 \times 6}$. Or, since to divide a number by 6 is to find $\frac{1}{6}$ of of it, $\frac{12}{25} \div 6 = \frac{1}{6}$ of $\frac{12}{25} = \dfrac{12 \div 6}{25}$, or $\dfrac{12}{25 \times 6}$.

Divide

| | | |
|---|---|---|
| 11. $\frac{16}{21}$ by 8. | 14. $\frac{45}{47}$ by 15. | 17. $2\frac{1}{3}$ by 8. |
| 12. $\frac{14}{23}$ by 7. | 15. $\frac{60}{61}$ by 20. | 18. $5\frac{1}{2}$ by 12. |
| 13. $\frac{33}{40}$ by 11. | 16. $\frac{75}{77}$ by 25. | 19. $6\frac{2}{3}$ by 10. |

## PRINCIPLE AND RULES.

**98.** PRINCIPLE.—*A fraction is divided by dividing its numerator or multiplying its denominator.*

**99.** RULES.—1. To divide a fraction by an integer, *Divide the numerator or multiply the denominator.*

2. To divide a mixed number by an integer, (1) *Reduce the mixed number to an improper fraction and divide as above;* or, (2) *Divide the integral part and then the fraction, and unite the quotients.*

## Case II.

### Integers divided by Fractions.

20. How many times is $\frac{2}{5}$ of a cent contained in 4 cents?

SOLUTION.—In four cents there are 20 fifths of a cent, and 2 fifths of a cent are contained in 20 fifths of a cent 10 times.

21. If a fruit jar hold $\frac{3}{4}$ of a gallon, how many jars will hold 6 gallons? 12 gallons? 18 gallons?

22. If $\frac{5}{8}$ of a yard of silk will make a vest, how many vests will 5 yards make? 7 yards? 10 yards?

23. If a yard of cloth cost $\$\frac{5}{6}$, how many yards can be bought for \$10? For \$15? For \$20?

24. How many times is $\frac{3}{4}$ contained in 8? $\frac{3}{4}$ in 12? $\frac{2}{3}$ in 9? $\frac{2}{3}$ in 15? $\frac{5}{6}$ in 9? $\frac{5}{6}$ in 12?

25. How many times is $\frac{3}{8}$ contained in 12? $\frac{7}{8}$ in 15?

26. Show that $8 \div \frac{3}{5} = \frac{8 \times 5}{3}$.

### WRITTEN PROBLEMS.

27. What is the quotient of $25 \div \frac{7}{8}$?

$$\text{PROCESS: } 25 \div \frac{7}{8} = \frac{25 \times 8}{7} = 28\tfrac{4}{7}, \textit{ Ans.}$$

NOTE.—It will be noticed that the integer is multiplied by the denominator of the fraction and the product divided by its numerator.

What is the quotient of

| | | |
|---|---|---|
| 28. $21 \div \frac{7}{12}$? | 31. $100 \div \frac{30}{37}$? | 34. $75 \div 6\frac{1}{4}$? |
| 29. $42 \div \frac{14}{15}$? | 32. $96 \div \frac{17}{30}$? | 35. $120 \div 3\frac{1}{3}$? |
| 30. $72 \div \frac{24}{25}$? | 33. $125 \div \frac{21}{50}$? | 36. $225 \div 5\frac{1}{2}$? |

**100.** RULES.—To divide an integer by a fraction, 1. *Multiply the integer by the denominator of the fraction, and divide the product by the numerator.* Or,

2. *Divide the integer by the numerator, and multiply the quotient by the denominator.*

## Case III.

### Fractions divided by Fractions.

37. How many times is $\frac{2}{5}$ of an inch contained in $\frac{4}{5}$ of an inch? $\frac{2}{5}$ of an inch in $\frac{6}{5}$ of an inch?

38. How many times $\frac{3}{8}$ in $\frac{6}{8}$? $\frac{3}{8}$ in $\frac{9}{8}$? $\frac{5}{8}$ in $\frac{10}{8}$?

39. How many times $\frac{4}{9}$ in $\frac{8}{9}$? $\frac{4}{9}$ in $\frac{12}{9}$? $\frac{5}{7}$ in $\frac{10}{7}$? $\frac{5}{7}$ in $\frac{15}{7}$? $\frac{6}{11}$ in $\frac{9}{11}$? $\frac{6}{11}$ in $\frac{18}{11}$?

40. How many times is $\frac{1}{3}$ contained in $\frac{3}{4}$? $\frac{1}{4}$ in $\frac{2}{3}$?

SUGGESTION.—Change the fractions to twelfths.

41. How many times $\frac{2}{3}$ in $\frac{5}{6}$? $\frac{3}{5}$ in $\frac{3}{4}$? $\frac{1}{3}$ in $\frac{7}{8}$? $\frac{2}{7}$ in $\frac{9}{14}$?

42. $\frac{3}{10}$ in $\frac{4}{5}$? $\frac{3}{8}$ in $\frac{7}{12}$? $\frac{3}{10}$ in $\frac{11}{15}$? $\frac{2}{3}$ in $\frac{3}{7}$? $\frac{2}{5}$ in $\frac{5}{8}$?

43. Show that the quotient of two fractions having a common denominator, equals the quotient of their numerators.

### WRITTEN PROBLEMS.

44. Divide $\frac{7}{8}$ by $\frac{3}{5}$.

PROCESS: $\frac{7}{8} \div \frac{3}{5} = \frac{7 \times 5}{8 \times 3} = \frac{35}{24} = 1\frac{11}{24}$, *Ans.*

Since $\frac{7}{8} = \frac{7 \times 5}{40}$, and $\frac{3}{5} = \frac{3 \times 8}{40}$, $\frac{7}{8} \div \frac{3}{5} = \frac{7 \times 5}{40} \div \frac{3 \times 8}{40} = \frac{7 \times 5}{3 \times 8}$. It is thus seen that inverting the terms of the divisor, and taking the product of the numerators for the numerator, and the product of the denominators for the denominator, is the same as reducing the fractions to a common denominator, and dividing the numerator of the dividend by the numerator of the divisor.

NOTE.—That $\frac{7}{8} \div \frac{3}{5} = \frac{7 \times 5}{8 \times 3}$ may also be thus explained: $\frac{3}{5} = 3$ times $\frac{1}{5}$, and since $\frac{7}{8} \div \frac{1}{5} = \frac{7 \times 5}{8}$, $\frac{7}{8} \div \frac{3}{5} = \frac{1}{3}$ of $\frac{7 \times 5}{8} = \frac{7 \times 5}{8 \times 3}$.

What is the quotient of

| | | |
|---|---|---|
| 45. $\frac{7}{10} \div \frac{14}{15}$? | 49. $3\frac{1}{2} \div 2\frac{1}{3}$? | 53. $\frac{7}{12} \div \frac{4}{5}$ of $\frac{15}{16}$? |
| 46. $\frac{12}{13} \div \frac{8}{11}$? | 50. $5\frac{1}{2} \div 3\frac{1}{4}$? | 54. $\frac{5}{6}$ of $\frac{9}{10} \div \frac{3}{8}$ of 4? |
| 47. $\frac{9}{20} \div \frac{18}{25}$? | 51. $6\frac{2}{3} \div 12\frac{1}{2}$? | 55. $\frac{7}{11}$ of $3\frac{2}{3} \div \frac{2}{9}$ of $2\frac{1}{4}$? |
| 48. $\frac{18}{25} \div \frac{9}{10}$? | 52. $16\frac{2}{3} \div 3\frac{1}{3}$? | 56. $\frac{12}{13} \div \frac{1}{9}$ of $\frac{5}{6}$ of $3\frac{3}{5}$? |

57. If a family use $\frac{4}{9}$ of a barrel of flour in a month, how long will $2\frac{1}{3}$ barrels last?

58. If a bushel of corn cost $\$\frac{5}{8}$, how many bushels can be bought for $\$6\frac{1}{2}$? For $\$9\frac{3}{4}$?

59. If 13 yards of silk cost $\$17\frac{1}{3}$, how many yards can be bought for $\$48\frac{4}{5}$? For $\$62\frac{1}{2}$?

60. If a man walk $3\frac{3}{10}$ miles an hour, in how many hours will he walk $20\frac{1}{4}$ miles?

61. At $\$33\frac{1}{3}$ an acre, how many acres of land can be bought for $\$841\frac{2}{3}$?

62. By what must $\frac{3}{5}$ be multiplied that the product may be $26\frac{2}{3}$?

63. Divide the product of $6\frac{1}{2}$ multiplied by $3\frac{1}{3}$ by the quotient of $4\frac{1}{2} \div 5\frac{1}{3}$?

## PRINCIPLES AND RULES.

**101.** PRINCIPLES.—1. *The quotient of two fractions having a common denominator, equals the quotient of their numerators.*

2. *The multiplying of both dividend and divisor by the same number does not change the value of the quotient.*

**102.** RULES.—To divide a fraction by a fraction, 1. *Reduce the fractions to a common denominator, and divide the numerator of the dividend by the numerator of the divisor.* Or,

2. *Invert the terms of the divisor, and then multiply the numerators together and also the denominators.* Or,

3. *Multiply both dividend and divisor by the least common multiple of the denominators of the fractions, and divide the resulting dividend by the resulting divisor.*

NOTES.—1. The third rule depends on the second principle; and, since multiplying two fractions by their least common multiple changes them to integers, the new dividend and divisor are always integral. Thus, multiplying both fractions by 24, the *l. c. m.*, $\frac{5}{8} \div \frac{7}{12} = 15 \div 14 = 1\frac{1}{14}$; multiplying by 6, the *l. c. m.*, $6\frac{2}{3} \div 5\frac{1}{2} = 40 \div 33 = 1\frac{7}{33}$. Compound fractions should first be reduced to simple fractions.

2. It is not necessary that the pupil be made equally familiar with these three methods of dividing one fraction by another. He should thoroughly master one of them.

## COMPLEX FRACTIONS.

**64.** Reduce the complex fraction $\frac{\frac{4}{5}}{\frac{6}{7}}$, to its simplest form.

PROCESS: $\frac{\frac{4}{5}}{\frac{6}{7}} = \frac{4}{5} \div \frac{6}{7} = \frac{4 \times 7}{5 \times 6} = \frac{14}{15}$, *Ans.*

Reduce to the simplest form

| | | | |
|---|---|---|---|
| 65. $\dfrac{\frac{3}{8}}{\frac{9}{10}}$ | 69. $\dfrac{16\frac{2}{3}}{25}$ | 73. $\dfrac{\frac{5}{6} \text{ of } \frac{6}{7}}{\frac{2}{3} \text{ of } \frac{3}{8}}$ | 77. $\dfrac{\frac{3}{8} + \frac{3}{10}}{\frac{3}{8} - \frac{3}{10}}$ |
| 66. $\dfrac{\frac{6}{7}}{24}$ | 70. $\dfrac{25}{16\frac{2}{3}}$ | 74. $\dfrac{\frac{2}{7} \text{ of } 2\frac{1}{3}}{5\frac{1}{2}}$ | 78. $\dfrac{\frac{5}{7} - \frac{1}{3}}{\frac{3}{4} + \frac{8}{9}}$ |
| 67. $\dfrac{15}{\frac{5}{9}}$ | 71. $\dfrac{12\frac{1}{2}}{\frac{5}{8}}$ | 75. $\dfrac{\frac{3}{8} \text{ of } \frac{4}{9}}{\frac{5}{12}}$ | 79. $\dfrac{\frac{7}{8}}{\frac{3}{4}} \times \dfrac{\frac{5}{9}}{\frac{1}{6}}$ |
| 68. $\dfrac{2\frac{1}{3}}{3\frac{1}{2}}$ | 72. $\dfrac{\frac{5}{8}}{12\frac{1}{2}}$ | 76. $\dfrac{\frac{6}{11}}{\frac{4}{7} \text{ of } \frac{14}{33}}$ | 80. $\dfrac{\frac{2}{9}}{\frac{3}{4}} \div \dfrac{\frac{3}{4}}{\frac{5}{9}}$ |

**103.** A complex fraction expresses an unexecuted division, the numerator being the dividend and the denominator the divisor. It is reduced to its simplest form *by performing the division as expressed.*

NOTES.—1. A complex fraction may be changed to a fraction with integral terms, *by multiplying both of its terms by the least common multiple of the denominators of its fractions.* (Art. 102, Note 1.) Compound fractions must first be reduced to simple fractions.

2. Let the above problems also be solved by this method.

## NUMBERS PARTS OF OTHER NUMBERS.

### MENTAL PROBLEMS.

1. If $\frac{1}{3}$ of a barrel of flour cost \$3, what will a barrel cost?

2. If $\frac{1}{5}$ of a ream of note paper cost 75 cents, what will a ream cost?

3. Charles gave Henry 7 marbles, which were $\frac{1}{8}$ of all he had: how many marbles had Charles?

4. 15 is $\frac{1}{6}$ of what number?
5. 16 is $\frac{1}{10}$ of what number?
6. $12\frac{1}{2}$ is $\frac{1}{8}$ of what number?
7. $16\frac{2}{3}$ is $\frac{1}{12}$ of what number?
8. $22\frac{2}{5}$ is $\frac{1}{9}$ of what number?
9. 24 is $\frac{2}{5}$ of what number?

SOLUTION.—If 24 is $\frac{2}{5}$ of a number, $\frac{1}{5}$ is $\frac{1}{2}$ of 24, which is 12. If 12 is $\frac{1}{5}$ of a number, $\frac{5}{5}$ is 5 times 12, or 60. Hence, 24 is $\frac{2}{5}$ of 60.

10. 27 is $\frac{3}{8}$ of what number?
11. 45 is $\frac{5}{12}$ of what number?
12. 64 is $\frac{8}{15}$ of what number?
13. $27\frac{1}{2}$ is $\frac{3}{5}$ of what number?
14. $46\frac{2}{3}$ is $\frac{9}{10}$ of what number?
15. $37\frac{1}{2}$ is $\frac{8}{9}$ of what number?
16. $87\frac{1}{3}$ is $\frac{7}{12}$ of what number?
17. 45 is $\frac{5}{7}$ of how many times 9?
18. 63 is $\frac{7}{8}$ of how many times 12?
19. 80 is $\frac{8}{15}$ of how many times 20?
20. 108 is $\frac{12}{25}$ of how many times 15?
21. What part of 4 is 1? What part of 4 is 3?
22. What part of 6 is 5? 9 is 8? 12 is 6?
23. 11 is 7? 16 is 12? 20 is 15? 18 is 12? 30 is 15?
24. 7 is what part of 21? 8 of 32? 9 of 27?
25. 13 of 39? 16 of 72? 15 of 25? 60 of 90?
26. $\frac{1}{4}$ is what part of $\frac{3}{4}$? $\frac{1}{5}$ of $\frac{4}{5}$? $\frac{2}{7}$ of $\frac{5}{7}$? $\frac{5}{9}$ of $\frac{7}{9}$?
27. $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{2}{3}$ of $\frac{3}{4}$? $\frac{3}{5}$ of $\frac{5}{6}$? $\frac{4}{7}$ of $\frac{9}{14}$? $\frac{5}{6}$ of $\frac{11}{12}$?
28. $\frac{2}{3}$ of 11? $\frac{3}{7}$ of 4? $\frac{5}{6}$ of 10? $\frac{4}{5}$ of 8? $\frac{5}{7}$ of 10?
29. $5\frac{1}{2}$ of $16\frac{1}{2}$? $6\frac{2}{3}$ of $33\frac{1}{3}$? $12\frac{1}{2}$ of $37\frac{1}{2}$? $33\frac{1}{3}$ of $16\frac{2}{3}$?
30. $3\frac{2}{3}$ of $6\frac{1}{2}$? $5\frac{1}{2}$ of $2\frac{3}{4}$? $2\frac{1}{4}$ of $3\frac{1}{8}$? $6\frac{1}{4}$ of $12\frac{1}{2}$?

## PRINCIPLE AND RULE.

**104.** PRINCIPLE.—*Only like numbers can be compared.*

**105.** RULE.—To find what part one number is of another, *Divide the number denoting the part by the number denoting the whole.*

## REVIEW OF FRACTIONS.

### MENTAL PROBLEMS.

1. A boy having \$$\frac{7}{8}$ gave \$$\frac{2}{5}$ for a knife: how much money had he left?

2. If $\frac{5}{7}$ be added to a certain fraction, the sum will be $\frac{9}{10}$: what is the fraction?

3. A laborer spends $\frac{3}{5}$ of his wages for board and $\frac{1}{8}$ for clothing: what part has he left?

4. A man did $\frac{1}{3}$ of a piece of work the first day, $\frac{1}{4}$ of it the second day, $\frac{1}{6}$ of it the third day, and the remainder the fourth day: what part of the work did he do the fourth day?

5. A man bought a farm, paying $\frac{2}{5}$ of the price down, $\frac{1}{3}$ of it the first year, $\frac{1}{6}$ the second year, and the remainder the third year: what part did he pay the third year?

6. A man is 42 years of age, and $\frac{2}{7}$ of his age equals the age of his son: how old is the son?

7. A man bought a cow for \$$33\frac{1}{3}$ and sold her for $\frac{5}{6}$ of what she cost: how much did he lose?

8. If a yard of velvet cost \$$8\frac{1}{3}$, what will $\frac{3}{5}$ of a yard cost?

9. Jane's age is $16\frac{2}{3}$ years, and Mary's age is $\frac{3}{4}$ of Jane's: how old is Mary?

10. A man owning $\frac{5}{6}$ of a mill sells $\frac{2}{3}$ of his share: what part of the mill does he still own?

11. Charles bought $\frac{3}{4}$ of a pound of candy and gave his sister $\frac{3}{8}$ of a pound, and his playmate $\frac{2}{3}$ of what remained: what part of a pound had he left?

12. A wife is 35 years of age, and her age is $\frac{5}{7}$ of the age of her husband: how old is her husband?

13. The difference between $\frac{5}{6}$ and $\frac{3}{5}$ of a certain number is 14: what is the number?

14. A farmer sold 50 sheep, which were $\frac{2}{5}$ of his flock: how many sheep had he before the sale?

15. When Charles is $\frac{2}{5}$ older than he now is, he will be 21 years of age: how old is he?

16. A farmer sold $\frac{3}{5}$ of his farm for \$1645: at this rate, what was the value of the farm?

17. A man sold $\frac{3}{7}$ of his farm and had 64 acres left: how many acres had he at first?

18. A man sold a horse for \$90, which was $\frac{1}{5}$ more than it cost him: what was the cost of the horse?

19. A lady paid \$30 for a cloak, which was $\frac{3}{7}$ more than she paid for a dress: what was the cost of the dress?

20. $\frac{3}{7}$ of 42 is $\frac{2}{11}$ of what number?

21. A man is 45 years old, and $\frac{2}{5}$ of his age is $\frac{3}{7}$ of the age of his wife: how old is his wife?

22. Samuel is $\frac{3}{5}$ as old as Charles, and Harry, who is 9 years old, is $\frac{3}{4}$ as old as Charles: how old are Charles and Samuel?

23. A man gave \$150 for a watch and chain, and the chain cost $\frac{3}{7}$ as much as the watch: what did each cost?

24. If to A's age there be added $\frac{2}{3}$ and $\frac{2}{5}$ of his age, the sum will be 62 years: what is A's age?

25. A farmer's sheep are in 4 fields; the first contains $\frac{2}{9}$ of all, the second $\frac{1}{6}$, the third $\frac{1}{4}$, and the fourth 52 sheep: how many sheep in the 4 fields?

26. A saddle cost \$35, and $\frac{2}{7}$ of the cost of the saddle was $\frac{5}{9}$ of the cost of a bridle: what was the cost of the bridle?

27. If to $\frac{5}{6}$ of a man's age 15 years be added, the sum will be $\frac{5}{4}$ of his age: how old is he?

28. The distance from Cleveland to Columbus is 138 miles, $\frac{15}{23}$ of which is $\frac{3}{4}$ of the distance from Columbus to Cincinnati: what is the distance from Columbus to Cincinnati?

29. $\frac{6}{7}$ is $\frac{3}{8}$ of what number?

30. If $\frac{2}{3}$ of the value of a house equals $\frac{4}{5}$ of the value of a lot, and the value of both is \$4400, what is the value of each?

31. If $\frac{2}{9}$ of A's money equals $\frac{6}{7}$ of B's, and both together have \$340, how much has each?

32. If $\frac{3}{4}$ of A's age is $\frac{3}{5}$ of B's, and $\frac{2}{3}$ of B's is 20 years: what is the age of each?

33. If $\frac{2}{7}$ of a yard of velvet cost $\$2\frac{2}{3}$, what will $\frac{4}{9}$ of a yard cost?

34. How many pounds of honey, at $\$\frac{3}{8}$ a pound, can be bought for \$3?

35. How many bushels of apples, at $\$\frac{3}{5}$ a bushel, can be bought for $\$16\frac{2}{3}$?

36. If a barrel hold $2\frac{3}{4}$ bushels, how many barrels will be required to pack 55 bushels of apples?

37. If $5\frac{1}{2}$ lb. of sugar cost \$1, how much will $49\frac{1}{2}$ lb. cost?

38. If $\frac{3}{4}$ of a yard of silk cost $\$1\frac{1}{5}$, how many yards can be bought for $\$10\frac{2}{3}$?

39. If $3\frac{2}{3}$ yards of cloth cost $\$5\frac{1}{2}$, what will $6\frac{1}{4}$ yards cost?

40. If a train of cars run $\frac{3}{5}$ of a mile in $1\frac{2}{7}$ minutes, how many miles will it run in 15 minutes?

41. If 4 pounds of coffee cost $\$\frac{8}{9}$, what will $7\frac{1}{2}$ pounds cost?

42. If $12\frac{1}{2}$ tons of hay will feed 5 horses a year, how many tons will feed 8 horses a year?

43. If a rod 5 feet long casts a shadow $8\frac{1}{3}$ feet long, what is the length of a pole whose shadow, at the same time of day, is $17\frac{1}{2}$ feet?

44. If 3 men can do a piece of work in $10\frac{3}{4}$ days, how long will it take 8 men to do it?

45. If a barrel of flour will supply 12 persons $4\frac{3}{5}$ weeks, how long will it supply 7 persons?

46. A can do a job of work in 12 days, and B in 10 days: how long will it take both to do it?

47. A and B can do a certain work in 8 days, and A can do it in 12 days: in what time can B do it?

48. A and B can mow a field in 10 days, and A can mow only $\frac{2}{3}$ as much as B: how long would it take each to mow the field?

49. How is the value of a proper fraction affected by

adding the same number to both of its terms? By subtracting the same number? (Illustrate, taking $\frac{3}{5}$.)

50. How is the value of an improper fraction, greater than 1, affected by adding the same number to both of its terms? By subtracting the same number? (Illustrate.)

### WRITTEN PROBLEMS.

51. Add $\frac{5}{6}$, $\frac{7}{8}$, $\frac{3}{10}$ of $\frac{5}{12}$, and $3\frac{1}{6}$.

52. From $\frac{2}{3}$ of $1\frac{1}{8}$ take $\frac{5}{7}$ of $\frac{14}{15}$.

53. From the sum of $27\frac{3}{8}$ and $20\frac{2}{3}$ take their difference.

54. Multiply $\frac{13}{15}$ by 35; 35 by $\frac{13}{15}$; $\frac{13}{15}$ by $\frac{10}{11}$; $3\frac{1}{2}$ by $2\frac{1}{3}$.

55. Divide $\frac{16}{21}$ by 32; 32 by $\frac{16}{21}$; $\frac{16}{21}$ by $\frac{7}{8}$; $4\frac{1}{2}$ by $3\frac{1}{5}$.

56. $\frac{11}{12} + \frac{7}{15} =$ what? $\frac{11}{12} - \frac{7}{15}$? $\frac{11}{12} \times \frac{7}{15}$? $\frac{11}{12} \div \frac{7}{15}$?

57. Multiply $2045\frac{3}{7}$ by 35; 806 by $84\frac{3}{5}$; $30\frac{1}{4}$ by $16\frac{2}{3}$.

58. Divide $347\frac{5}{6}$ by 15; 692 by $21\frac{5}{8}$; $19\frac{4}{9}$ by $16\frac{2}{3}$.

59. A farm is divided into five fields, containing respectively $21\frac{3}{4}$ A., $34\frac{2}{3}$ A., $45\frac{7}{8}$ A., $56\frac{5}{6}$ A., and $29\frac{1}{2}$ A.: how many acres in the farm?

60. There are $30\frac{1}{4}$ sq. yd. in a square rod: how many square rods in $786\frac{1}{2}$ sq. yd.?

61. A man travels $5\frac{3}{4}$ miles an hour: how long will it take him to make a journey of $75\frac{2}{3}$ miles?

62. At $\$8\frac{2}{3}$ a ton, how many tons of hay can be bought for $\$108\frac{1}{3}$?

63. If $\frac{4}{15}$ of an acre of land cost $68, what will $12\frac{2}{7}$ acres cost?

64. If $\frac{5}{8}$ of a yard of velvet cost $\$8\frac{4}{7}$, how many yards can be bought for $\$196\frac{4}{7}$?

65. If a number be diminished by $\frac{5}{7}$ of $\frac{12}{25}$ of itself, the remainder will be 69: what is the number?

66. A pedestrian walked $\frac{5}{12}$ of his journey the first day, $\frac{3}{8}$ of it the second day, and then had 24 miles to travel: how long was the journey?

67. A man pays $350 a year for house rent, which is $\frac{14}{55}$ of his income: what is his income?

68. A man bequeathed to his wife $4860, which was $\frac{12}{35}$ of his estate: what was the value of the estate?

69. A graded school enrolls 208 boys, and $\frac{7}{15}$ of the pupils are girls: how many pupils are enrolled in the school?

70. A man owning $\frac{5}{9}$ of a ship sells $\frac{3}{7}$ of his share for \$3480: at this rate, what is the value of the ship?

71. A owning $\frac{3}{5}$ of a mill, sold $\frac{2}{5}$ of his share to B, and $\frac{1}{2}$ of what he then owned to C for \$460: what was the value of the mill at the rate of C's purchase?

72. A owns $\frac{7}{12}$ of a section of land; B, $\frac{7}{16}$ of a section; and C, $\frac{3}{10}$ as much as both A and B: what part of a section does C own?

73. A bought $\frac{2}{3}$ of a factory for \$21840, and sold $\frac{3}{7}$ of his share to B, and $\frac{2}{5}$ of it to C: what part of the factory did A then own?

74. A and B together own 396 acres of land, and $\frac{3}{5}$ of A's farm equals $\frac{3}{4}$ of B's: how many acres does each own?

75. A stock of goods is owned by three partners, A owning $\frac{3}{8}$, B $\frac{5}{11}$, and C the remainder; the goods were sold at a profit of \$6160: what was each partner's share?

76. $\frac{5}{8}$ of a stock of goods was destroyed by fire, and $\frac{3}{5}$ of the remainder was damaged by water, and the uninjured goods were sold at cost for \$5280: what was the cost of the entire stock of goods?

77. A man paid $\frac{2}{5}$ of his money for a farm, $\frac{1}{3}$ of what remained for repairs, $\frac{1}{3}$ of what then remained for stock, $\frac{1}{2}$ of what then remained for utensils, and then had left \$650: how much money had he at first?

78. A merchant tailor has $67\frac{2}{5}$ yards of cloth, from which he wishes to cut an equal number of coats, pants, and vests: how many of each can he cut if they contain $3\frac{3}{4}$, $2\frac{7}{8}$, and $1\frac{4}{5}$ yards respectively?

79. An estate was divided between two brothers and a sister; the elder brother received $\frac{3}{8}$ of the estate, the younger $\frac{3}{10}$, and the sister the remainder, which was \$450 less than the elder brother received: what was the value of the estate? t was each brother's share?

# SECTION IX.

# DECIMAL FRACTIONS.

## NUMERATION AND NOTATION.

1. If a unit be divided into ten equal parts, what is one part called?

2. If a tenth of a unit be divided into ten equal parts, what is one part? What is $\frac{1}{10}$ of $\frac{1}{10}$?

3. If a hundredth of a unit be divided into ten equal parts, what is one part? What is $\frac{1}{10}$ of $\frac{1}{100}$?

4. What part of a tenth is a hundredth? What part of a hundredth is a thousandth?

5. How do the fractions $\frac{3}{10}$, $\frac{3}{100}$, and $\frac{3}{1000}$ compare with each other in value? $\frac{7}{10}$, $\frac{7}{100}$, and $\frac{7}{1000}$?

**106.** Since the fractional units, tenths, hundredths, thousandths, etc., decrease in value *tenfold*, they are expressed, like the orders of integers, on *a scale of ten.* This is done by extending the orders to the right of units, and calling the first fractional order *tenths*, the second *hundredths*, the third *thousandths*, etc. A period is placed at the left of the order of tenths. Thus, $\frac{5}{10}$ is written .5; $\frac{5}{100}$ is written .05; $\frac{5}{1000}$ is written .005, etc.

Copy and read

| (6) | (7) | (8) | (9) | (10) | (11) |
|---|---|---|---|---|---|
| .4 | .03 | .002 | .06 | .07 | .005 |
| .7 | .05 | .004 | .006 | .004 | .4 |
| .6 | .08 | .006 | .08 | .8 | .07 |
| .9 | .09 | .007 | .5 | .09 | .009 |

12. How many tenths and hundredths in .25? In .45? .63? .78? .84? .69? .39?

13. How many tenths, hundredths, and thousandths in .325? In .246? .307? .405? .056?

14. How many tenths, hundredths, and thousandths in .045? In .407? .008? .065? .607? .325?

15. How many hundredths in $\frac{15}{100}$? In $\frac{8}{100}$? .34? .42?

16. How many thousandths in $\frac{25}{1000}$? In $\frac{125}{1000}$? .325? .065? .205? .008? .046?

**107.** When the right-hand figure of a decimal denotes hundredths, the whole decimal denotes hundredths, and when the right-hand figure denotes thousandths, the whole decimal denotes thousandths. Thus, .25 is read 25 *hundreths*; .325 is read 325 *thousandths.*

Copy and read

| (17) | (18) | (19) | (20) | (21) |
|---|---|---|---|---|
| .15 | .016 | .245 | .8 | .007 |
| .42 | .024 | .354 | .63 | .038 |
| .36 | .045 | .403 | .086 | .462 |
| .50 | .083 | .587 | .369 | .507 |
| .06 | .007 | .067 | .504 | .45 |

**108.** When fractions denoting tenths, hundredths, thousandths, etc., are expressed like integers, on the decimal scale, they are said to be expressed *decimally.*

Express decimally

| (22) | (23) | (24) | (25) | (26) |
|---|---|---|---|---|
| $\frac{3}{10}$ | $\frac{4}{1000}$ | $\frac{45}{1000}$ | $\frac{75}{100}$ | $\frac{18}{1000}$ |
| $\frac{7}{10}$ | $\frac{6}{1000}$ | $\frac{63}{1000}$ | $\frac{43}{1000}$ | $\frac{208}{1000}$ |
| $\frac{6}{100}$ | $\frac{14}{1000}$ | $\frac{215}{1000}$ | $\frac{7}{100}$ | $\frac{355}{1000}$ |
| $\frac{8}{100}$ | $\frac{56}{100}$ | $\frac{407}{1000}$ | $\frac{7}{1000}$ | $\frac{43}{1000}$ |
| $\frac{12}{100}$ | $\frac{40}{100}$ | $\frac{500}{1000}$ | $\frac{106}{1000}$ | $\frac{5}{1000}$ |

27. What is the name of the third decimal order? The fourth? The fifth? The sixth?

28. What does each significant figure of .0034 denote? Of .00275? Of .03405? Of .000325? Of .030056?

Copy and read

| (29) | (30) | (31) | (32) |
|---|---|---|---|
| .246 | .0635 | .00647 | .0307 |
| .0246 | .00635 | .000647 | .03007 |
| .708 | .3464 | .04056 | .030007 |
| .0708 | .03464 | .004056 | .034005 |
| .3425 | .32875 | .32453 | .450605 |

**109.** When a decimal fraction is expressed decimally, the right-hand figure is written in the order indicated by the name of the decimal. Thus, $\frac{325}{100000}$ is written .00325.

Express decimally

| (33) | (34) | (35) | (36) |
|---|---|---|---|
| $\frac{3}{100}$ | $\frac{6}{10000}$ | $\frac{7}{100000}$ | $\frac{29}{1000000}$ |
| $\frac{75}{100}$ | $\frac{33}{10000}$ | $\frac{37}{100000}$ | $\frac{609}{1000000}$ |
| $\frac{8}{1000}$ | $\frac{405}{10000}$ | $\frac{208}{100000}$ | $\frac{4045}{1000000}$ |
| $\frac{28}{1000}$ | $\frac{3042}{10000}$ | $\frac{3056}{100000}$ | $\frac{33033}{1000000}$ |
| $\frac{356}{1000}$ | $\frac{5007}{10000}$ | $\frac{38045}{100000}$ | $\frac{204056}{1000000}$ |

Express decimally

| (37) | (38) |
|---|---|
| 7 tenths; | 42 ten-thousandths; |
| 24 hundredths; | 506 ten-thousandths; |
| 29 thousandths; | 4008 ten-thousandths; |
| 405 thousandths; | 65 hundred-thousandths; |
| 65 millionths; | 6007 hundred-thousandths; |
| 5064 millionths; | 54008 hundred-thousandths; |
| 40056 millionths. | 3004 hundred-thousandths. |

39. Eighty-five thousandths.
40. Four hundred and seven thousandths.
41. Ninety-five ten-thousandths.
42. Six hundred and forty-four ten-thousandths.
43. Seven thousand and eighty-two ten-thousandths.
44. Fifty-seven hundred-thousandths.
45. Seven hundred and eight hundred-thousandths.

46. Nine thousand and forty-eight hundred-thousandths.
47. Six hundred and four millionths.
48. Seven thousand six hundred and forty-three millionths.
49. Forty thousand and sixty-three millionths.

**110.** An integer and a decimal may be written together as one number, as $6\frac{5}{10}$ or 6.5; $25\frac{7}{100}$ or 25.07. In reading such mixed decimal numbers, the integer and the decimal are connected by *and*. Thus, 4.5 is read 4 *and* 5 tenths.

50. Read 45.6; 30.25; 204.045; 84.0307.
51. Read 2005.045; 408.00075; 3040.0046; 50060.00705.
52. Read 400.045; 500.0063; 7000.0084; 60000.00006.

SUGGESTION.—In such cases read the integer as units; as, four hundred *units* and forty-five thousandths. The omission of the word units changes the mixed number to a pure decimal.

53. Read 5600.0084; 40508.0307; 75000.000605.
54. Read 300000.000003; 35000000.000035.
55. Write decimally $56\frac{3}{100}$; $604\frac{35}{1000}$; $400\frac{305}{1000000}$.
56. Write decimally $207\frac{35}{1000000}$; $2560\frac{4056}{10000000}$.
57. Three hundred units and three hundred and forty-eight millionths.

## DEFINITIONS, PRINCIPLES, AND RULES.

**111.** A ***Decimal Fraction*** is a fraction whose denominator is some power of ten.

The word *decimal* is derived from *decem*, a Latin word meaning *ten*. It is applied to this class of fractions because the successive fractional units or orders decrease *tenfold*, or on the scale of ten.

NOTE.—The powers of ten are 10, 100, 1000, etc. (Art. 388.)

**112.** Decimal fractions may be expressed in three ways:

1. By words; as, three tenths, twelve hundredths.
2. By writing the denominator under the numerator, in the form of a common fraction; as, $\frac{3}{10}$, $\frac{12}{100}$.

3. By omitting the denominator and writing the fraction in a decimal form; as, .3, .012. The denominator is understood.

Three tenths, $\frac{3}{10}$, and .3, each express the same *decimal* fraction, but the term decimal is usually applied to decimal fractions when expressed by the third method. Since common fractions may have 10, 100, etc., for a denominator, it follows that decimal fractions are a class of common fractions.

**113.** The ***Decimal Point*** is a period placed at the left of the order of tenths, to designate the decimal orders.

**114.** A ***Mixed Decimal*** is a decimal ending at the right with a common fraction; as, $.6\frac{2}{3}$, $.033\frac{1}{3}$.

**115.** A ***Mixed Decimal Number*** is an integer and a decimal written together as one number. It is called more simply a *Mixed Number.*

The orders on the left of the decimal point are *integral,* and those on the right are *decimal.* The decimal orders are called *Decimal Places.*

**116.** The following table gives the names of a few integral and decimal orders, and shows the relation between them:

| Hundred-millions. | Ten-millions. | Millions. | Hundred-thousands. | Ten-thousands. | Thousands. | Hundreds. | Tens. | Units. | *Decimal point.* | Tenths. | Hundredths. | Thousandths. | Ten-thousandths. | Hundred-thousandths. | Millionths. | Ten-millionths. | Hundred-millionths. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | . | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| *Integral Orders.* | | | | | | | | | | *Decimal Orders.* | | | | | | | |

**117.** PRINCIPLES.—1. *The denominator of a decimal fraction is* 1 *with as many ciphers annexed as there are decimal places in the fraction.*

2. *The value of the successive decimal orders decreases tenfold from left to right, and increases tenfold from right to left.* Hence,

3. *The removal of a decimal figure one place to the right* DECREASES *its value tenfold, and its removal one place to the left* INCREASES *its value tenfold.*

4. *The name of a decimal is the same as the name of its right-hand order.* Hence,

5. *A decimal is read precisely as it would be were the denominator expressed.*

**118.** RULES.—1. To read a decimal, *Read it as though it were an integer, and add the name of the right-hand order.*

2. To write a decimal, *Write it as an integer, and so place the decimal point that the right-hand figure shall stand in the order denoted by the name of the decimal.*

NOTE.—When the number does not fill all the decimal places, supply the deficiency by prefixing decimal ciphers.

### WRITTEN PROBLEMS.

Express decimally

58. Two hundred and five ten-thousandths.

59. Forty thousand and thirty-four millionths.

60. Two thousand and four hundred-thousandths.

61. Six hundred and fifteen ten-millionths.

62. Six hundred units and fifteen ten-thousandths.

63. Fifteen and fifteen thousandths.

64. Three hundred thousand three hundred and three hundred-millionths.

65. Five million and eighty-five ten-millionths.

66. Twelve hundred-thousandths.

67. Four hundred units and four hundred and sixty-five millionths.

68. Twenty-five and twenty-five thousandths.

69. Five thousand units and five thousandths.

70. Three hundred and seventy-five and three hundred and seventy-five billionths.

71. Thirty thousand and forty-six hundred-thousandths.

72. One million and forty-five billionths.

73. Eighty thousand and forty and three hundred and six ten-thousandths.

74. Fifteen thousand units and fifteen ten-thousandths.

75. Seventy-five and five thousand and forty-three millionths.

76. One million units and one millionth.

## REDUCTION OF DECIMALS.

### Case I.

**Decimals reduced to Lower or Higher Orders.**

1. How many tenths in 6 units? In 15 units? In 24 units?

2. How many hundredths in 5 tenths? In .6? .8? .7?

3. How many thousandths in .06? In .24? .47? .55?

4. How many tenths in .60? In .70? .90? .600? .700? .800? .5000? 1.50?

5. How many hundredths in .240? In .420? .560? .4500? .8500? .35000? .0700?

#### WRITTEN PROBLEMS.

6. Reduce .875 to millionths.

PROCESS: .875 = .875000

7. Reduce .0674 to ten-millionths.

8. Reduce .075 to hundred-thousandths.

9. Reduce 62.7 to thousandths.

10. Reduce 5.33 to ten-thousandths.

11. Reduce 3. to hundredths.

12. Reduce 45. to ten-thousandths.

13. Reduce .04500 to thousandths.

PROCESS: .04500 = .045

14. Reduce 5.24000 to hundredths.

**119.** Principles.—1. *Annexing ciphers to a decimal fraction multiplies both of its terms by the same number, and hence does not change its value.* (Art. 85.)

2. *Cutting off ciphers from the right of a decimal fraction divides both of its terms by the same number, and hence does not change its value.* (Art. 81.)

Note.—The annexing of decimal ciphers to an integer does not change its value. Thus, 12. = 12.0, or 12.00; that is, 12 units = 120 tenths = 1200 hundredths, etc.

## Case II.

### Decimals reduced to Common Fractions.

15. How many fifths in $\frac{4}{10}$? $\frac{6}{10}$? .2? .8?

16. How many fourths in $\frac{25}{100}$? $\frac{50}{100}$? $\frac{75}{100}$? .25? .50? .75?

17. How many twentieths in $\frac{15}{100}$? $\frac{85}{100}$? .20? .25? .55? .75? .95?

### WRITTEN PROBLEMS.

18. Reduce .625 to a common fraction in its lowest terms.

Process: $.625 = \frac{625}{1000} = \frac{25}{40} = \frac{5}{8}$, *Ans.*

Reduce to common fractions in lowest terms

| | | |
|---|---|---|
| 19. .125 | 25. .004 | 31. 62.025 |
| 20. .75 | 26. .5625 | 32. 37.625 |
| 21. .075 | 27. .0125 | 33. $56.37\frac{1}{2}$ |
| 22. .0625 | 28. .3525 | 34. $247.33\frac{1}{3}$ |
| 23. .1625 | 29. 3.525 | 35. $16.66\frac{2}{3}$ |
| 24. .2250 | 30. 37.75 | 36. $214.00\frac{1}{4}$ |

**120.** Rule.—To reduce a decimal to a common fraction, *Omit the decimal point and supply the denominator, and then reduce the common fraction to its lowest terms.*

Note.—When the denominator is written the fraction is both decimal and common.

## Case III.

### Common Fractions reduced to Decimals.

37. How many tenths in $\frac{1}{2}$? In $\frac{1}{5}$? $\frac{2}{5}$? $\frac{3}{5}$?
38. How many hundredths in $\frac{1}{4}$? $\frac{3}{4}$? $\frac{3}{5}$? $\frac{4}{5}$?
39. How many hundredths in $\frac{1}{20}$? $\frac{3}{20}$? $\frac{7}{20}$? $\frac{11}{20}$?
40. How many hundredths in $\frac{1}{25}$? $\frac{4}{25}$? $\frac{8}{25}$? $\frac{12}{25}$?

#### WRITTEN PROBLEMS.

41. Change $\frac{3}{125}$ to a decimal.

PROCESS.

```
125 ) 3.00 ( .024, Ans.
      2 50
        500
        500
```

Since $\frac{3}{125} = \frac{1}{125}$ of 3, and since $3 = 3.000$ (Art. 119, Note), $\frac{1}{125}$ of $3 = \frac{1}{125}$ of $3.000 = .024$. Or, $\frac{3}{125} = \frac{1}{125}$ of 3 units, and 3 units $= 3000$ thousandths, and $\frac{1}{125}$ of 3000 thousandths $= 24$ thousandths $= .024$.

Reduce to decimal fractions

| | | | |
|---|---|---|---|
| 42. $\frac{5}{8}$ | 48. $\frac{32}{25}$ | 54. $\frac{13}{40}$ | 60. $12\frac{3}{20}$ |
| 43. $\frac{9}{16}$ | 49. $\frac{87}{24}$ | 55. $\frac{7}{400}$ | 61. $25\frac{4}{125}$ |
| 44. $\frac{3}{75}$ | 50. $\frac{12}{125}$ | 56. $\frac{23}{250}$ | 62. $37\frac{13}{80}$ |
| 45. $\frac{25}{32}$ | 51. $\frac{3}{40}$ | 57. $\frac{4}{1250}$ | 63. $\frac{5}{600}$ |
| 46. $\frac{64}{125}$ | 52. $\frac{7}{80}$ | 58. $\frac{1}{750}$ | 64. $\frac{23}{300}$ |
| 47. $\frac{80}{125}$ | 53. $\frac{19}{200}$ | 59. $\frac{21}{480}$ | 65. $\frac{14}{111}$ |

**121.** RULE.—To reduce a common fraction to a decimal, *Annex decimal ciphers to the numerator and divide by the denominator, and point off as many decimal places in the quotient as there are annexed ciphers.*

NOTES.—1. When a sufficient number of decimal places is obtained, the remainder may be discarded, or the quotient may be expressed as a mixed decimal.

2. When the denominator of a common fraction in its lowest terms contains other prime factors than 2 and 5, the process will not terminate.

3. When the quotient repeats the same figure, or the same set of figures, as in problems 63, 64, and 65, it is called a *Repeating Decimal*, or a *Circulating Decimal*, and the figure or figures repeated are called a *Repetend*. (Art. 431.)

## ADDITION OF DECIMALS.

1. Add 16.25, 48.037, 90.0033, and .864.

PROCESS.

| |
|---:|
| 16.25 |
| 48.037 |
| 90.0033 |
| .864 |
| 155.1543, *Ans.* |

Since only like orders can be added (Art. 27), write the figures of the same order in the same column. Since ten units of any order make one unit of the next higher order, begin at the right and add as in simple numbers. Place the decimal point at the left of the 1 tenth.

2. Add .375, 80.06, 45.0084, .00755, and 84.635.

3. Add 84.08, 16.075, 2.9, 1.96, 1.003, and 5.0008.

4. Add \$15.34, \$65.048, \$9.083, \$12., \$16.66$\frac{2}{3}$, \$18.06, \$95.37$\frac{1}{2}$, and \$35.75.

5. Add 26.37$\frac{1}{2}$, 19.08$\frac{1}{4}$, 23.042$\frac{1}{5}$, 38.5, 6.00$\frac{3}{4}$, and 7$\frac{1}{12}$.

6. Add 256 thousandths, 3005 millionths, 207 ten-thousandths, 34 ten-millionths, and 94 hundred-thousandths.

7. Add fifteen thousandths, eighty-one ten-thousandths, fifty-six millionths, seventeen ten-millionths, and two hundred and five hundred-thousandths.

8. How many rods of fence will inclose a field, the four sides of which are respectively 46.6 rd., 50.65 rd., 24.33$\frac{1}{3}$ rd., and 27 rd.?

9. Five bars of silver weigh respectively .75 lb., 1.15 lb., .86$\frac{1}{3}$ lb., 1.34 lb., and .9 lb.: what is their total weight?

10. The average amount of rain in San Francisco in the winter months is 11.25 inches; in the spring, 8.81 inches; in the summer, .03 inches; and in the autumn, 2.75 inches: what is the amount for the year?

**122.** RULES.—To add decimals, 1. *Write the numbers so that figures of the same order shall stand in the same column.*

2. *Add as in the addition of integers, and place the decimal point at the left of the tenths' order in the amount.*

NOTE.—If a mixed decimal does not contain as many decimal places as either of the other numbers, change the terminal common fraction to a decimal, and continue the division until the requisite number of decimal places is secured.

## SUBTRACTION OF DECIMALS.

1. From 47.625 take 28.7.

| 1st Process. | 2d Process. |
|---|---|
| 47.625 | 47.625 |
| 28.700 | 28.7 |
| 18.925 | 18.925 |

2. From 46.7 take 29.825.

| 1st Process. | 2d Process. |
|---|---|
| 46.700 | 46.7 |
| 29.825 | 29.825 |
| 16.875 | 16.875 |

Reduce the decimals to a like order (Art. 119), and since units can only be taken from like units, write the numbers so that figures of the same order shall stand in the same column; and since ten units of any decimal order make one unit of the next higher order, subtract as in simple numbers. Place the decimal point at the left of the tenths' order.

Note.—A comparison of the two processes shows that it is unnecessary to fill the vacant orders with ciphers.

3. From 4.05 take 2.0075.
4. From .6¼ take .0087½.
5. From 12. take .0005.
6. From six tenths take six thousandths.
7. From forty-four thousandths take forty-four millionths.
8. From 301 ten-thousandths take 4005 millionths.
9. From 50065 ten-millionths take 1307 billionths.
10. A man walked 33.7 miles the first day and 28.75 miles the second: how much farther did he walk the first day than the second?
11. The average amount of rain at Cincinnati in the summer months is 13.7 inches, and in the winter months it is 11.15 inches: what is the difference?
12. The mean height of the barometer at Boston is 29.934 inches, and at Pekin it is 30.154 inches: what is the difference?

**123.** Rules.—To subtract decimals, 1. *Write the numbers so that figures of the same order shall stand in the same column.*

2. *Subtract as in the subtraction of integers, and place the decimal point at the left of the tenths' order in the remainder.*

## MULTIPLICATION OF DECIMALS.

1. How much is 7 times $\frac{1}{10}$? 7 times $\frac{4}{10}$? 8 times $\frac{7}{10}$?
2. How much is 8 times $\frac{1}{100}$? 8 times $\frac{7}{100}$? 6 times $\frac{9}{100}$?
3. What is the product of $\frac{1}{10} \times \frac{1}{10}$? $\frac{5}{10} \times \frac{6}{10}$? $\frac{8}{10} \times \frac{9}{10}$?
4. What is the product of $\frac{1}{10} \times \frac{1}{100}$? $\frac{4}{10} \times \frac{7}{100}$?
5. What is the product of $\frac{1}{100}$ by $\frac{1}{100}$? $\frac{7}{100}$ by $\frac{5}{100}$?
6. What is the denominator of the product when tenths are multiplied by units? Tenths by tenths? Tenths by hundredths? Hundredths by hundredths? Hundredths by thousandths?
7. What is the denominator of the product of any two fractions whose denominators are powers of 10?

### WRITTEN PROBLEMS.

8. Multiply .625 by .23.

PROCESS.

```
 .625
  .23
 1875
1250
.14375
```

Since $.625 = \frac{625}{1000}$, and $.23 = \frac{23}{100}$, $.625 \times .23 = \frac{625}{1000} \times \frac{23}{100} = \frac{14375}{100000} = .14375$. Hence, $.625 \times .23 = .14375$. Since thousandths multiplied by hundredths produce *hundred-thousandths*, the product contains *five* decimal places, or as many as both of the factors.

Multiply

| | | |
|---|---|---|
| 9. 6.5 by .75 | 14. 4.36 by .27 | 19. .085 by 30. |
| 10. .043 by 6.5 | 15. 64. by .032 | 20. 2.56 by 250. |
| 11. .0432 by 5.4 | 16. 30.3 by .018 | 21. 3.24 by $.33\frac{1}{3}$ |
| 12. .048 by 24. | 17. .056 by 24. | 22. 5.75 by $8\frac{3}{5}$ |
| 13. 5.6 by .056 | 18. 50. by .08 | 23. $16\frac{3}{4}$ by .045 |

24. Multiply sixteen thousand by sixteen thousandths.
25. Multiply 205 millionths by 46 thousandths.
26. Multiply 6.25 by 10. By 100.

PROCESS.

$6.25 \times 10 = 62.5$
$6.25 \times 100 = 625.$

Since the removal of a decimal figure one place to the left increases its value tenfold (Art. 117, Pr. 3), the removal of the decimal point one place to the right multiplies 6.25 by 10, and the removal of the point two places to the right multiplies by 100.

27. Multiply 3.406 by 100. By 1000.
28. Multiply .00048 by 1000. By 100000.
29. Multiply .0000256 by 10000. By 1000000.

### PRINCIPLES AND RULES.

**124.** PRINCIPLES.—1. *The number of decimal places in the product equals the number of decimal places in both factors.*

2. *Each removal of the decimal point one place to the right, multiplies the decimal by* 10.

**125.** RULES.—1. To multiply one decimal by another, *Multiply as in the multiplication of integers, and point off as many decimal places in the product as there are decimal places in both multiplicand and multiplier.*

NOTE.—If there be not enough decimal figures in the product, supply the deficiency by prefixing decimal ciphers.

2. To multiply a decimal by 10, 100, 1000, etc., *Remove the decimal point as many places to the right as there are ciphers in the multiplier.*

NOTE.—If there be not enough decimal places in the product, supply the deficiency by annexing ciphers.

## DIVISION OF DECIMALS.

1. How many times are 5 tenths contained in 10 tenths? 7 tenths in 35 tenths?

2. How many times are 7 hundredths contained in 21 hundreths? 7 hundredths in 35 hundredths?

3. What is $\frac{9}{10} \div \frac{3}{10}$? $\frac{27}{100} \div \frac{9}{100}$? $\frac{75}{1000} \div \frac{25}{1000}$?

4. What is .8 ÷ .4? .21 ÷ .07? .084 ÷ .012?

5. What is $\frac{3}{10} \div \frac{6}{100}$? $\frac{12}{100} \div \frac{6}{1000}$? $\frac{15}{100} \div \frac{15}{1000}$?

SUGGESTION.—Reduce the fractions to a common denominator.

6. What is .3 ÷ .15? .25 ÷ .125? .12 ÷ .012?

7. Of what order is the quotient when tenths are divided by tenths? Hundredths by hundredths? Thousandths by thousandths?

8. Of what order is the quotient when any order is divided by a like order? When any number is divided by a like number?

## WRITTEN PROBLEMS.

9. Divide 8.05 by .35

PROCESS.

```
.35) 8.05 ( 23., Ans.
     7 0
     1 05
     1 05
```

35 hundredths are contained in 805 hundredths, a like number, 23 times, and hence $8.05 \div .35 = 23$. *The quotient is units.*

10. Divide 80.5 by .35

PROCESS.

```
.35 ) 80.50 ( 230., Ans.
      70
      10 5
      10 5
         0
```

By annexing a decimal cipher to 80.5, which does not change its value (Art. 119), the dividend and divisor are made like numbers, and hence *their quotient is units.* $80.50 \div .35 = 230$.

11. Divide .805 by .35

PROCESS.

```
.35 ) .805 ( 2.3, Ans.
       70
       105
       105
```

Since .35 and .80, the first partial dividend, are like numbers, the first quotient figure (2) denotes *units;* and if the first figure denotes units, the second must denote tenths. Hence, $.805 \div .35 = 2.3$.

The pointing in all the cases in the division of decimals, may also be explained on the principle, that the dividend is the product of the divisor and quotient, and hence it *must contain as many decimal places as both divisor and quotient.*

In the 9th example, the divisor and dividend contain an equal number of decimal places, and hence there are no decimal places in the quotient.

In the 10th example, the divisor contains one more decimal place than the dividend, and hence a decimal place must be added to the dividend before the division is possible.

In the 11th example, the divisor contains *two* decimal places and the dividend *three*, and hence the quotient contains *one* decimal 'ace.

Divide

| | | | |
|---|---|---|---|
| 12. | 32.4 by 1.8 | 25. | 6.241 by .0079 |
| 13. | 2.56 by .64 | 26. | 67.5 by .075 |
| 14. | .288 by .036 | 27. | .675 by 75. |
| 15. | 82.5 by 2.75 | 28. | 6.75 by 750. |
| 16. | 62.5 by .025 | 29. | 256. by .075 |
| 17. | 9. by .45 | 30. | .256 by 250. |
| 18. | 4.53 by .0302 | 31. | .0025 by 50. |
| 19. | .3 by .0125 | 32. | 25. by .00125 |
| 20. | .625 by 12.5 | 33. | .001 by 100. |
| 21. | .0256 by .32 | 34. | 100 by .001 |
| 22. | 17.595 by 8.5 | 35. | .045 by 900. |
| 23. | 3.3615 by 12.45 | 36. | \$13.50 by \$.37½ |
| 24. | .031812 by 4.82 | 37. | \$12. by \$.06¼ |

38. Divide twenty-four thousandths by sixteen millionths.
39. Divide seventy-eight by thirty-four thousandths.
40. Divide fifteen millionths by six hundredths.
41. Divide 45.7 by 10. By 100.

PROCESS.

$45.7 \div 10 = 4.57$
$45.7 \div 100 = .457$

Since the removal of a decimal figure one place to the right decreases its value tenfold (Art. 117, Pr. 3), the removal of the decimal point one place to the *left* divides a decimal by 10, and the removal of the point two places to the left divides it by 100.

42. Divide 483.75 by 100. By 1000.
43. Divide 54.50 by 100. By 10000.
44. Divide .005 by 1000. By 100.

## PRINCIPLES AND RULES.

**126.** PRINCIPLES.—1. *Since the dividend is the* PRODUCT *of the divisor and quotient, it contains as many decimal places as both divisor and quotient.* Hence,

2. *The quotient must contain as many decimal places as the number of decimal places in the dividend exceeds the number of decimal places in the divisor.* Hence,

3. *When the divisor and dividend contain the same number of decimal places, the quotient is units.*

4. *The dividend must contain as many decimal places as the divisor before division is possible.*

5. *Each removal of the decimal point one place to the left divides a decimal by* 10.

**127.** RULES.—1. To divide one decimal by another, *Divide as in the division of integers, and point off as many decimal places in the quotient as the number of decimal places in the dividend exceeds the number in the divisor.*

NOTES.—1. When the divisor contains more decimal places than the dividend, supply the deficiency in the dividend by *annexing* decimal ciphers.

2. When the quotient has not enough decimal figures, supply the deficiency by *prefixing* decimal ciphers.

3. When there is a remainder, the division may be continued by annexing ciphers, each cipher thus annexed adding one decimal place to the dividend. Sufficient accuracy is usually secured by carrying the division to four or five decimal places.

2. To divide a decimal by 10, 100, 1000, etc., *Remove the decimal point as many places to the left as there are ciphers in the divisor.*

### REVIEW PROBLEMS.

1. Reduce $\frac{3}{125}$ to a decimal.
2. Reduce $\frac{7}{2500}$ to a decimal.
3. Change .325 to a common fraction.
4. Change .0045 to a common fraction.
5. From the sum of 67.5 and .54 take their difference.
6. From the sum of 64.5 and .015 take their product.
7. Multiply $6.25 + .075$ by $6.25 - .075$.
8. Divide .0512 by $.032 \times .005$.
9. From $25.6 \div .064$ take $32.4 \times .015$.
10. What is the value of $\$5.33 \times 2.5 \div .075$?
11. What is $.08\frac{1}{4} \times 1.2\frac{1}{2} \div .006\frac{1}{4} \times .016$?
12. Multiply 15 millionths by 7 million.
13. Divide 16 ten-millionths by 25 thousandths.

Divide 205 millions by 41 ten-thousandths.

# SECTION X.

# UNITED STATES MONEY.

## PRELIMINARY DEFINITIONS.

**128.** ***United States Money*** is the legal currency of the United States. It is also called *Federal Money*.

**129.** The denominations used in business and accounts, are *dollars, cents*, and *mills*. A dollar equals 100 cents, and a cent equals 10 mills.

The figures denoting dollars are separated from those denoting cents by a period, called a *Separatrix* or *Decimal Point*, and they are preceded by the character, $, called the *Dollar Sign*.

**130.** The first two figures at the right of dollars denote *cents*, and the third figure denotes *mills*. The two figures denoting cents express *hundredths* of a dollar, and the figure denoting mills expresses *tenths* of a cent, or *thousandths* of a dollar. The three figures denoting cents and mills may be read together as so many *thousandths* of a dollar.

NOTES.—1. United States Money consists of *Coin* and *Paper Money*. Coin is called Specie Currency or *Specie*, and paper money is called *Paper Currency*.

2. The principal gold coins are the fifty-dollar piece, double eagle ($20), eagle ($10), half-eagle, quarter-eagle, three-dollar piece, and dollar.

The silver coins are the dollar, half-dollar, quarter-dollar, dime, half-dime, and three-cent piece.

The nickel coins are the five-cent piece, three-cent piece, and cent.

The copper coins (old) are the two-cent piece and cent.

3. Gold and silver coins are alloyed, to make them harder and more durable. The gold coins contain 9 parts of gold and 1 part of an alloy, composed of copper and silver; and the silver coins, except the three-cent pieces, contain 9 parts of silver and 1 part of copper. Nickel and copper coins are made of pure metal.

4. Paper money consists of notes issued by the United States, called *Treasury Notes*, and bank notes issued by banks.

5. Treasury notes of a value less than $1, as fifty cents, twenty-five cents, fifteen cents, ten cents, five cents, and three cents, are called *Fractional Currency*.

## 131. NOTATION AND REDUCTION.

1. Express in words, $75.50; $105.08; $1000.45; $15080.; $.87; $.375; $5.

2. Express in words, $37.507; $250.075; $80.005; $.075; $2080.375; $100.058; $.065.

3. Read decimally, $70.25; $140.05; $387.60; $560.09; $84.37; $.08.

4. Read decimally, $.255; $16.455; $300.056; $475.005; $1005.375; $240.061; $.005.

### WRITTEN PROBLEMS.

5. Write, in figures, ten dollars fifty cents.
6. Write forty dollars sixty cents five mills.
7. Write 100 dollars 37 cents 4 mills.
8. Write 430 dollars 5 cents; 25 dollars 5 mills.
9. Write 75 cents 6 mills; 6 cents 5 mills
10. Write 10 mills; 10 cents 4 mills.
11. How many cents in $25? $100? $350?
12. How many mills in $47? $150? $165?
13. How many mills in $.75? $.625? $.017?
14. How many cents in $5.37? $16.85? $40.08?
15. How many mills in $.37½? $4.62½? $10?
16. Reduce 1500 cents to dollars.
17. Reduce 15000 mills to dollars.
18. Reduce 450 mills to cents.
19. Reduce $25.08 to mills.
20. Reduce $100.01 to cents; to mills.

## ADDITION AND SUBTRACTION.

1. A man paid $7.50 for a pair of boots, and $5.50 for a hat: how much did he pay for both?

2. A lady paid $15 for a shawl, $5.75 for a hat, $2.25 for a pair of gloves, and $4 for a pair of gaiters: what was the amount of her purchases?

3. A drover bought cows at $36.50 a head, and sold them at $40 a head: how much did he gain?

4. A man bought a coat for $24.25, and a vest for $4.50, and handed the merchant three $10 bills: how much money did he receive back?

5. A mechanic earns $20 a week, and his family expenses amount to $16.75 a week: how much has he left?

6. A bookseller bought a set of maps for $17, and a set of charts for $6.50, and sold both sets for $28.50: how much did he gain?

### WRITTEN PROBLEMS.

7. What is the sum of $.65, $15.44, $60.62½, $100, $94.05, and $.87½?

8. From $100.15 take $62.37½.

9. To the sum of $308.60 and $190.125 add their difference.

10. From the sum of $2750. and $1680.62½ take their difference.

11. A merchant's sales for a week were as follows: Monday, $125.60; Tuesday, $98.50; Wednesday, $190.30; Thursday, $215.; Friday, $175.80; Saturday, $247.90: what was the amount of his sales for the week?

12. A man exchanged three city lots, valued respectively at $900, $1200, and $750, for a farm valued at $3075, paying the difference in money: how much money did he pay?

13. A man receiving a salary of $1600 a year, pays $325 for house rent, $450.80 for provisions, $200.60 for clothing, and $245 for all other expenses: how much has he left?

14. A man deposits in a bank, at different times, \$75, \$230.80, \$180.40, and \$95, and he draws out \$40, \$87.50, \$331.45, \$20.15, and \$18.60: what is his bank balance?

**132.** RULE.—To add or subtract sums of money, *Write units of the same denomination in the same column, add or subtract as in simple numbers, and separate dollars and cents by a period, and prefix the dollar sign.*

## MULTIPLICATION AND DIVISION.

1. A mechanic earns \$2.50 a day: how much will he earn in 6 days? 10 days?
2. What will 8 barrels of flour cost, at \$7.25 a barrel? At \$6.50 a barrel?
3. What will 20 yards of carpeting cost, at \$1.75 a yard? At \$2.25 a yard?
4. A drover paid \$38.70 for 9 sheep: what did they cost apiece?
5. A man paid \$42 for 8 tons of coal: what did it cost per ton?
6. If a man earn \$39 in 6 days: how much will he earn in 10 days? In 20 days?
7. At 25 cents a dozen, how many dozens of eggs can be bought for \$4.50?

### WRITTEN PROBLEMS.

8. A farmer sold 45 hogs at \$22.45 apiece: how much did he receive for them?
9. A miller sold 237 pounds of flour, at \$7.62½ a barrel: what amount did he receive?
10. A man sold a farm of 260 acres, at \$33⅓ per acre: what was the amount received?
11. A farm containing 125 acres was sold for \$5093.75: what was the price per acre?
12. How many carriages, at \$125 apiece, can be bought for \$8000? For \$7500?

13. At $\$12.37\frac{1}{2}$ a ton, how many tons of hay can be bought for $4653? For $1163.25?

14. A farmer sold 3 hogs, weighing respectively 278, 309, and 327 pounds, at $\$.07\frac{1}{2}$ a pound: how much did he receive?

15. A farmer sold in one year 536 pounds of butter, at 30 cts. a pound; 1200 pounds of cheese, at $16\frac{2}{3}$ cts.; and 19 tons of hay, at $8.75 a ton: how much did he receive?

16. A grocer bought 540 pounds of coffee for $81, and 420 pounds of tea for $525; he sold the coffee at 18 cts. a pound, and the tea at $1.60 a pound: how much did he gain?

**133.** RULES.—1. To multiply or divide sums of money by an abstract number, *Multiply or divide as in simple numbers, separate dollars and cents in the result by a period, and prefix the dollar sign.*

2. To divide one sum of money by another, *Reduce both numbers to the same denomination, and divide as in simple numbers.*

## ABBREVIATED METHODS.

### LEDGER COLUMNS.

**134.** A ***Ledger*** is a book in which business men keep a summary of accounts.

The items on a ledger page often make long columns of figures, which are added or footed with absolute accuracy.

**135.** Let the pupil foot the following ledger columns by adding two or more columns at once, being as careful to obtain accurate results as he would be in actual business. (See Art. 22.)

| (1) | (2) | (3) | (4) |
|---|---|---|---|
| $1.25 | $19.50 | $75.50 | $1912.88 |
| 8.14 | 20.00 | 184.30 | 806.40 |
| 2.75 | 12.45 | 111.10 | 1000.00 |
| .65 | 14.52 | 43.95 | 1250.86 |
| .75 | 25.48 | 263.55 | 943.82 |
| 8.37 | 40.50 | 100.00 | 607.55 |
| 12.50 | 8.60 | 90.00 | 400.33 |
| 4.65 | 9.35 | 7.15 | 148.67 |
| .83 | .65 | 13.48 | 249.50 |
| 7.16 | .73 | 2.75 | 2040.00 |
| 10.28 | .84 | 52.30 | 4508.70 |
| 1.20 | 12.10 | 900.25 | 3406.30 |
| .95 | .86 | 625.80 | 1280.75 |
| .48 | .93 | 314.87 | 1300.00 |
| 13.47 | 2.95 | 64.50 | 877.77 |
| 23.00 | 14.63 | 49.87 | 620.14 |
| 3.08 | 9.82 | 302.58 | 8.60 |
| 6.15 | 12.60 | 10.10 | 7.45 |
| 24.92 | 19.30 | 100.98 | 13.33 |
| .83 | 22.33 | 78.60 | 286.45 |
| .92 | 9.81 | 44.50 | 1300.80 |
| .45 | 8.76 | 77.88 | 1440.00 |
| 14.86 | 12.57 | 320.65 | 986.70 |
| 5.80 | 18.19 | 19.10 | 87.80 |
| 7.26 | 7.63 | 8.50 | 137.40 |
| 12.00 | 14.60 | 436.75 | 1500.00 |
| 5.00 | 4.85 | 135.20 | 885.73 |
| 4.37 | 9.63 | 44.88 | 236.40 |
| 6.45 | 12.83 | 65.90 | 13483.86 |
| 17.83 | 18.10 | 6.01 | 11456.20 |
| 2.65 | 7.63 | 7.83 | 88.00 |
| 1.50 | 2.20 | 4.22 | 24.30 |
| .85 | .35 | 3.25 | 16.50 |
| 12.20 | .75 | .85 | 9.85 |
| 4.65 | 8.50 | .62 | 100.00 |
| 8.15 | 4.65 | 1.25 | 40.60 |

SUGGESTION.—The partial footings obtained by each summary, should be written upon a separate piece of paper. This will permit the re-adding of any column or set of columns, as the case may be, without the trouble of re-adding the preceding columns, and it will ·o avoid the defacing of the page by erasures and corrections.

## ALIQUOT PARTS.

**136.** When the price of an article is an aliquot part of a dollar, the cost of any number of such articles may be found more readily than by multiplying.

**137.** The aliquot parts of a dollar commonly used in business, are:

| | |
|---|---|
| 50 cts. = $\frac{1}{2}$ of $1.00 | $12\frac{1}{2}$ cts. = $\frac{1}{8}$ of $1.00 |
| 25 " = $\frac{1}{4}$ of 1.00 | $6\frac{1}{4}$ " = $\frac{1}{16}$ of 1.00 |
| 20 " = $\frac{1}{5}$ of 1.00 | $33\frac{1}{3}$ " = $\frac{1}{3}$ of 1.00 |
| 10 " = $\frac{1}{10}$ of 1.00 | $16\frac{2}{3}$ " = $\frac{1}{6}$ of 1.00 |

The following aliquot parts of aliquot parts of a dollar are frequently used:

| | |
|---|---|
| 25 cts. = $\frac{1}{2}$ of 50 cts. | $16\frac{2}{3}$ cts. = $\frac{1}{2}$ of $33\frac{1}{3}$ cts. |
| $12\frac{1}{2}$ " = $\frac{1}{4}$ of 50 " | $12\frac{1}{2}$ " = $\frac{1}{2}$ of 25 " |
| $6\frac{1}{4}$ " = $\frac{1}{8}$ of 50 " | $6\frac{1}{4}$ " = $\frac{1}{4}$ of 25 " |

### MENTAL PROBLEMS.

1. What will 56 pounds of grapes cost, at $12\frac{1}{2}$ cts. a pound?

SOLUTION.—At $1 a pound, 56 pounds will cost $56, and at $12\frac{1}{2}$ cts., which is $\frac{1}{8}$ of $1, 56 pounds will cost $\frac{1}{8}$ of $56, which is $7.

2. What will 120 spellers cost, at 25 cts. apiece? At $33\frac{1}{3}$ cts.?

3. What is the cost of 96 dozens of eggs, at $16\frac{2}{3}$ cts. a dozen? At 20 cts.? At 25 cts.?

4. What will 240 pounds of sugar cost, at $12\frac{1}{2}$ cts. a pound? At $16\frac{2}{3}$ cts.? At 20 cts.?

5. At $16\frac{2}{3}$ cents a dozen, how many dozens of eggs can be bought for $15?

SOLUTION.—At $16\frac{2}{3}$ cents a dozen, $1 will buy 6 dozens of eggs, and $15 will buy 15 times 6 dozens, or 90 dozens.

6. At $12\frac{1}{2}$ cts. a pound, how many pounds of lard can be bought for $12? For $25?

7. How many pounds of butter, at $33\frac{1}{3}$ cts. a pound, can be bought for $15? For $33?

8. At $6\frac{1}{4}$ cts. a quart, how many quarts of currants can be bought with 30 quarts of cherries, at 10 cts. a quart?

### WRITTEN PROBLEMS.

9. What will 348 yards of carpeting cost, at $1.62$\frac{1}{2}$ cts. a yard?

PROCESS.

$1.62$\frac{1}{2}$ = $1 + 50 cts. + $12\frac{1}{2}$ cts.

| | | |
|---|---|---|
| | $348 | = cost at $1 a yard. |
| $\frac{1}{2}$ | 174 | = " " 50 cts. a yard. |
| $\frac{1}{4}$ | 43.50 | = " " $12\frac{1}{2}$ " " |
| | $565.50 | = " " $1.62$\frac{1}{2}$ " |

10. What will 1600 bushels of oats cost, at $37\frac{1}{2}$ cts. a bushel? At 45 cts. a bushel? At $62\frac{1}{2}$ cts.?

11. What will 2464 bushels of wheat cost, at $1.25 a bushel? At $1.37$\frac{1}{2}$? At $1.62$\frac{1}{2}$?

12. What will 1250 yards of carpeting cost, at $1.37$\frac{1}{2}$ a yard? At $1.50? At $1.87$\frac{1}{2}$?

13. What will 640 bottles of ink cost, at $87\frac{1}{2}$ cents a bottle? At $62\frac{1}{2}$ cts.? At 75 cts.?

14. At 25 cts. a dozen, how many dozens of eggs can be bought for $42? For $105? For $60.50?

15. At $33\frac{1}{3}$ cts. a yard, how many yards of cloth can be bought for $750? For $120?

16. What will 5 lb. 10 oz. of butter cost, at 35 cts. a pound?

PROCESS.

| | |
|---|---|
| $ .35 | = cost of 1 lb. |
| $1.75 | = " " 5 " |
| .175 | = " " 8 oz. ($\frac{1}{2}$ lb.) |
| .044 | = " " 2 " ($\frac{1}{8}$ lb.) |
| $1.969 | = " " 5 lb. 10 oz. |

17. What will 9 lb. 13 oz. of cheese cost, at 15 cts. a pound? At 18 cts.? At 20 cts.?

18. What will 16 gal. 3 qt. of sirup cost, at $1.75 a gallon? At $1.62$\frac{1}{2}$? At $1.90?

19. What will 7 bu. 3 pk. 4 qt. of cherries cost, at $4.25 a bushel? At $3.50? At $4.50?

20. What will 2 pk. 7 qt. of chestnuts cost, at $3.50 a bushel? At $2.75? At $2.62½? At $3.12½?

### DEFINITION AND RULES.

**138.** An ***Aliquot Part*** of a number is any integer or mixed number which will exactly divide it.

**139.** Rules.—1. To find the cost of a number of articles when the price is an aliquot part of a dollar, *Find the cost at* $1, *and take the aliquot parts of the result.*

2. To find the number of articles which can be purchased for a given sum of money when the price is an aliquot part of a dollar, *Find the number of articles that can be purchased for* $1, *and multiply the result by the given sum of money.*

## BILLS.

**140.** Each of the following bills should be neatly made out on paper, in proper form, and receipted.

(1)

Cincinnati, O., Jan. 1, 1870.

Thomas Knight,

*Bought of* Baker, Smith & Co.

| | | | | | |
|---|---|---|---|---|---|
| 1869 | | | | | |
| Nov. 18, | 48 lb. | Castile Soap, | @ 16⅔c. | . . . . | $8.00 |
| " " | 25 " | Starch, | @ 6¼ | . . . . | 1.56 |
| " 30, | 65 " | Sugar, | @ 15 | . . . . | 9.75 |
| " " | 33 gal. | Vinegar, | @ 20 | . . . . | 6.60 |
| Dec. 12, | 16 lb. | Rio Coffee, | @ 23 | . . . . | 3.68 |
| " " | 5 " | Star Candles, | @ 20 | . . . . | 1.00 |
| " " | 56 " | Butter, | @ 33⅓ | . . . . | 18,67 |
| " 15, | 10 " | Cheese, | @ 15 | . . . . | 1.50 |
| | | | | | $50.76 |

*Received Payment,*

Baker, Smith & Co.
*per* Coons.

(2)

ST. LOUIS, March 3, 1870.

JAMES COOPER & BRO.,

*To* CHARLES CAMP & CO., *Dr.*

| | | | | |
|---|---|---|---|---|
| To 37 bls. Flour, Ex., | @ $4.50 | . . . . . | $ | |
| " 23 " " Fy., | @ 5.25 | . . . . | | |
| " 25 " Green Apples, | @ 2.12½ | . . . . . | | |
| " 14 bxs. Lemons, | @ 7.50 | . . . . | | |
| " 5 " Raisins, | @ 4.75 | . . . . . | | |
| | | | $ | |

*Received Payment,*

What is the amount due?

(3)

CLEVELAND, O., Nov. 24, 1869.

DR. WILLIAM JONES,

*To* CHARLES C. WILHELM, *Dr.*

| | | | |
|---|---|---|---|
| To 24 Days' Work, | @ $2.75 | . . . . . | $ |
| " 21 lb. Nails, | @ 6¼ | . . . . | |
| " 540 ft. Pine Lumber, | @ 4.50 per 100 | . . . | |
| " 4 M. Shingles, | @ 8.33⅓ | . . . . | |
| | | | $ |

*Cr.*

| | | |
|---|---|---|
| By Cash, Oct. 16, | . . . . . . . | $25 |
| " " " 23, | . . . . . . | 44 |
| " " Medical Services to date | . . . . | 15 |
| | | $ |

*Received Payment, per due-bill,*

CHARLES C. WILHELM.

What is the amount of the due-bill?

4. Henry Smith bought of John Clarke, of Louisville, Ky., as follows: Mch. 10, 1870, 7 pair calf boots @ $5.75; 6 pair ladies' gaiters @ $3.25; 10 pair children's shoes @ $1.75; Apr. 1st, 12 pair coarse boots @ $3.12½; 6 pair calf

shoes @ \$3.30; Apr. 12, 7 pair ladies' slippers, \$1.33$\frac{1}{3}$; 3 pair calf boots, \$5.62$\frac{1}{2}$. Make out and receipt the above bill as clerk of John Clarke.

5. Robert Sterns & Co. bought of Dudley & Bro., Detroit, Mich., Dec. 20, 1869, as follows: 5 doz. ink-stands @ \$2.12$\frac{1}{2}$; 9 boxes steel pens @ \$.87$\frac{1}{2}$; 8 reams note paper @ \$3.50; 5 dozen spellers @ \$2.33$\frac{1}{3}$; and 2 dozen copy books @ \$1.80. They sold Dudley & Bro. 3 sets outline maps @ \$8.25, and paid them \$15 in money. Make out the above bill and receipt by due-bill.

6. Mrs. C. B. Jones bought of Cole, Steele & Co., of Indianapolis, as follows: Nov. 12, 1869, 23 yds. calico @ 16$\frac{2}{3}$c.; 45 yds. sheeting @ 20c.; Dec. 7th, 12 yds. silk @ \$1.62$\frac{1}{2}$; 8 handkerchiefs @ 45c.; 2 pair kid gloves @ \$1.87$\frac{1}{2}$. Make out and receipt the above bill.

## DEFINITIONS.

**141.** An ***Account*** is a record of business transactions between two parties, with specifications of debts and credits.

The party owing the debts specified, is called the *Debtor*, and the party to whom they are due, is called the *Creditor*.

**142.** A ***Bill*** is a written statement of an account. It is drawn by the creditor against the debtor, and gives the time and place of the transaction, and the names of the parties.

When the debtor has made payments on the account, or has charges against the creditor, such payments or charges are called *Credits*. They are entered as in Bill 3.

**143.** A bill is receipted by writing the words "*Received Payment*" at the bottom, and affixing the creditor's name. This may be done by the creditor, or by a clerk, agent, or any other authorized person.

If the debtor is not able to pay a bill when presented, it may be accepted by writing the word "*Accepted*" across its face, with date and signature. When a bill is paid by a

promissory note or due-bill, the fact may be added to the words "*Received Payment,*" as in Bill 3.

**144.** A ***Bill of Goods*** is a written statement of goods sold, with the amount and price of each article, and the entire cost. It is also called an *Invoice.*

When sales are made at different times, the date is written at the left, as in Bill 1.

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## SECTION XI.

# MENSURATION.

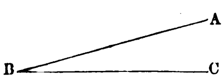
### I. SURFACES.—DEFINITIONS.

**145.** A ***Line*** is length.

**146.** A ***Straight Line*** is a line having the same direction throughout its whole extent.

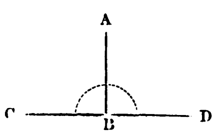
NOTE.—The word line is commonly used to denote a straight line.

**147.** An ***Angle*** is the divergence of two lines meeting at a common point. The point of meeting is called the *vertex.*




Thus the divergence of the lines B A and B C is the angle A B C, and the point B is its vertex.

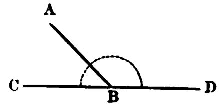
**148.** When a line so meets another line as to make the two adjacent angles equal, each angle is a *Right Angle,* and the first line is *perpendicular* to the second.




Thus the two equal adjacent angles A B C and A B D are right angles, and the line A B is perpendicular to the line C D.

**149.** An ***Obtuse Angle*** is greater than a right angle, and an ***Acute Angle*** is less than a right angle.

Thus the angle A B D is an obtuse angle, and the angle A B C is an acute angle. The line A B is an *oblique line.*

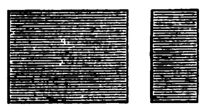



**150.** A ***Surface*** is that which has length and width, but not depth or thickness.

**151.** A ***Plane Surface*** is a surface such that all possible straight lines connecting each two points of it, lie wholly within the surface. It is also called a *Plane.*

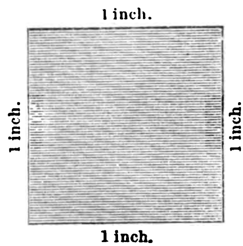
NOTE.—To determine whether the surface of a table is a plane, take a ruler with a straight edge and apply it to the surface in many different directions. If the edge rests uniformly upon the surface, it is a plane.

**152.** A ***Rectangle*** is a plane figure bounded by four straight lines and having four right angles.



**153.** A ***Square*** is a rectangle with its four sides equal.

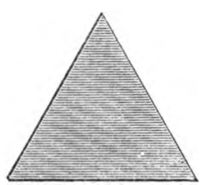
A ***Square Inch*** is a square each side of which is an inch in length.

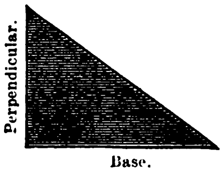



The figure represents a square inch of real size.

A *square foot, square yard, square rod,* etc., are squares whose sides are respectively 1 foot, 1 yard, 1 rod, etc., in length.

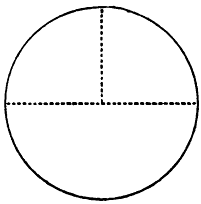
**154.** A ***Triangle*** is a plane figure bounded by three straight lines and having three angles.






**155.** A ***Right-angled Triangle*** is a triangle having a right angle. One of the sides including the right angle is called the *Base*, and the other the *Perpendicular* or *Altitude*.

**156.** A ***Circle*** is a portion of a plane bounded by a curved line, all points of which are equally distant from a point within, called the *center*.



The curved line which bounds a circle is its *Circumference.*

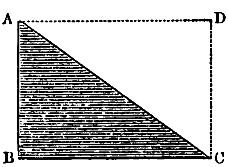
One-half of a circumference is a *Semi-circumference;* one-fourth is a *Quadrant;* and any portion is an *Arc.*

**157.** The ***Diameter*** of a circle is a straight line passing through the center and terminating on both sides in the circumference. One-half of a diameter is a *Radius.*

All the diameters of a circle are equal, and all the radii are equal.

The circumference of a circle is 3.1416 (nearly $3\frac{1}{7}$) times the diameter.

**158.** The ***Area*** of a plane figure is its extent of surface, or superficial contents. It is expressed by some unit of measure as a square inch, a square foot, etc.




**159.** The area of a right-angled triangle is one-half the area of a rectangle with the same base and altitude. The triangle A B C is one-half of the rectangle A B C D.

**160.** The area of a circle equals the product of the circumference by the one-half of the radius.

NOTE.—This may be illustrated by dividing a circle by diameters into eighths, and considering each a *triangle.*

### MENTAL PROBLEMS.

1. How many square inches in a piece of paper 4 inches long and 1 inch wide? 4 inches long and 2 inches wide?

2. How many square feet in a piece of zinc 4 feet long and 3 feet wide? 4 feet long and 4 feet wide?

3. How many square inches in a pane of glass 12 inches square? Then how many square inches in a square foot?

4. How many square feet in a piece of oil-cloth 7 feet long and 3 feet wide? 8 ft. long and 6 ft. wide?

5. How many square feet in a square yard?

6. How many square feet in the floor of a room 20 by 15 ft.? 30 by 24 ft.? 50 by 30 ft.?

NOTE.—The dimensions of a plane figure are usually expressed by writing the word "by," or the sign "×," between the figures denoting the length and width.

7. How many square yards in a pavement 40 by 5 yd.? 50 × 4 yd.? 80 × 5 yd.?

8. How many square miles in a township 5 miles square? 6 miles square?

9. How many square inches in a right-angled triangle, whose base is 8 inches and whose altitude is 6 inches?

10. The diameter of a circle is 10 feet: what is its circumference?

### WRITTEN PROBLEMS.

11. How many square feet in a floor $37\frac{1}{2}$ by 23 ft.?

12. How many square yards in a walk 124.5 by 3.25 yd.?

13. How many square feet in the walls of a room 24 by $18\frac{3}{4}$ ft. and $10\frac{1}{2}$ ft. high? What is the area of the ceiling?

14. How many square chains in a farm 134 chains long and 52.5 chains wide?

15. How many square feet in a city lot $62\frac{1}{2}$ ft. front by 208 ft. deep?

16. A garden containing 3267 square yards is $49\frac{1}{2}$ yards wide: how long is it?

17. A street containing 800 square rods is $33\frac{1}{3}$ rods long: how wide is it?

18. How many yards of carpeting, $\frac{3}{4}$ of a yard wide, will cover a room 15 by $8\frac{1}{4}$ yd.?

19. How many square yards in a triangular garden whose base is 54.5 yards, and altitude 33.2 yards?

20. A triangle contains 270 sq. in., and the base is 36 in.: what is its altitude?

21. The diameter of a circle is 12 inches: how many square inches in its area?

22. How many square feet in a circle whose diameter is 20 feet?

**161.** RULES.—1. To find the area of a rectangle, *Multiply the length by the width.*

2. To find either side of a rectangle, *Divide the area by the other side.*

3. To find the area of a triangle, *Multiply the base by one half the altitude.*

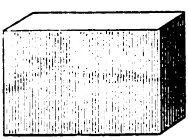
4. To find the area of a circle, *Multiply the circumference by one fourth of the diameter.*

NOTE.—The two dimensions must be expressed in the same denomination.

## II. SOLIDS.—DEFINITIONS.

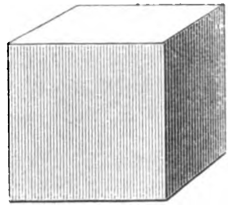
**162.** A ***Solid*** is that which has length, width, and depth or thickness. It is also called a *Volume* or *Body.*

A line has only length; a surface has length and width; and a solid has length, width, and depth.



**163.** A ***Rectangular Solid*** is a body bounded by six rectangular surfaces.

The surfaces bounding a solid are called *Faces*, and the sides of these faces are called *Edges.* A rectangular solid has twelve edges. The face on which a solid is supposed to rest is called its *Base.*



**164.** A ***Cube*** is a body bounded by six equal squares. All its edges are equal.

A ***Cubic Inch*** is a cube whose edges are each one inch in length.

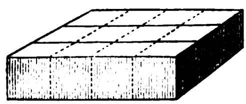
A *cubic foot*, *cubic yard*, *cubic rod*, etc., are each cubes whose edges are respectively 1 foot, 1 yard, 1 rod, etc.

**165.** A ***Cylinder*** is a solid whose two bases are equal and parallel circles.

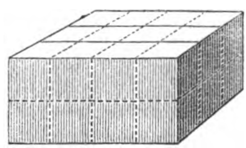
**166.** The volume of a body is called its *Solid Contents*, or *Capacity*. It is expressed in some unit of measure, as a cubic inch, a cubic foot, etc.

### MENTAL PROBLEMS.

1. How many cubic inches in a rectangular solid, 4 inches long, 1 inch wide, and 1 inch thick?



2. How many cubic inches in a rectangular solid, 4 inches long, 3 inches wide, and 1 inch thick?



3. How many cubic inches in a rectangular solid, 4 inches long, 3 inches wide, and 2 inches thick?

4. How many cubic feet in a block of marble 6 ft. long, 3 ft. wide, and 2 ft. thick? 10 ft. long, 5 ft. wide, and 4 ft. thick?

5. How many cubic feet in a cubic yard?

6. How many cubic feet in a bin 6 ft. long, 3 ft. wide, and 3 ft. deep? 8 ft. long, 5 ft. wide, and 2 ft. deep?

7. How many cubic yards in a room 5 yd. long, 4 yd. wide, and 3 yd. high?

## WRITTEN PROBLEMS.

8. How many cubic feet in a block of granite 16 ft. long, 8 ft. wide, and 5 ft. thick?

PROCESS.

$$\begin{array}{r} 16 \text{ cu. ft.} \\ \underline{8} \\ 128 \text{ cu. ft.} \\ \underline{5} \\ 640 \text{ cu. ft.} \end{array}$$

A block 16 ft. long, 1 ft. thick, and 1 ft. wide, contains 16 cu. ft.; and a block 16 ft. long, 1 ft. thick, and 8 feet wide, contains 8 times 16 cu. ft., or 128 cu. ft.; and a block 16 ft. long, 8 feet wide, and 5 ft. thick, contains 5 times 128 cu. ft. = 640 cu. ft. Hence, solid contents = 16 cu. ft. $\times$ 8 $\times$ 5.

9. How many cubic feet in a pile of wood 45 ft. long, $3\frac{1}{2}$ ft. wide, and 7 ft. high?

10. How many cubic yards in a cubic rod?

11. How many cubic feet in a cube each of whose edges is $12\frac{1}{2}$ ft. in length?

12. A building, 65 ft. by 44 ft., has a foundation wall 12 ft. deep and 2 ft. thick: how many cubic feet in the foundations?

13. A pile of wood, containing 840 cu. ft., is 30 ft. long and $3\frac{1}{2}$ ft. wide: how high is the pile?

14. If 27 bricks make a cubic foot, how many bricks will make a wall 45 ft. long, 27 ft. high, and $2\frac{1}{3}$ ft. thick?

15. How many cans, 6 by 4 by 2 in., can be placed in a box 30 by 18 by 20 in the clear?

16. The base of a cylinder is 12 inches in diameter, and its altitude is 25 inches: how many cubic inches in its solid contents?

**167.** RULES.—1. To find the solid contents of a rectangular solid, *Multiply the length, width, and thickness together.*

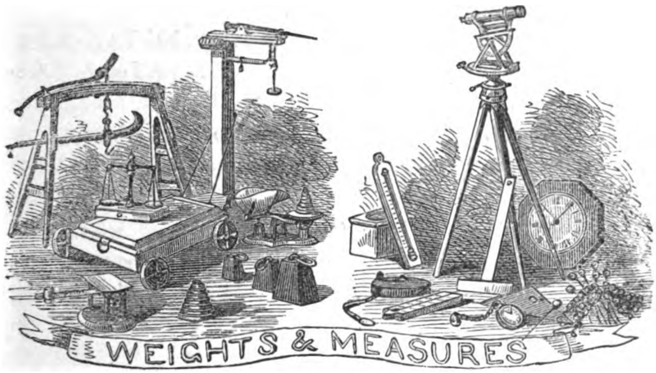
2. To find the length, width, or thickness of a rectangular solid, *Divide the solid contents by the product of the other two dimensions.*

NOTE.—The three dimensions must be expressed in the same denomination.

3. To find the solid contents of a cylinder, *Multiply the area of the base by the altitude.*

# SECTION XII.

# DENOMINATE NUMBERS.




## REDUCTION.

### Case I.

**Reduction of Denominate Integers and Mixed Numbers.**

1. How many mills in 9 cents? In $12\frac{1}{2}$ cents? $62\frac{1}{2}$ cents? 100 cents?

2. How many cents in 7 dimes? $25\frac{1}{2}$ dimes? 45.4 dimes? 56.8 dimes? 75.3 dimes?

3. How many dollars in 50 dimes? 120 dimes? 145 dimes? 1250 dimes? 1625 dimes?

4. How many dollars in 800 cents? 2400 cents? 1365 cents? 2235 cents?

5. How many farthings in 9 pence? 72 pence? $90\frac{1}{2}$ pence? 24.5 pence?

NOTE.—For tables see appendix.

6. How many pence in $8\frac{2}{3}$ shillings? $10\frac{1}{2}$ s.? $33\frac{1}{3}$ s.? 2.5 s.? 6.5 s.?

7. How many shillings in 15 £? 2.5 £? 16.4 £?

8. How many pence in 22 far.? 48 far.? 105 far.? 201 far.?

9. How many pounds in 120 s.? 360 s.? 720 s.?

10. How many shillings in 72 d.? 144 d.? 25.2 d.? 34.8 d.? 52.92 d.? 73.44 d.?

11. How many drams in 8 oz. avoir.? 20 oz.? 4.5 oz.?

12. How many ounces in 5 lb. avoir.? $10\frac{1}{2}$ lb.? 2.5 lb.?

13. How many pounds in 64 oz. avoir.? 19.2 oz.? 4.8 oz.?

14. How many grains in 5 pwt.? $10\frac{1}{2}$ pwt.? 2.5 pwt.?

15. How many pwt. in 7 oz.? 6.5 oz.? $12\frac{2}{5}$ oz.?

16. How many ounces of gold in 7 lb.? $12\frac{2}{3}$ lb.? 1.5 lb.? 4.5 lb.? 12.5 lb.?

17. How many pounds of gold in 48 oz.? 14.4 oz.? 2.52 oz.? 4.68 oz.? 62.4 oz.?

18. How many scruples in 12 ʒ? $8\frac{2}{3}$ ʒ? 14.5 ʒ?

19. How many drams in 15 ℥? $12\frac{3}{4}$ ℥? 11.5 ℥?

20. How many ounces in 9 ℔? 5.5 ℔? 10.5 ℔?

21. How many inches in $8\frac{1}{3}$ ft.? $15\frac{1}{2}$ ft.? $33\frac{1}{3}$ ft.?

22. How many yards in 12 rd.? 1.6 rd.? 3.2 rd.?

23. How many rods in 11 yd.? 33 yd.? 6.6 yd.?

24. How many miles in 18 fur.? 13.6 fur.? 7.2 fur.?

25. How many sq. ft. in $3\frac{1}{3}$ sq. yd.? $16\frac{2}{3}$ sq. yd.?

26. How many square yards in 12.6 sq. ft.? 49.5 sq. ft.? 1.71 sq. ft.? 56.7 sq. ft.?

27. How many quarts in 17 pk.? $12\frac{1}{2}$ pk.? $30\frac{1}{4}$ pk.?

28. How many gallons in 35 qt.? 14.8 qt.? 2.56 qt.?

29. How many weeks in 365 days? 25.2 days?

30. How many years in 192 mo.? 25.2 mo.? 100 mo.?

## WRITTEN PROBLEMS.

31. Reduce 5 £ 6 s. 3 d. to pence; 1275 d. to pounds.

PROCESS:

```
  5 £  6 s. 3 d.
 20
 ----
 106 s.
  12
 ----
1275 d., Ans.
```

PROCESS:

```
12 ) 1275
     ----
20 ) 106  3 d.
     ----
      5 £  6 s.

5 £ 6 s. 3 d., Ans.
```

32. Reduce 38 lb. 11 oz. 7 dr. to drams.
33. Reduce 12 bu. 5 qt. to pints.
34. Reduce 13 mi. 5 fur. 3 yd. to yards.
35. Reduce 11 A. 3 R. 22 P. to perches.
36. Reduce 503 pt. to bushels.
37. Reduce 324 gi. to gallons.
38. Reduce 10280 ft. to miles.
39. Reduce 12460″ to degrees.
40. Reduce 30684 sec. to higher denominations.
41. How many pence in £45? In £237$\frac{2}{3}$?
42. How many perches in 95 A.? 320$\frac{3}{4}$ A.?
43. How many hundred-weight in 4085 oz. avoir.?
44. How many miles in 12840 ft.?
45. Reduce 13 mi. 5$\frac{1}{2}$ fur. to inches.
46. Reduce 113420 inches to miles.
47. Reduce 3450 cubic feet of wood to cords.
48. Reduce 5124 quarts to bushels.
49. Reduce 16 common years to hours.
50. How many seconds were in the year 1868?
51. Reduce 4 common yr. 45 d. to minutes.
52. Reduce 3.7 bushels to pints.
53. Reduce 4.5 rods to feet.
54. Reduce 3.65 lb. Troy to ounces.
55. Reduce 15° 40′ 36″ to seconds.
56. Reduce 588487″ to degrees.
57. Reduce 12.3 miles to feet.
58. Reduce 365$\frac{1}{4}$ days to weeks.
59. Reduce 706.35 perches to acres.
60. How many acres in 12$\frac{3}{5}$ sq. miles?

**168.** RULES.—I. To reduce a denominate number from a higher to a lower denomination,

1. *Multiply the number of the highest denomination by the number of units of the next lower which equals a unit of the higher, and to the product add the number of the lower denomination, if any.*

2. *Proceed in like manner with this and each successive*

*result thus obtained, until the number is reduced to the required denomination.*

NOTE.—The successive denominations of the compound number should be written in their proper orders, and the vacant denominations, if any, filled with ciphers.

II. To reduce a denominate number from a lower to a higher denomination,

1. *Divide the given denominate number by the number of units of its own denomination which equals one unit of the next higher, and place the remainder, if any, at the right.*
2. *Proceed in like manner with this and each successive quotient thus obtained, until the number is reduced to the required denomination.*
3. *The last quotient, with the several remainders annexed in proper order, will be the answer required.*

NOTE.—The above rules also apply to the reduction of denominate fractions, both common and decimal. (Art. 169.)

## Case II.

### Reduction of Denominate Fractions.

1. What part of a peck is $\frac{1}{16}$ of a bushel? $\frac{3}{16}$ bu.?

SOLUTION.— $\frac{1}{16}$ bu. $=\frac{1}{16}$ of 4 pk. $=\frac{4}{16}$ pk. or $\frac{1}{4}$ pk., and $\frac{3}{16}$ bu. $=$ 3 times $\frac{1}{4}$ pk. $=\frac{3}{4}$ pk. Hence, $\frac{3}{16}$ bu. $=\frac{3}{4}$ pk.

2. What part of a quart is $\frac{1}{12}$ of a peck? $\frac{5}{12}$ pk.?
3. What part of a day is $\frac{2}{15}$ of a week? $\frac{8}{15}$ w.?
4. What part of an hour is $\frac{1}{30}$ of a day? $\frac{7}{30}$ d.?
5. What part of an inch is $\frac{3}{40}$ of a foot? $\frac{11}{40}$ ft.?
6. What decimal part of an inch is .03 of a foot?

SOLUTION.— .03 ft. = .03 of 12 in., or 12 times .03 in. = .36 in.

7. What decimal of an hour is .05 of a day? .025 d.?
8. What decimal of a day is .12 of a week? .012 w.?
9. What decimal of a quart is .125 of a peck? .35 pk.?
10. What part of an inch is $\frac{2}{25}$ of a foot? .08 ft.?
11. What part of a pint is $\frac{3}{40}$ of a gallon? .06 gal.?

12. What part of a foot is $\frac{5}{8}$ of an inch?

SOLUTION.— $\frac{5}{8}$ in. $= \frac{5}{8}$ of $\frac{1}{12}$ ft. $= \frac{5}{96}$ ft.

13. What part of a week is $\frac{5}{12}$ of a day? $\frac{6}{13}$ d.?
14. What part of an hour is $\frac{7}{10}$ of a minute? $\frac{20}{7}$ min.?
15. What part of a gallon is $\frac{4}{5}$ of a pint? $\frac{5}{6}$ pt.?
16. What part of a pound avoir. is $\frac{4}{5}$ of an ounce?
17. What decimal of a foot is .48 of an inch?

SOLUTION.— .48 in. = .48 of $\frac{1}{12}$ ft. $= \frac{1}{12}$ of .48 ft. = .04 ft.

18. What decimal of a bushel is .12 of a peck? 3.6 pk.?
19. What decimal of a week is .49 of a day? 6.3 d.?
20. What decimal of a pound Troy is .144 of an ounce? 2.52 oz.? 38.4 oz.? .72 oz.? 9.6 oz.?
21. What decimal of a ream is .8 of a quire? 2.8 quires?
22. What part of a dime is $\frac{5}{8}$ of a cent? .625 ct.?
23. What part of a shilling is $\frac{3}{5}$ of a penny? .6 d.? .18 d.? 2.4 d.? 1.44 d.?
24. What part of a gallon is $\frac{11}{12}$ of a pint? .64 pt.?

## WRITTEN PROBLEMS.

25. Reduce $\frac{7}{18000}$ of a day to the fraction of a minute.

PROCESS: $\frac{7}{18000}$ d. $= \frac{7 \times 24}{18000}$ h. $= \frac{7 \times 24 \times 60}{18000}$ min. $= \frac{14}{25}$ min.

26. Reduce $\frac{13}{3200}$ of a pound avoirdupois to the fraction of a dram.
27. Reduce $\frac{13}{24}$ of a yard to inches.
28. Reduce $\frac{11}{12}$ of a pound Troy to pennyweights.
29. Reduce .005 of a pound to the decimal of a penny.
30. Reduce .0065 of a week to the decimal of an hour.
31. Reduce 9.6 pwt. to the decimal of a pound Troy.
32. Reduce 3.96 inches to the decimal of a rod.
33. Reduce 30.8 rods to the decimal of a mile.
34. Reduce .096 of a bushel to the decimal of a pint.
35. Reduce $\frac{20}{33}$ of a rod to the fraction of a league.

36. Reduce $\frac{15}{28}$ of a degree to the fraction of a circumference.
37. Reduce $\frac{17}{25}$ of a day to minutes.
38. Reduce $\frac{16}{21}$ of a week to hours.
39. Reduce $\frac{8}{9}$ of a minute to the fraction of a day.
40. Reduce 11.2 perches to the decimal of an acre.
41. Reduce 13.62 cords to cord feet.
42. Reduce .037 lb. avoirdupois to drams.
43. Reduce $56\frac{5}{8}$ lb. Troy to grains.
44. Reduce $\frac{11}{120}$ of a gallon to the fraction of a pint.
45. Reduce 2.43 miles to feet.
46. Reduce 777.6 pence to pounds.
47. Reduce 1.408 ft. to the decimal of a mile.
48. Reduce $\frac{7}{15}$ of an hour to the fraction of a day.
49. Reduce .012 of a mile to yards.
50. Reduce $\frac{4}{5}$ of a yard to the decimal of a mile.

**169.** Rule.—To reduce denominate fractions from a higher to a lower denomination, or from a lower to a higher, *Proceed as in the reduction of denominate integers.*

Note.—Denominate fractions are reduced to a lower denomination by multiplying, and to a higher denomination by dividing, the same as denominate integers; but in reduction descending there are no units of a lower order to add, and in reduction ascending there are no remainders.

## Case III.

### Reduction of Denominate Fractions to Lower Integers.

1. How many months in $\frac{2}{3}$ of a year? $\frac{3}{4}$ of a year? $\frac{5}{6}$ of a year?
2. How many hours in $\frac{5}{6}$ of a day? $\frac{7}{8}$ of a day? $\frac{11}{12}$ of a day?
3. How many minutes in $\frac{7}{12}$ of an hour? $\frac{7}{15}$ of an hour? $\frac{9}{20}$ of an hour?
4. How many yards in $\frac{7}{11}$ of a rod? $\frac{2}{3}$ of a rod? $\frac{7}{10}$ of a rod?
5. How many quarts in .75 of a peck? 1.25 pk.?

6. How many months in .25 of a year? $.33\frac{1}{3}$ yr.?
7. How many days in .35 of a week? 4.5 w.? 7.3 w.?
8. How many pecks and quarts in .85 of a bushel?

SOLUTION.— .85 bu. = .85 of 4 pk. = 3.4 pk., and .4 pk. = .4 of 8 qt. = 3.2 qt. Hence, .85 bu. = 3 pk. 3.2 qt.

9. How many feet and inches in .75 of a yard?
10. How many quarts and pints in $\frac{3}{5}$ of a gallon?
11. How many days and hours in $\frac{7}{8}$ of a week?
12. How many pecks and quarts in .55 of a bushel?

## WRITTEN PROBLEMS.

13. Reduce $\frac{7}{16}$ of a day and .415 of an hour each to integers of lower denominations.

PROCESS.

$\frac{7}{16}$ da. $= \frac{7}{16}$ of 24 h. $= \frac{7 \times 24}{16}$ h. $= 10\frac{1}{2}$ h.

$\frac{1}{2}$ h. $= \frac{1}{2}$ of 60 min. $= \frac{60}{2}$ min. $=$ 30 min.

$\frac{7}{16}$ da. = 10 h. 30 min.

PROCESS.

$$\begin{array}{r} .415\text{ h.} \\ \underline{60} \\ 24.900\text{ min.} \\ \underline{60} \\ 54.000\text{ sec.} \end{array}$$

.415 h. = 24 min. 54 sec.

Reduce to integers of lower denomination

14. $\frac{7}{9}$ of a mile.
15. $\frac{5}{12}$ of a week.
16. $\frac{7}{15}$ of a lb. Troy.
17. $\frac{13}{16}$ of a rod.
18. $\frac{11}{12}$ of an acre.
19. $\frac{7}{8}$ of a cord.
20. .85 of a lb. avoir.
21. .325 of a ton.
22. $.08\frac{1}{3}$ of a yard.
23. .9375 of a gallon.
24. .5625 of a cwt.
25. .0135 of a cord.

**170.** RULE.—To reduce a denominate fraction to integers of lower denominations,

1. *Multiply the fraction by the number of units of the next lower denomination, which equals a unit of its denomination.*

2. *Proceed in like manner with the fractional part of the product and of each succeeding product, until the lowest denomination is reached.*

3. *The integral parts of the several products, written in proper order, will be the lower integers sought.*

NOTE.—When the last product contains a fraction, it should be united with the integer of the lowest denomination, forming a mixed number.

## Case IV.

### Reduction of Integers of Lower Denominations to Fractions of Higher Denominations.

1. What part of a dollar is 25 cents? 50 cts.?
2. What part of a foot is 8 inches? 10 in.?
3. What part of a day is 9 hours? 15 h.?
4. What part of a yard is 2 ft. 6 in.?

SOLUTION.—1 yd. = 36 in., and 2 ft. 6 in. = 30 in.; 1 in. = $\frac{1}{36}$ of a yd., and 30 in. = $\frac{30}{36}$ yd. = $\frac{5}{6}$ yd. Hence, 2 ft. 6 in. = $\frac{5}{6}$ yd.

5. What part of a gallon is 3 qt. 1 pt.?
6. What part of a bushel is 2 pk. 5 qt.?
7. What part of a rod is 3 yd. 2 ft.?
8. What part of a barrel (31 gal.) is 15 gal. 2 qt.?
9. What part of 3 pecks is 2 pk. 4 qt.?
10. What part of 5 yards is 2 yd. 2 ft.?

SUGGESTION.—Each of the above answers should be expressed both as a common fraction and as a decimal.

### WRITTEN PROBLEMS.

11. Reduce 15 w. 5 da. to the fraction of a common year.

PROCESS.

15 w. 5 da. = 110 da.

$\frac{110}{365}$ yr. = $\frac{22}{73}$ yr., *Ans.*

12. Reduce 1 yd. 2 ft. 6 in. to the fraction of a rod.
13. Reduce 1 pk. 2 qt. $1\frac{1}{5}$ pt. to the fraction of a bushel.
14. Reduce 9 oz. $2\frac{2}{7}$ dr. to the fraction of a pound.
15. Reduce 9 h. 36 min. to the decimal of a year.
16. Reduce 2 pk. 3 qt. 1.2 pt. to the decimal of a bushel.
17. Reduce 13 s. 4 d. to the decimal of a pound Sterling.

18. Reduce 1 R. 14 P. to the decimal of an acre.
19. Reduce 8 oz. 8 pwt. to the decimal of a pound Troy.
20. Reduce 1 fur. 18 rd. 1 yd. to the decimal of a mile.
21. What part of 1 bu. 3 pk. is 5 pk. 6 qt.?
22. What part of 3 w. 4 da. is 3 da. 8 h.?
23. What part of 12 A. 2 R. is 1 A. 2 R. 10 P.?
24. What part of 3 barrels of flour is 110 lb. 4 oz.?

**171.** RULE.—To reduce a denominate number, simple or compound, to the fraction of a higher denomination, *Reduce the number which is a part and the number which is a whole to the same denomination, and write the former result as a numerator and the latter as a denominator of a fraction.*

NOTES.—1. The answer may be expressed decimally by changing the common fraction to a decimal.

2. When the whole is a unit and the part a compound number, the process may be somewhat shortened *by reducing the number of the lowest denomination to a fraction of the next higher, prefixing the higher number, if any, and then reducing this result to a fraction of the next higher denomination, and so on, until the required fraction is reached.* Thus, in the 16th problem above, the 1.2 pt. = .6 qt.; and 3.6 qt. = .45 pk.; and 2.45 pk. = .6125 bu.

## DEFINITIONS.

**172.** A ***Denominate Number*** is a number composed of concrete units of one or several denominations. It may be an integer, a mixed number, or a fraction.

**173.** Denominate numbers are either *Simple* or *Compound.*

A ***Simple Denominate Number*** is composed of units of the same denomination; as, 7 quarts.

A ***Compound Denominate Number*** is composed of units of several denominations; as, 5 bu. 3 pk. 7 qt. It is also called a *Compound Number.*

NOTE.—Every compound number is necessarily denominate.

**174.** Denominate numbers express *Currency, Measure,* and *Weight.*

***Currency*** is the circulating medium used in trade and commerce as a representative of value.

***Measure*** is the representation of extent, capacity, or amount.

***Weight*** is a measure of the force called gravity, by which bodies are drawn toward the earth.

**175.** The following diagram represents the three general classes of denominate numbers, their subdivisions, and the tables included under each:

- I. CURRENCY,
  - 1. Coin,
  - 2. Paper Money.
    - 1. United States Money,
    - 2. English Money.
- II. MEASURE,
  - 1. Of extension,
    - 1. Lines and arcs,
      - 1. Long Measure,
      - 2. Circular Measure.
    - 2. Surfaces: Square Measure.
    - 3. Capacity,
      - 1. Cubic Measure,
      - 2. Wood Measure,
      - 3. Dry Measure,
      - 4. Liquid Measure.
  - 2. Of duration: Time Measure.
- III. WEIGHT,
  - 1. Avoirdupois Weight,
  - 2. Troy Weight,
  - 3. Apothecaries Weight.

NOTE.—For tables see appendix.

**176.** The ***Reduction*** of a denominate number is the process of changing it from one denomination to another without altering its value.

**177.** Reduction is of two kinds, *Reduction Descending* and *Reduction Ascending.*

***Reduction Descending*** is the process of changing a denominate number from a higher to a lower denomination.

***Reduction Ascending*** is the process of changing a denominate number from a lower to a higher denomination.

## MENTAL PROBLEMS.

1. How many half-pint bottles can be filled with $2\frac{1}{2}$ gallons of sweet oil?

2. A boy bought $\frac{3}{4}$ of a bushel of chestnuts for \$2, and sold them at 10 cents a quart: how much did he gain?

3. If a workman can do a job of work in 120 hours, how many days will it take him if he work 8 hours a day?

4. How much will $\frac{3}{5}$ of a cwt. of sugar cost, at $16\frac{2}{3}$ cents a pound?

5. If a man spend $\frac{1}{3}$ of each day in sleep, how many hours will he sleep in the last three months of the year?

6. If a man walk 10 hours a day, at the rate of 3.3 miles an hour, how far will he walk in 6 days?

7. How many square inches in the surface of a brick 8 inches long, 4 inches wide, and 2 inches thick?

8. How many square feet in a board 12.6 ft. long and 8 inches wide?

9. How many solid feet in a plank 16 feet long, $1\frac{1}{2}$ feet wide, and 4 inches thick?

10. A man paid \$36 for a stack of hay containing $4\frac{1}{2}$ tons, and sold it at 50 cents a hundred: how much did he gain?

## WRITTEN PROBLEMS.

11. How many yards of carpeting, $\frac{3}{4}$ of a yard wide, will carpet a room 27 feet long and $21\frac{1}{3}$ feet wide?

12. How many acres in a street $2\frac{1}{4}$ miles long and 5 rods wide?

13. What would be the cost of a township of land 6 miles square, at \$10.50 an acre?

14. A rectangular field is 60 rods long and $37\frac{1}{2}$ rods wide: how many boards, each 12 feet long, will inclose it with a fence 5 boards high?

15. At $\$5.62\frac{1}{2}$ a cord, what will be the cost of a pile of wood 85 ft. 6 in. long, 6 ft. 4 in. high, and 4 ft. wide?

16. How many bricks, 4 by 8 in., will it take to pave a walk 16 feet wide and $6\frac{1}{2}$ rods long?

17. How many gold rings, each weighing 3.2 pwt., can be made from a bar of gold weighing .75 of a pound?

18. An octavo book contains 480 pages: how many reams of paper will it take to print an edition of 1200 copies, making no allowance for waste?

19. How many perches of masonry in the wall of a cellar 45 feet long, 34 feet wide, 8 feet high, and $2\frac{1}{6}$ feet thick?

NOTE.—In measuring walls of cellars and buildings, masons take the distance round the *outside* of the walls (the girth) for the length, thus measuring each corner twice.

20. How many perches of stone in the walls of a fort 120 feet square, the walls being $33\frac{1}{3}$ feet high and, on an average, 11 feet thick?

21. What will it cost to excavate a cellar 40 ft. long, 21 ft. 6 in. wide, and 4 ft. deep, at $1.75 a cubic yard?

22. A bin is 8 ft. long, $3\frac{1}{2}$ ft. wide, and 4 ft. deep: how many bushels of grain ($2150\frac{2}{5}$ cu. in.) will it hold?

23. A circular park is 165 yards in diameter: how many acres does it contain?

24. How many cubic feet in the capacity of a circular well $3\frac{1}{3}$ ft. in diameter and 20 ft. deep?

25. A cylindrical cistern is 5 ft. in diameter and 6 ft. 4 in. deep: how many gallons of water will it hold?

26. A congressional township is 6 miles square, and is divided into 36 sections: how many acres in a section?

27. A tract of land is 4 miles long and $2\frac{1}{4}$ miles wide: how many sections does it contain? How many acres?

28. A speculator bought $3\frac{1}{4}$ sections of land at $4.50 an acre, and sold them at $6.25 an acre: how much did he gain?

29. A man sold a farm containing a quarter of a section of land, for $3280: what did he receive per acre?

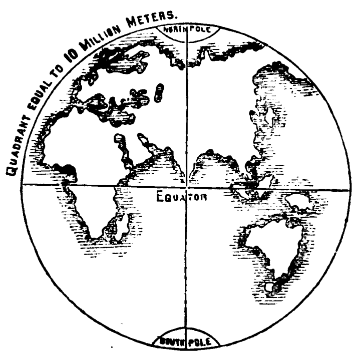
## THE METRIC SYSTEM.

**178.** ***The Metric System*** is a system of weights and measures based on the decimal scale.

The system was first adopted by France, and it is now in general use in nearly all the countries of Europe. The use of the system in the United States was legalized by Congress in 1866, and it is employed, to some extent, in several departments of the government service. It has long been used by the Coast Survey.

The convenience and accuracy of the system have secured its very general adoption in the sciences and arts, but it is not probable that it will soon come into general use in business transactions.

**179.** The ***Meter*** is the primary unit of the system. It is the *ten-millionth* part of the distance on the earth's surface from the equator to the pole.




The ***Liter*** (*lē′-ter*) is the unit of the measures of capacity. It is the thousandth part of a cubic meter.

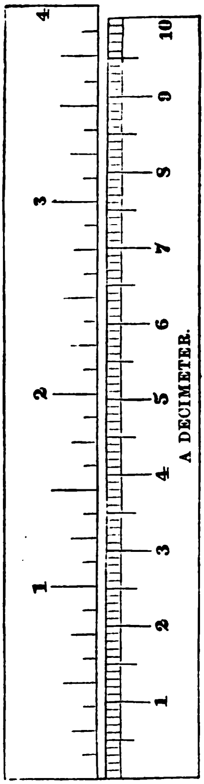
The ***Gram*** is the unit of weights. It is the weight of the thousandth part of a liter of water at its greatest density.

**180.** The meter, liter, and gram are each multiplied by 10, 100, 1000, and 10000, giving *multiple units*, and they are also each divided by 10, 100, 1000, giving the decimal *subdivisions* of tenths, hundredths, thousandths, etc.

**181.** The multiples are named by prefixing to the name of the primary unit, or base, the Greek numerals, *Deka* (10), *Hecto* (100), *Kilo* (1000), and *Myria* (10000); and the subdivisions are named by prefixing the Latin words, *Deci* (10th), *Centi* (100th), and *Milli* (1000th).

## METRIC TABLES.

### 182.—I. MEASURES OF LENGTH.




*The Unit is a* METER = 39.37 *inches, nearly.*

| Denominations. | | Values. | | Equivalents. |
|---|---|---|---|---|
| Myriameter | = | 10000 meters | = | 6.2137 mi. |
| Kilometer | = | 1000 meters | = | 0.6214 mi. |
| Hectometer | = | 100 meters | = | $328\frac{1}{12}$ ft. |
| Decameter | = | 10 meters | = | 393.7 in. |
| *Meter* | = | 1 meter | = | 39.37 in. |
| Decimeter | = | .1 meter | = | 3.937 in. |
| Centimeter | = | .01 meter | = | 0.3937 in. |
| Millimeter | = | .001 meter | = | 0.0394 in. |

*Decimal Scale.*

| Myriameter. | Kilometer. | Hectometer. | Decameter. | Meter. | | Decimeter. | Centimeter. | Millimeter. |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | . | 0 | 0 | 0 |

Ten units of any denomination of the above table equal one unit of the next higher denomination, and, hence, the successive denominations correspond to successive orders of figures in the decimal system: the meter denoting units; the decameter, tens, etc.

The correspondence between the metric denominations and those of United States Money is also noticeable. The *millimeter* corresponds to mills; the *centimeter* to *cents;* the *decimeter* to *dimes;* the *meter* to *dollars,* etc.

The above diagram shows that a decimeter is a little less than four inches, and that a centimeter is a little more than $\frac{3}{8}$ of an inch.

NOTE.—As no abbreviations for the names of the metric units have been agreed upon in this country, the names are given in full in this work. The tables of equivalents need not be memorized by the pupil.

## 183.—II. MEASURES OF SURFACE.

*The Unit is an* ARE, *or a Square Decameter.*

| Denominations. | Values. | Equivalents. |
|---|---|---|
| Hectare | = 10000 sq. meters | = 2.471 acres. |
| Are (*air*) | = 100 sq. meters | = 119.6 sq. yards. |
| Centiare | = 1 sq. meter | = 1.196 sq. yards. |

Since 100 units of each denomination in the above table equal one of the next higher, each occupies two orders of figures. The centiares correspond, in this respect, to *cents*, which occupy two places.

*Decimal Scale.*

| Hectare. | | Are. | | | Centiare. |
|---|---|---|---|---|---|
| 0 | 0 | 0 | . | 0 | 0 |

The above table is used in measuring land.

The primary unit for the measuring of small surfaces is a *square meter.*

NOTE.—Centiare is also written *Centare.*

## 184.—III. MEASURES OF CAPACITY.

*The Unit is a* LITER, *or a Cubic Decimeter.*

| Denominations. | Values. | Equivalents. *Dry Measure.* | *Liquid Measure.* |
|---|---|---|---|
| Kiloliter | = 1000 liters | = 1.308 cu. yd. | = 264.17 gallons. |
| Hectoliter | = 100 liters | = 2.8375 bu. | = 26.417 gallons. |
| Decaliter | = 10 liters | = 9.08 qt. | = 2.6417 gallons. |
| *Liter* | = 1 liter | = 0.908 qt. | = 1.0567 quarts. |
| Deciliter | = .1 liter | = 6.1022 cu. in. | = 0.845 gill. |
| Centiliter | = .01 liter | = 0.6102 cu. in. | = 0.338 fluid ounce. |
| Milliliter | = .001 liter | = 0.061 cu. in. | = 0.27 fluid dram. |

The kiloliter equals a *cubic meter*, the liter a *cubic decimeter*, and the milliter a *cubic centimeter.*

The Kiloliter is called a *Stere* (*Stair*), and is the principal measure of wood, stone, etc. One-tenth of a stere is a *Decistere,* and 10 steres are a *Decastere.*

The liter is used in measuring liquids, and the hectoliter in measuring grains.

NOTE.—When the three dimensions of a regular solid are expressed in *decimeters,* their product will be the contents in *liters.*

### 185.—IV. WEIGHTS.

*The Unit is a* GRAM = 15.432 *grains.*

| Denominations. | | Values. | Equivalents in Av. Weight. |
|---|---|---|---|
| Millier, or tonneau | = | 1000000 grams | = 2204.6 pounds |
| Quintal | = | 100000 grams | = 220.46 pounds |
| Myriagram | = | 10000 grams | = 22.046 pounds |
| Kilogram, or Kilo | = | 1000 grams | = 2.2046 pounds |
| Hectogram | = | 100 grams | = 3.5274 ounces |
| Decagram | = | 10 grams | = 0.3527 ounce |
| Gram | = | 1 gram | = 15.432 gr. Troy. |
| Decigram | = | .1 gram | = 1.5432 gr. Troy. |
| Centigram | = | .01 gram | = 0.1543 gr. Troy. |
| Milligram | = | .001 gram | = 0.0154 gr. Troy. |

A millier equals the weight of a *cubic meter* of water at its greatest density; a kilogram equals a *liter* of water; the gram, a cubic centimeter of water; and the milligram, a cubic millimeter of water. The kilogram, called, for brevity, *Kilo,* is the ordinary weight of commerce.

### 186.—V. METRIC EQUIVALENTS OF COMMON DENOMINATIONS.

*Long Measure.*

An inch = .0254 meter.
A foot = .3048 meter.
A yard = .9144 meter.
A rod = 5.029 meters.
A mile = 1.6094 kilometers.

*Square Measure.*

A sq. inch = .000645 sq. meter.
A sq. foot = .0929 sq. meter.
A sq. yard = .8362 sq. meter.
A sq. rod = .2529 are.
An acre = .4047 hectare.
A sq. mile = 259 hectares.

*Cubic Measure.*

A cu. inch = .0164 liter.
A cu. foot = .2832 hectoliter.
A cu. yard = .7646 stere.
A cord = 3.6245 steres.

*Weight.*

A grain = .0648 gram.
A pound av. = .4536 kilogram.
A pound Troy = .373 kilogram.
A ton = .907 tonneau.

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A gallon = 3.786 liters.
A bushel = .3524 hectoliters.

The new nickel 5 cent piece weighs 5 grams, and is 2 centimeters, or $\frac{1}{50}$ of a meter, in diameter.

## MENTAL PROBLEMS.

1. How many meters in a decameter? In a hectometer? A kilometer? A myriameter?

2. What part of a meter is a decimeter? A centimeter? A millimeter?

3. Name the metric units of length, in order, from the lowest to the highest.

4. How many liters in a hectoliter? In a decaliter? A kiloliter?

5. Name the metric units of capacity from the highest to the lowest?

6. What part of a gram is a centigram? A decigram? A milligram?

7. How many meters in 5 decameters? 44 decameters? 225 decameters? 34.6 decameters?

8. How many liters in 3 hectoliters? 37 hectoliters? 22.5 hectoliters? 7.45 hectoliters?

9. How many grams in 8 kilograms? 24 kilograms? 3.25 kilograms? .456 kilogram?

10. What decimal part of a meter is a centimeter? 15 centimeters? 72 centimeters?

11. What decimal part of a gram is a milligram? 24 milligrams? 245 milligrams?

12. When the metric units are expressed on the decimal scale, which order, from the decimal point, is the decimeter? The millimeter? The centimeter?

13. Which order, from the decimal point, is the decaliter? The kiloliter? The hectoliter?

14. Read the several orders in 324.56 meters as metric units.

*Ans.* 3 hectometers 2 decameters 4 meters 5 decimeters and 6 centimeters.

15. Read the several orders in 504.046 grams as metric units.

16. Read 4080.57 liters in metric units.

## WRITTEN PROBLEMS.

17. Write 5 kilograms 7 hectograms 6 decagrams 5 grams and 6 centigrams on the decimal scale as grams.

*Ans.* 5765.06 grams.

18. Write 6 hectoliters 4 decaliters 3 liters and 5 deciliters on the decimal scale as liters.

19. How many meters in 6 kilometers 7 decameters and 5 decimeters?

20. How many grams in 6 kilograms 4 decagrams and 8 centigrams?

21. Reduce 234.56 hectograms to grams.

PROCESS: $234.56 \times 100 = 23456$. *Ans.* 23456 grams.

22. Reduce 345.8 centigrams to grams.

PROCESS: $345.8 \div 100 = 3.458$. *Ans.* 3.458 grams.

23. Reduce 45.06 kiloliters to liters.
24. Reduce 35.4 hectoliters to liters.
25. Reduce 84.5 ares to square meters.
26. Reduce 132.4 centimeters to meters.
27. Reduce 24000 millimeters to meters.
28. Reduce 434.5 centiliters to liters.
29. Reduce 3,225 quintals to grams.
30. Reduce 746.35 decagrams to kilograms.
31. How many yards in 220 meters?

PROCESS: 39.37 in. $\times 220 \div 12 \div 3 = 240.59 +$. *Ans.* 240.59 yd.

32. How many miles in 44.5 kilometers?
33. How many inches in 24 centimeters?
34. How many bushels in 250 hectoliters of wheat?
35. How many gallons in $37\frac{1}{2}$ liters of sirup?
36. How many pounds of butter in 150 kilos?
37. How many liters in 35 cubic feet?
38. How many steres in 20 cords of wood?
39. How many ares in $\frac{3}{4}$ of an acre?
40. How many meters in 1760 feet?

# SECTION XIII.

# COMPOUND NUMBERS.

## ADDITION AND SUBTRACTION.

1. What is the sum of 7 cwt. 44 lb. 6 oz. 11.5 dr.; 12 cwt. 13 lb. 7.6 dr.; 23 cwt. 56 lb. 12 oz.; and 27 cwt. 14 oz. 8.4 dr.?

PROCESS.

| | cwt. | lb. | oz. | dr. |
|---|---|---|---|---|
| | 7 | 44 | 6 | 11.5 |
| | 12 | 13 | 0 | 7.6 |
| | 23 | 56 | 12 | 0. |
| | 27 | 00 | 14 | 8.4 |
| 3 T. | 10 | 15 | 1 | 11.5 |

Since only like numbers can be added, write the numbers of the same denomination in the same columns. The sum of the drams is 27.5 dr. = 1 oz. 11.5 dr. Write the 11.5 dr. under drams, and add the 1 oz. with the ounces. Proceed in like manner until the numbers of the several denominations are added.

2. Add 16 mi. 7 fur. 27 rd. 3 yd. 2 ft. $8\frac{1}{2}$ in.; 13 mi. 4 fur. 5 yd. 1 ft. $7\frac{3}{4}$ in.; 27 mi. 35 rd. 4 yd. $5\frac{1}{3}$ in.; and 6 fur. 24 rd. 3 yd. 2 ft.

3. Add 13 w. 6 d. 13 h. 48 min.; 8 w. 13 h. 51 min. 37 sec.; 12 w. 5 d. 22 h. 16 min. 44 sec.; 1 w. 10 h. 15 min.; and 1 d. 10 h. 26 sec.

4. Add 24 lb. 10 oz. 17 pwt. 22 gr.; 16 lb. 19 pwt.; 10 oz. 15 pwt. 21 gr.; 45 lb. 9 oz. 18 gr.; and 13 lb. 11 oz. 18 pwt. 23 gr.

5. Add 15 bu. 3 pk. 7 qt.; 27 bu. 5 qt. 1 pt.; 8 bu. 2 pk. 1 pt.; 47 bu. 3 pk.; 12 bu. 2 pk. 1 qt. 1 pt.; and 19 bu. 1 pk. 3 qt.

6. Add 16° 32′ 43″; 28° 47′ 53″; 25° 53″; 4 s. 48′ 48″; 11 s. 16° 36′ 59″; and 5 s. 18° 7′ 8″.

7. How many cords of wood in three piles, the first being 23 ft. long, 4 ft. wide, and 7 ft. high; the second, 28 ft. long, 4 ft. wide, and $6\frac{1}{2}$ ft. high; and the third, 17 ft. long, 8 ft. wide, and $7\frac{1}{4}$ ft. high?

8. From 13 rd. 3 yd. 1 ft. 6.4 in. take 9 rd. 4 yd. 11.5 in.

PROCESS.

| rd. | yd. | ft. | in. |
|---|---|---|---|
| 13 | 3 | 1 | 6.4 |
| 9 | 4 | 0 | 11.5 |
| 3 | $4\frac{1}{2}$ | 0 | 6.9 |
| | $\frac{1}{2}$ = | 1 | 6 |
| 3 | 4 | 2 | .9 |

Write the subtrahend under the minuend, placing the numbers of the several denominations in columns, as in compound addition. Since 11.5 in. is greater than 6.4 in., add 12 in. to 6.4 in. and then subtract. To balance the 12 in. added to the minuend, add 1 ft. (12 in.) to the subtrahend (Art. 30, Pr. 3), or, if preferred, subtract 1 ft. from the minuend. Proceed in like manner until the difference between the numbers of the several denominations is found. Reduce the $\frac{1}{2}$ yd. to feet and inches, and add the result to the 0 ft. 6.9 in. of the remainder.

9. From 30 mi. 6 fur. 14 rd. 3 yd. 1 ft. 4 in. take 25 mi. 36 rd. 4 yd. 2 ft. 10 in.

10. From 33 rd. 1 yd. 2 ft. 11 in. take 16 rd. 3 yd. 8 in.

11. From 104° 11′ 20″ take 83° 43′ 36″.

12. Boston is 71° 4′ 9″ W. longitude, and San Francisco is 122° 26′ 15″ W. longitude: what is their difference in longitude?

13. From the sum of 245 A. 2 R. 27 P. and 187 A. 3 R. 34 P. take their difference.

14. A note was given July 23, 1863, and it was paid Nov. 16, 1868: how long did it run?

15. A man was born Sept. 12, 1827, and his eldest son was born Apr. 6, 1855: what is the difference in their ages?

16. Baltimore is situated 76° 37′ W., and Vienna 16° 23′ E.: what is their difference in longitude?

17. A ship in latitude 37° 20′ north, sails 15° 45′ south; then 12° 36′ north; then 18° 40′ south: what is her latitude?

## DEFINITIONS AND RULES.

**187.** A ***Compound Number*** is a number composed of units of several denominations. (Art. 173.)

**188.** Compound numbers are of the *same kind* when their corresponding terms denote units of the same denomination; as, 3 bu. 2 pk., and 6 bu. 3 pk. 5 qt.

**189.** ***Compound Addition*** is the process of finding the sum of two or more compound numbers of the same kind.

**190.** RULE.—To add compound numbers,

1. *Write the compound numbers to be added so that units of the same denomination shall stand in the same column.*

2. *Add the column of the lowest denomination, and divide the sum by the number of units of that denomination, which equals a unit of the next higher denomination; write the remainder under the column added, and add the quotient with the next column.*

3. *In like manner add the remaining columns, writing the sum of the highest column under it.*

NOTE.—In both simple and compound addition, *the sum of each column is divided by the number of units of that denomination, which equals one of the next higher denomination.* In simple addition this divisor is 10; in compound addition it is a varying number, since the several denominations are expressed on a *varying scale.*

**191.** ***Compound Subtraction*** is the process of finding the difference between two compound numbers of the same kind.

**192.** RULE.—To subtract one compound number from another,

1. *Write the subtrahend under the minuend, placing terms of the same denomination in the same column.*

2. *Beginning at the right, subtract each successive term of the subtrahend from the corresponding term of the minuend, and write the difference beneath.*

3. *If any term of the subtrahend be greater than the corresponding term of the minuend, add to the term of the minuend as many units of that denomination as equal one of the next higher, and from the sum subtract the term of the subtrahend, writing the difference beneath.*

4. *Add one to the next term of the subtrahend, and proceed as before.*

NOTE.—Instead of adding one to the next term of the subtrahend, one may be subtracted from the next term of the minuend.

## MULTIPLICATION AND DIVISION.

1. Multiply 15 rd. 3 yd. 1 ft. 7 in. by 11.

PROCESS.

| | | | | |
|---|---|---|---|---|
| | 15 rd. | 3 yd. | 1 ft. | 7 in. |
| | | | | 11 |
| 4 fur. | 11 rd. | 5 yd. | 2 ft. | 5 in. |

Since the value of the units of the successive denominations increases from right to left, begin at the right hand. 11 times 7 in. = 77 in. = 6 ft. 5 in. Write the 5 in. under inches, and reserve the 6 ft. to add with the product of feet. Proceed in like manner until the numbers of the several denominations are multiplied.

2. If a man can build 7 rd. 11 ft. 6 in. of fence in a day, how much can 15 men build?

3. How many bushels of wheat in 18 bins, each containing 124 bu. 3 pk. 5 qt.?

4. How much hay in 13 stacks, each containing 4 T. 13 cwt. 56 lb.?

5. What is the weight of 12 silver spoons, each weighing 2 oz. 13 pwt. 14 gr.?

6. Divide 19 mi. 4 fur. 20 rd. 2 yd. 9 in. by 7.

PROCESS.

| | | | | | | |
|---|---|---|---|---|---|---|
| 7 ) | 19 mi. | 4 fur. | 20 rd. | 2 yd. | 0 ft. | 9 in. |
| | 2 mi. | 6 fur. | 14 rd. | 1 yd. | 2 ft. | $8\frac{1}{7}$ in. |

Since the value of the units of the successive denominations decreases from left to right, begin at the left hand. $\frac{1}{7}$ of 19 mi. = 2 mi. with 5 mi. remaining. Write the 2 mi. under miles, and reduce the 5 mi. to furlongs, and add the 4 fur. which gives 44 fur. $\frac{1}{7}$ of 44 fur. = 6 fur. with 2 fur. remaining. Reduce the 2 fur. to rods, add the 20 rd., take $\frac{1}{7}$ of the result, and proceed in like manner until the numbers of all the denominations are divided.

7. Divide 27 mi. 3 fur. 25 rd. 12 ft. 6 in. by 12.

8. A ship sailed 39° 12′ 40″ in 21 days: how many degrees did it average each day?

9. If 15 equal bars of silver contain 24 lb. 8 oz. 16 pwt., what is the weight of each bar?

10. If 12 equal bins hold 430 bu. 2 pk. of wheat, how much wheat is there in each bin?

11. From 13 w. 5 d. 18 h. 40 min. take 7 w. 23 h. 45 min., and divide the difference by 15.

12. Add 4 fur. 23 rd. 3 yd. 2 ft. and 7 fur. 16 rd. 1 ft., and divide the sum by 22.

13. From the sum of 56 lb. 13 oz. 9 dr. and 47 lb. 15 oz. 15 dr. take their difference, and divide the result by 9.

14. How many rings, each weighing 4 pwt. 15 gr., can be made from a bar of gold weighing 1 lb. 10 oz.?

SUGGESTION.—Reduce both divisor and dividend to the same denomination.

15. How many kegs, each containing 5 gal. 1 qt., can be filled from a cask holding 63 gal.?

16. How many rotations will a wheel 12 ft. 6 in. in circumference make in rolling $\frac{5}{8}$ of a mile?

17. How many lengths of fence, each 11 ft. 6 in., will inclose a square field each side of which is 20 rd. 5 yd.?

18. How many barrels, each holding 2 bu. 3 pk., will hold 132 bushels of apples?

19. How many axes, each weighing 3 lb. 3 oz., can be made from a ton of iron?

20. How many steps, 2 ft. 6 in. each, will a man take in walking round a field 45 rods square?

21. The length of a solar year is 365 d. 5 h. 48 min. 48 sec.: how much time is $\frac{7}{12}$ of a solar year?

## DEFINITIONS AND RULES.

**193.** ***Compound Multiplication*** is the process of taking a compound number a given number of times. The multiplier is always an abstract number.

**194.** RULE.—To multiply a compound number,

1. *Write the multiplier under the lowest denomination of the multiplicand.*

2. *Beginning at the right, multiply each term of the multiplicand in order, and reduce each product to the next higher*

*denomination, writing the remainder under the term multiplied, and adding the quotient to the next product.*

NOTE.—In both simple and compound multiplication, *the successive products are each divided by the number of units of their denomination, which equals one of the next higher denomination.*

**195.** ***Compound Division*** is the process of dividing a compound number into equal parts.

**196.** RULES.—I. To divide a compound number,

1. *Write the divisor at the left of the dividend, as in simple division.*

2. *Beginning at the left, divide each term of the dividend in order, and write the quotient under the term divided.*

3. *If the division of any term give a remainder, reduce the remainder to the next lower denomination, to the result add the number of that denomination in the dividend, and then divide as above.*

NOTE.—When the divisor is a large number, the successive terms of the quotient may be written at the right of the dividend, as in long division.

II. To divide a compound number by another of the same kind, *Reduce both compound numbers to the same denomination, and then divide as in simple division.*

NOTE.—This is not properly compound division, since the compound numbers are reduced to *simple* numbers before dividing.

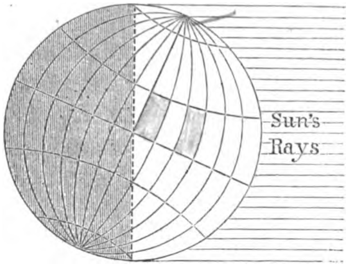
## LONGITUDE AND TIME.

**197.** ***Longitude*** is distance east or west from a given meridian. It is measured in degrees, minutes, and seconds. Thus, 15° 24′ 40″ east longitude denotes a position 15° 24′ 40″ east of the meridian from which longitude is reckoned.

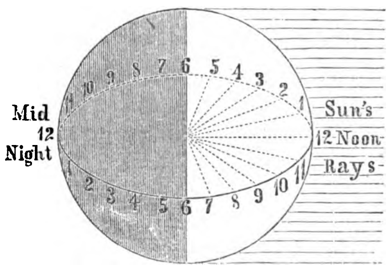
Since every circle is divided into 360 degrees, the length of a degree depends upon the size of the circle of which it is a part.

The length of a degree of longitude depends upon the latitude of the parallel on which it is measured. It is greatest at the equator, where it is 69½ miles; and least at the poles, where it is nothing.

**198.** The earth rotates on its axis from west to east once every twenty-four hours, and the illuminated space between any two meridians passes under the sun's rays in the same length of time. A degree of surface at the equator passes under the sun's rays in the same time as a degree at any latitude between the equator and the polar circle.




**199.** When the vertical rays of the sun are on the meridian of any place, it is noon, or 12 o'clock, at that place; and since the sun's rays pass over the earth's surface *from east to west*, it is *after* noon at all places east of this meridian, and *before* noon at all places west of it. When it is noon at Cincinnati, it is after noon at New York, and before noon at St. Louis.




If 24 clocks were placed 15° apart on any parallel of latitude between the polar circles, the difference in time between any two consecutive clocks would be *one hour;* and the 24 clocks would together represent every hour of the day. The figures in the diagram represent the location of the clocks (15° apart), and also the hour of the day corresponding to noon on the meridian.

**MENTAL PROBLEMS.**

1. The earth rotates on its axis once every 24 hours: what part of a rotation does it make in 1 hour?

2. How many degrees of the earth's surface pass under the sun's rays in 24 hours? In 1 hour?

3. How many degrees of longitude make a difference of 1 hour in time?

4. When it is noon at Washington, what is the hour of day 15° east of Washington? 15° west of Washington?

5. When it is 6 o'clock at Boston, what is the hour of day 30° east of Boston? 45° west of Boston?

6. If 15° of longitude give a difference of 1 hour in time, how much longitude will give a difference of 1 minute in time?

7. When it is 4 o'clock at Cincinnati, what is the time 15′ east of Cincinnati? 45′ west of Cincinnati?

8. When it is 9 o'clock at Chicago, what is the time 15° 15′ east of Chicago? 15° 45′ west of Chicago?

9. If 15′ difference in longitude gives a difference of 1 minute in time, what difference in longitude will give a difference of 1 second in time?

10. What difference in longitude gives a difference of 1 hour in time? 1 minute? 1 second?

11. The difference in time between two cities is 2 hours: what is the difference in their longitude? Which has the earlier time?

12. The difference in time between New York and St. Louis is 1 h. $2\frac{1}{3}$ min.: what is the difference in their longitude?

13. A gentleman left Boston and traveled until his watch was 1 h. 3 min. too slow: how far had he traveled, and in which direction?

14. Two captains observed an eclipse of the moon, one seeing it at 9 P. M. and the other at $11\frac{1}{2}$ P. M.: what was the difference in their longitude?

## WRITTEN PROBLEMS.

15. The difference in longitude between two places is 31° 45′ 30″: what is the difference of time?

PROCESS.

| | | | | |
|---|---|---|---|---|
| 15 ) | 31° | 45′ | 30″ | Divide by 15 as in compound division. |
| | 2 h. | 7 min. | 2 sec. | |

16. The difference in longitude between two cities is 5° 31′: what is the difference in time?

17. The longitude of Cincinnati is 84° 26′ W., and San Francisco is 122° 26′ 15″ W.: when it is noon at Cincinnati what is the time at San Francisco?

18. Philadelphia is 75° 10′ W.: when it is noon at San Francisco what is the time at Philadelphia?

19. Boston is 71° 4′ 9″ W.: when it is 7 P. M. at Boston what is the time at Cincinnati? At San Francisco?

20. Berlin is 13° 23′ 53″ E.: when it is noon at Boston what is the time at Berlin?

21. The difference in time between two cities is 1 h. 35 min. 12 sec.: what is their difference in longitude?

PROCESS.

| | | |
|---|---|---|
| 1 h. 35 min. 12 sec. | | Multiply by 15 as in compound multiplication. |
| 15 | | |
| 23° 48′ 0″, *Ans.* | | |

22. The difference in time in the observations of an eclipse, on two vessels at sea, is 2 h. 15 min. 10 sec.: what is their difference in longitude?

23. An eclipse was observed at New York, 74° W., at 9.30 P. M., and the time of its observation on a vessel in the Atlantic Ocean, was 11.45 P. M.: what was the longitude of the vessel?

24. The difference in time between the chronometers of two observatories is 45 min. 30 sec., and the longitude of the observatory having the earlier or faster time is 85° 40′ W.: what is the longitude of the other observatory?

25. The distance from Boston to Chicago is about 890 miles, and a degree of longitude at Boston contains about 52 miles: when it is noon at Boston what is the time at Chicago?

26. The distance from Washington to St. Louis is about 690 miles, and a degree of longitude at Washington contains about 53 miles: when it is 9 o'clock at St. Louis what is the time at Washington?

27. The difference in the longitude of two vessels, at the time of the observation of an eclipse, was 25° 30′: what was their difference in time?

28. How much earlier does the sun rise at Baltimore, which is 76° 37′ W., than at Cincinnati?

29. How much later does the sun set at Chicago, which is 87° 35′ W., than at Boston?

30. How much later does the sun set at San Francisco than at Cincinnati?

31. How much earlier does the sun rise at New York than at San Francisco?

32. How much later does the sun set at Boston than at Berlin?

## TABLE AND RULES.

**200.** TABLE: 15° difference in long. gives a difference of 1 h. in time.

15′ difference in long. gives a difference of 1 m. in time.

15″ difference in long. gives a difference of 1 sec. in time.

**201.** RULES.—1. To find the difference in time corresponding to any difference in longitude, *Divide the difference in longitude, expressed in degrees, minutes, and seconds, by* 15, *and the respective quotients will be hours, minutes, and seconds of time.*

2. To find the difference in longitude corresponding to any difference in time, *Multiply the difference in time, expressed in hours, minutes, and seconds, by* 15, *and the respective products will be degrees, minutes, and seconds of longitude.*

3. To find the time at one place when the time at another place and their difference of time are known, *When the second place is* EAST *of the first,* ADD *their difference of time; when it is* WEST *of the first,* SUBTRACT *their difference of time.*

# SECTION XIV.

# PERCENTAGE.

## NOTATION AND DEFINITIONS.

**202.** One per cent. of a number is one hundredth of it, two per cent. is two hundredths; and, generally, any per cent. of a number is so many *hundredths* of it.

1. How many hundredths of a number is 4 per cent. of it? 7 per cent. of it? 15 per cent. of it?
2. How many hundredths of a number is $3\frac{1}{3}$ per cent. of it? $12\frac{1}{2}$ per cent.? $33\frac{1}{3}$ per cent.?
3. How many hundredths of a number is $\frac{1}{2}$ of one per cent. of it? $\frac{3}{7}$ of one per cent.? $\frac{2}{5}$ of one per cent.?
4. How many hundredths of a number is 115 per cent. of it? 135 per cent.? 180 per cent.?
5. What per cent. of a number is .05 of it? .09 of it? .15 of it? .35 of it?
6. What per cent. of a number is $.03\frac{1}{2}$ of it? $.33\frac{1}{3}$ of it? $.00\frac{3}{7}$ of it? $.00\frac{1}{3}$ of it?
7. What per cent. of a number is $\frac{115}{100}$ of it? $\frac{140}{100}$ of it? 1.25 of it? 1.65 of it?
8. How many hundredths of a number is 45 % of it? 110 %? 125 %? 170 %? 215 %?

NOTE.—The character % is often used instead of the words "per cent." Thus, 15 % denotes 15 *per cent.*

9. What fractional part of a number is 10 % of it? 20 %? 25 %? 50 %?

SOLUTION.— 50 % of a number is $\frac{50}{100}$, or $\frac{1}{2}$ of it.

10. What fractional part of a number is $12\frac{1}{2}$ % of it? $16\frac{2}{3}$ %? $33\frac{1}{3}$ %? $66\frac{2}{3}$ %?

11. What fractional part of 100 % is 25 %? 50 %? 75 %? $66\frac{2}{3}$ %?

### WRITTEN EXERCISES.

12. Express decimally 1 %; 3 %; 6 %; 7 %; 8 %; 9 %.
13. Express decimally 15 %; 20 %; 25 %; 33 %; 45 %.
14. Express decimally 112 %; 125 %; 150 %; 220 %.
15. Express decimally $6\frac{1}{2}$ %; $3\frac{1}{4}$ %; $12\frac{1}{2}$ %; $24\frac{3}{5}$ %.

NOTE.—The fractional part of one per cent. may be expressed decimally, as thousandths, ten-thousandths, etc. Thus, $6\frac{1}{4}$ % = $.06\frac{1}{4}$, or .0625.

16. Express decimally $7\frac{3}{5}$ %; $16\frac{2}{3}$ %; $10\frac{1}{2}$ %; $30\frac{1}{4}$ %.
17. Express decimally $\frac{1}{2}$ %; $\frac{3}{5}$ %; $\frac{1}{3}$ %; $\frac{3}{7}$ %; $\frac{7}{15}$ %.
18. Express decimally 5 %; $\frac{3}{10}$ %; $7\frac{3}{10}$ %; $\frac{1}{4}$ %; $20\frac{1}{4}$ %.

### DEFINITIONS.

**203.** Any ***Per Cent.*** of a number or quantity is so many hundredths of it.

NOTE.—The term *per cent.* is a contraction of the Latin *per centum*, which means *by the hundred.*

**204.** The ***Rate Per Cent.*** is the fraction denoting the number of hundredths taken.

The rate per cent. is expressed numerically either as a common fraction or as a decimal.

**205.** The ***Rate*** is the number of hundredths.

NOTE.—In this work, the terms *Per Cent.*, *Rate Per Cent.*, and *Rate*, are not used as synonymous. In the statement, "12 is 6 per cent. of 200," 12 is considered the *per cent.*; $\frac{6}{100}$, or .06, as the *rate per cent.*; and 6 as the *rate*. The rate is the *numerator* of the fraction denoting the rate per cent.

**206.** The character % is called the *Per Cent. Sign*, and is read *per cent.*

**207.** ***Percentage*** embraces all numerical operations in which *one hundred* is the basis of computation.

## THE FOUR CASES OF PERCENTAGE.

**208.** Four numbers are considered in percentage, and such is the relation between them that, if any two of them are given, the other two may be found.

These four numbers are:

1. The ***Base,*** or the number of which the per cent. is found.

2. The ***Rate Per Cent.,*** or the fraction denoting the number of hundredths of the base taken.

3. The ***Per Cent.,*** or the part of the base corresponding to the rate per cent. It is also called the *Percentage.*

4. The ***Amount*** or ***Difference,*** or the number obtained by adding the per cent. to, or subtracting it from, the base.

### Case I.

**The Base and the Rate Per Cent. given, to find the Per Cent.**

1. How much is 5% of 800?

SOLUTIONS.—1. Since $5\% = \frac{5}{100}$, $5\%$ of $800 = \frac{5}{100}$ of 800, which is 40. Or,

2. $1\%$ of $800 = \frac{1}{100}$ of 800, which is 8; and $5\%$ of $800 = 5$ times 8, which is 40.

2. What is 6% of 1200? 8% of 250? 9% of 4000?

3. What is 5% of \$300? 8% of \$450? 12% of \$500?

4. What is 6% of 500 miles? 10% of 250 miles? 15% of 600 miles? 20% of 300 miles?

#### WRITTEN PROBLEMS.

5. What is 8% of 674.50?

| 1st Process. | 2d Process. |
|---|---|
| \$674.50 | $\$6.745 = 1\%$ |
| .08 | 8 |
| \$53.96 00 | $\$53.960 = 8\%$ |

NOTE.—Let the pupil use *one* method until he is familiar with it.

What is

| | |
|---|---|
| 6. $5\%$ of 245? | 16. $\frac{1}{2}\%$ of \$540? |
| 7. $9\%$ of 360? | 17. $\frac{1}{10}\%$ of \$4000? |
| 8. $15\%$ of 1200? | 18. $\frac{3}{7}\%$ of 21700 ft.? |
| 9. $25\%$ of 37.5? | 19. $\frac{3}{4}\%$ of \$48.50? |
| 10. $33\%$ of \$150? | 20. $33\frac{1}{3}\%$ of 965 days? |
| 11. $8\%$ of \$37.50? | 21. $16\frac{2}{3}\%$ of \$.54? |
| 12. $3\%$ of \$180.25? | 22. $15\%$ of $\frac{5}{8}$? Of $\frac{9}{10}$? |
| 13. $1\frac{1}{3}\%$ of 1050 lb.? | 23. $66\frac{2}{3}\%$ of $\frac{7}{15}$? Of $2\frac{2}{3}$? |
| 14. $2\frac{1}{2}\%$ of 60.8 lb.? | 24. $12\%$ of .25? Of .45? |
| 15. $12\frac{1}{2}\%$ of 560 days? | 25. $6\frac{1}{4}\%$ of 50? Of .75? |

26. What is the difference between $33\%$ and $25\frac{1}{2}\%$ of 480 miles?

27. If $70\%$ of a certain ore is iron, how much iron is there in 3740 pounds of ore?

28. If $20\%$ of air-dried wood is water, how much water is there in $143\frac{3}{4}$ tons of wood?

29. A man receives \$1650 a year, and his expenses are $87\frac{1}{2}\%$ of his income: how much has he left?

30. A grain dealer owning 58500 bushels of wheat, shipped $37\frac{1}{2}\%$ of it by a steamer, $33\frac{1}{3}\%$ of it by a schooner, and the rest of it by railroad: how many bushels did he ship by each?

## FORMULAS AND RULES.

**209.** FORMULAS.—1. *Per cent.* $=$ *base* $\times$ *rate per cent.*
2. *Amount* $=$ *base* $+$ *per cent.*
3. *Difference* $=$ *base* $-$ *per cent.*

**210.** RULES.—To find a given per cent. of any number,

1. *Multiply the given number by the rate per cent. expressed decimally.* Or,

2. *Remove the decimal point two places to the left, and multiply the result by the rate.*

NOTE.—When the rate is an aliquot part of 100, the per cent. may be found by taking the same aliquot part of the base. Thus, $33\frac{1}{3}\%$ of $\$48 = \frac{1}{3}$ of \$48. The process in each of the succeeding cases may be shortened by using the fraction denoting the aliquot part.

## Case II.

### The Base and the Per Cent. given, to find the Rate Per Cent.

1. What per cent. of 16 is 4?

SOLUTIONS.—1. 1 is $\frac{1}{16}$ of 16, and 4 is $\frac{4}{16}$, or .25 (Art. 121). Hence, 4 is 25% of 16. Or,

2. 1% of 16 is .16, and 4 is as many per cent. of 16 as .16 is contained times in 4.00, which is 25.

2. What per cent. of $50 are $5? $20?
3. What per cent. of $300 are $12? $30?
4. What per cent. of 150 lb. are 75 lb.? 50 lb.?
5. What per cent. of 250 ft. are 50 ft.? 100 ft.?

#### WRITTEN PROBLEMS.

6. What per cent. of 62.5 is 15?

| 1ST PROCESS. | 2D PROCESS. |
|---|---|
| $15 \div 62.5 = .24$ | $1\%$ of $62.5 = .625$ |
| $.24 = 24\%$, *Ans.* | $15 \div .625 = 24$, *Rate.* |

NOTE.—The quotient obtained by the first process is the *rate per cent.*, and the quotient obtained by the second process is the *rate.*

What per cent. of

7. 75 is 4.5?
8. 125 is 25?
9. 120 is 40?
10. $450 are $90?
11. $192 are $32?
12. $760 are $19?
13. $1000 are $5?
14. $6 are 45 cts.?
15. 75 lb. are 16.5 lb.?
16. 20 ft. are 1.2 ft.?
17. $37\frac{1}{2}$ yd. are 5 yd.?
18. .75 is .15?
19. .60 is .45?
20. $\frac{5}{6}$ is $\frac{1}{3}$?
21. $\frac{4}{5}$ is $\frac{2}{3}$?
22. $2\frac{1}{2}$ is $\frac{3}{4}$?

23. A farmer had 320 sheep and sold 48 of them: what per cent. of the flock did he sell?

24. A gold ring is 22 carats fine: what per cent. of it is gold?

25. What per cent. of $45 is $16\frac{2}{3}\%$ of $150?

26. A regiment of 750 men lost 160 men in a certain battle: what per cent. of the regiment remained?

27. What per cent. of any number is $\frac{3}{5}$ of it? $\frac{3}{4}$ of it? $\frac{4}{15}$ of it? $\frac{9}{20}$ of it?

28. What per cent. of a number is $\frac{5}{8}$ of it? $\frac{3}{25}$ of it? $\frac{12}{25}$ of it? $\frac{1}{3}$ of $\frac{5}{8}$ of it?

## FORMULA AND RULES.

**211.** FORMULA.—*Rate % = per cent. ÷ base.*

**212.** RULES.—To find what per cent. one number is of another,

1. *Divide the number which is the per cent. by the base, and the quotient expressed in hundredths will be the rate per cent.* Or,

2. *Divide the number which is the per cent. by one per cent. of the base, and the quotient will be the rate.*

## Case III.

### Per Cent. and Rate Per Cent. given, to find the Base.

1. 45 is 15% of what number?

SOLUTIONS.—1. If 15%, or .15, of a number is 45, the number equals 45 ÷ .15, which is 300. Or,

2. If 45 is 15% of a number, 1% of it is $\frac{1}{15}$ of 45, which is 3, and 100%, or the number, is 100 times 3, which is 300.

2. 320 is 16% of what number?

3. 7.2 pounds are 12% of how many pounds?

4. Charles is 15 years old, and his age is 30% of his father's age: how old is his father?

5. A man's expenses are $28 a month, which is 70% of his wages: how much does he earn a month?

6. In a certain school 56 pupils study arithmetic, which is 28 per cent. of the whole number of pupils in school: how many pupils in the school?

## WRITTEN PROBLEMS.

7. A man owes \$4560, which is 30% of his estate: how much is his estate?

| 1st Process. | 2d Process. |
|---|---|
| \$4560 ÷ .30 = \$15200, *Ans.* | \$4560 ÷ 30 × 100 = \$15200, |

8. 256 is 35% of what number?
9. $133\frac{1}{3}$ is $16\frac{2}{3}$% of what number?
10. $107\frac{1}{2}$ is 15% of what number?
11. 540 sheep are 36% of how many sheep?
12. 5280 pounds are $66\frac{2}{3}$% of how many pounds?
13. \$189.80 are 104% of what sum of money?
14. \$88.66 are 110% of what sum of money?
15. The number of pupils in daily attendance in a certain school is 420, which is 80% of the number enrolled: how many pupils are enrolled?
16. The number of youth of school age in a certain city is 5220, which is 36% of the number of inhabitants: what is the population of the city?
17. A man spent 60% of his money for a suit of clothes, 25% of it for books, and had \$7.50 left: how much money had he at first?
18. A man invested \$5400 in railroad stock, which was $37\frac{1}{2}$% of his property: what was the value of his property?
19. In a storm, a ship's crew threw overboard 250 barrels of flour, which was 40% of the number of barrels on board: how many barrels of flour were left on board?
20. A man owning 60% of a factory, sold 40% of his share for \$9600: at this rate, what was the value of the factory?
21. The land surface of the earth is about 50000000 sq. miles, which is $33\frac{1}{3}$% of the water surface: what is the extent of the water surface?
22. The population of a certain city in 1860 was 64000, which is 80% of the population in 1870: what was the population in 1870?

### FORMULA AND RULES.

**213.** FORMULA.—*Base* = *per cent.* ÷ *rate per cent.*

**214.** RULES.—To find a number when a certain per cent. of it is given,

1. *Divide the number which is the per cent. by the rate per cent. expressed decimally.* Or,

2. *Divide the number which is the per cent. by the rate, and multiply the quotient by* 100.

## Case IV.

### The Amount or Difference and the Rate Per Cent. given, to find the Base.

1. 216 is 8 % more than what number?

SOLUTIONS.—1. If 216 is 8% more than a number, 216 is 108% or 1.08 of it, and hence the number equals 216 ÷ 1.08, which is 200. Or,

2. If 216 is 108% of a number, 1% of it is $\frac{1}{108}$ of 216, which is 2, and 100% is 100 times 2, which is 200.

2. 318 is 6 % more than what number?

3. $480 is 20 % more than what sum of money?

4. 560 pounds are 12 % more than how many pounds?

5. 184 is 8 % less than what number?

SUGGESTION.—If 184 is 8% less than a number, 184 is 100% — 8%, or 92% of it.

6. 285 is 5 % less than what number?

7. $356 are 11 % less than how many dollars?

8. 425 feet are 15 % less than how many feet?

9. A horse cost $160, which was 20 % less than the cost of a carriage: what was the cost of the carriage?

10. A school enrolls 230 boys, which is 15 % more than the number of girls enrolled: how many pupils in the school?

### WRITTEN PROBLEMS.

11. A farm was sold for $6390, which was 12½ % more than it cost: what was the cost of the farm?

| 1st Process. | 2d Process. |
|---|---|
| $100\% + 12\frac{1}{2}\% = 112\frac{1}{2}\% = 1.125$ | $\$6390 \div 112.5 = \$56.80 = 1\%$ |
| $\$6390 \div 1.125 = \$5680$, *Ans.* | $\$56.80 \times 100 = \$5680$, *Ans.* |

12. A man's expenses are \$400 a year, which is $31\frac{1}{4}\%$ less than his income: what is his income?

13. 276 is 15% more than what number?

14. What number increased by 30% of itself, equals 162.5?

15. What number diminished by $16\frac{2}{3}\%$ of itself, equals 2035.8?

16. A's farm contains 306 acres, which is 32% less than B's: how many acres in B's farm?

17. When gold was worth 25% more than currency, what was the gold value of \$150 in currency?

18. When gold was worth 50% more than currency, what was the value in gold of a dollar bill?

19. The number of pupils in daily attendance at a school is 570, which is 5% less than the number enrolled: how many pupils are enrolled?

20. The number of pupils enrolled in a certain town is 920, which is 15% more than the average number of pupils in daily attendance: what is the average daily attendance?

21. The population of a certain city in 1870 was 171572, which is 18% more than its population in 1860: what was the population in 1860?

## FORMULAS AND RULES.

**215.** Formulas.—1. *Base* = *amount* ÷ (1 + *rate* %).
2. *Base* = *difference* ÷ (1 — *rate* %).

**216.** Rules.—To find a number when another number is given, which is a given rate per cent., more or less,

1. *Divide the given number by* 1 *plus or minus the given rate per cent. expressed decimally.* Or,

2. *Divide the given number by* 100 *plus or minus the given rate, and multiply the quotient by* 100.

## REVIEW OF THE FOUR CASES.

**217.** The formulas of the four preceding cases of percentage are here presented together for comparison:

CASE I.—*Per cent.* = *base* × *rate per cent.*

CASE II.—*Rate per cent.* = *per cent.* ÷ *base.*

CASE III.—*Base* = *per cent.* ÷ *rate per cent.*

CASE IV.—*Base* = $\begin{cases} \textit{Amount} \div (1 + \textit{rate per cent}). \\ \textit{Difference} \div (1 - \textit{rate per cent}). \end{cases}$

NOTE.—The two formulas, *amount* = *base* + *per cent.*, and *difference* = *base* — *per cent.*, do not involve the operations of percentage, but simply the adding and subtracting of numbers.

### MENTAL PROBLEMS.

1. What is $12\frac{1}{2}\%$ of 640? $16\frac{2}{3}\%$ of 360?
2. What is $33\frac{1}{3}\%$ of 672? $66\frac{2}{3}\%$ of 321?
3. 15 is what per cent. of 60? Of 90?
4. $16\frac{2}{3}$ lb. is what per cent. of 50 lb.? Of 100 lb.?
5. $25\%$ of 120 is what per cent. of 90?
6. $33\frac{1}{3}\%$ of 150 is what per cent. of 250?
7. 80 is $12\frac{1}{2}\%$ of what number?
8. $20\%$ of 105 is $25\%$ of what number?
9. $33\frac{1}{3}\%$ of 225 is $15\%$ of what number?
10. 45 is what per cent. of $75\%$ of 120?
11. 360 is $20\%$ more than what number?
12. 60 is $33\frac{1}{3}\%$ more than what number?
13. $33\frac{1}{3}\%$ of 240 is $33\frac{1}{3}\%$ less than what number?
14. $25\%$ of 280 is $16\frac{2}{3}\%$ more than what number?
15. A man is 60 years of age, and $20\%$ of his age is $25\%$ of the age of his wife: how old is his wife?

### WRITTEN PROBLEMS.

16. The population of a certain city in 1860 was 63500, and the census of 1870 shows an increase of $17\frac{1}{5}\%$: what was the population in 1870?

17. A farm contains 480 acres, of which 30% is meadow, $25\frac{1}{2}$% pasture, $16\frac{2}{3}$% grain land, and the rest woodland: how many acres of each kind of land in the farm?

18. A merchant failed in business owing $10500 and having $6300 worth of property: what per cent. of his indebtedness can he pay?

19. A clerk, receiving a yearly salary of $950, pays $275 a year for board, $180 for clothing, and $150 for other expenses: what per cent. of his salary is left?

20. A lady pays $280 a year for board, $175 a year for clothing and other expenses, and lays up 35% of her income: what is her income?

21. A man's expenses are 80% of his income, and $33\frac{1}{3}$% of his income equals 10% of his property, which is valued at $27000: what are his expenses?

22. A merchant sold a stock of goods for $10811, and gained $13\frac{4}{5}$%: what was the cost of the goods?

23. A cargo of damaged corn was sold at auction for $9450, which was $33\frac{1}{3}$% less than cost: what was the cost of the corn?

24. An orchard contains 1200 trees, of which 45% are apple, 22% peach, $12\frac{1}{2}$% cherry, and the rest pear: how many trees of each kind in the orchard?

25. A owns $42\frac{1}{2}$% of a factory worth $35000, B owns 37% of it, and C owns the remainder: what is the value of each of their shares?

26. A man bequeathed $7560 to his wife, which was $62\frac{1}{2}$% of the sum bequeathed to his children, and the sum bequeathed to his wife and children was 80% of his estate: what was the value of the estate?

27. The population of a city in 1870 was 41064, which was 16% more than in 1860, and the population in 1860 was $6\frac{1}{4}$% less than in 1865: what was the population in 1865?

28. The number of deaths in a certain city in 1869 was 1950, which was equal to $3\frac{1}{4}$% of the population: what was the population?

## APPLICATIONS OF PERCENTAGE.

**218.** The principal applications of percentage are *Profit and Loss*, *Commission and Brokerage*, *Capital and Stocks*, *Insurance*, *Taxes*, *Customs*, *Bankruptcy*, *Interest*, *Discount*, *Exchange*, *Equation of Payments*, and *Equation of Accounts*.

All the problems are solved by the application of one or more of the four cases of percentage.

## PROFIT AND LOSS.

**219.** The ***Cost*** of an article is the price paid for it, or the total expense incurred in producing it.

**220.** The ***Selling Price*** of an article is the amount asked or received for it by the seller.

The *selling price* of the seller is the *cost* to the buyer, and *vice versa*.

**221.** When an article is sold for more than its cost, it is said to be sold at a *profit* or *gain*; when it is sold for less than its cost, it is said to be sold at a *loss* or *discount*. Hence,

**222.** ***Profit*** or ***Gain*** is the amount which the selling price of an article exceeds its cost.

**223.** ***Loss*** or ***Discount*** is the amount which the selling price of an article is less than its cost.

NOTE.—The terms *gain* and *loss* are not limited to business transactions. When any quantity undergoes an increase or decrease, from any cause, there is a gain or loss, and when such gain or loss can be expressed in hundredths, it may be computed by the principles of percentage.

### MENTAL PROBLEMS.

1. A merchant bought a piece of cloth for $80, and sold it at 25% profit: for how much did he sell it? (*Case I.*)
2. A dealer bought hats at $5 apiece, and sold them at 20% profit: what was the selling price?

3. Hats, costing \$5 apiece, were sold at a loss of 20%: what was the selling price?

4. At what price must flour, costing \$6 a barrel, be sold to gain $16\frac{2}{3}$%?

5. A grocer bought sugar at 12 cts., 16 cts., and 18 cts. a pound: for how much must each kind be sold to gain 20%? To gain 25%?

6. A merchant sells broadcloth, costing \$4, for \$5 a yard: what per cent. does he gain? (*Case II.*)

7. When broadcloth, costing \$5 a yard, is sold for \$4 a yard, what is the loss per cent.?

8. Teas costing \$1.20 and \$1.50 a pound, are sold respectively at \$1.50 and \$1.80 a pound: what is the gain per cent.?

9. A merchant sold velvet at a profit of \$2 a yard, and gained 20%: how much did it cost? (*Case III.*)

10. A dealer sold boots at \$1.50 a pair less than cost, and thereby lost $33\frac{1}{3}$%: what did they cost?

11. A grocer sold tea at 30 cents above cost, and gained $16\frac{2}{3}$%: what was the cost of the tea? What was the selling price?

12. A man sold a horse for \$90, and gained 20%: what was the cost of the horse? (*Case IV.*)

13. A man sold a horse for \$80, and lost 20%: what was the cost of the horse?

14. Sold butter at 40 cts. a pound, and gained 25%: how much did it cost?

15. A watch, costing \$80, was sold at a loss of 10%: for how much was it sold?

16. How must shoes, costing \$2, \$2.50, and \$3 a pair, be sold respectively to gain 25%? How must each kind be sold to gain 30%?

17. How must muslin that cost 10 cts., 15 cts., and 18 cts. a yard, be sold to gain 20%?

18. Sold tea at 90 cts. a pound, and gained 20%: what would have been my gain per cent. had I sold it at \$1 a pound?

### WRITTEN PROBLEMS.

19. A house and lot, which cost $6750, were sold at a gain of $12\frac{1}{2}\%$: for how much were they sold?

20. Carriages, costing $165, are sold at 18% profit: what is the gain on each carriage?

21. A man paid $4500 for a farm, and sold it for $5400: what was the gain per cent.?

22. A drover bought cattle at $65 a head, and sold them at $84.50 a head: what was the gain per cent.?

23. Carpeting, costing $1.75 a yard, is sold for $2: what is the gain per cent.?

24. A drover bought horses at $130 a head, expended $6 each in taking them to market, and then sold them at $153.50 a head: what was the gain per cent.?

25. A cargo of wheat costing $16500, being damaged, is sold for $13750: what was the loss per cent.?

26. A merchant sold a lot of goods at $12\frac{1}{2}\%$ profit, and gained $8160: what was the cost?

27. A grocer sold 82 barrels of apples at 22% profit, and gained $45.10: what was the cost per barrel?

28. A man sold a watch for $180, and lost $16\frac{2}{3}\%$: what was the cost of the watch?

29. A house and lot were sold for $7762.50, at a gain of 15%: what was the cost?

30. A dry goods firm sold $45000 worth of goods in a year; $\frac{2}{5}$ of the receipts were sales at 20% profit, $\frac{1}{2}$ at 25% profit, and the rest at $33\frac{1}{3}\%$ profit: what was the cost of all the goods?

31. Sold a piece of carpeting for $240, and lost 20%: what selling price would have given a gain of 20%?

32. A merchant sells goods at retail at 30% above cost, and at wholesale at 12% less than the retail price: what is his gain per cent. on goods sold at wholesale?

33. How must cloth, costing $3.50 a yard, be marked that a merchant may deduct 15% from the marked price and still make 15% profit?

34. A merchant marked a piece of silk at 25% above cost, and then sold it at 20% less than the marked price: did he gain or lose, and how much?

### FORMULAS AND RULES.

**224.** FORMULAS.—1. *Gain or loss* = *cost* × *rate* %.

2. *Rate per cent.* = *gain or loss* ÷ *cost.*

3. *Cost* = *gain or loss* ÷ *rate* %.

4. $\textit{Cost} = \textit{selling price} \div \begin{cases} 1 + \textit{rate}\,\%. \\ 1 - \textit{rate}\,\%. \end{cases}$

**225.** RULES.—1. To find the gain or loss when the cost and rate per cent. are given, *Multiply the cost by the rate per cent. expressed decimally.* (Form. 1.)

2. To find the rate per cent. when the cost and the gain or loss are given, *Divide the gain or loss by the cost, and the quotient expressed in hundredths will be the rate per cent.* (Form. 2.)

3. To find the cost when the gain or loss and the rate per cent. are given, *Divide the gain or loss by the rate per cent. expressed decimally.* (Form. 3.)

4. To find the cost when the selling price and the rate per cent. of gain or loss are given, *Divide the selling price by* 1 *plus or minus the rate per cent.* (Form. 4.)

NOTE.—Let the pupil review the above problems, solving those in which the rate is an aliquot part of 100, by using the fraction. (Art. 210, Note.)

## COMMISSION AND BROKERAGE.

**226.** An ***Agent*** is a person who transacts business for another.

**227.** A ***Factor*** is an agent who buys and sells goods intrusted to his possession and control. A mercantile factor is also called a *Commission Merchant.*

When a factor lives in a different country or part of the country from his employer, he is called a *Correspondent* or *Consignee.* The

goods shipped or consigned to a Consignee are called a *Consignment;* and the sender of the goods is called a *Consignor.*

**228.** A ***Broker*** is a person who buys and sells gold, bills of exchange, stocks, bonds, etc.; or an agent who buys and sells property in possession of others.

**229.** A ***Collector*** is an agent who collects debts, taxes, duties, etc. (Arts. 263, 267.)

**230.** ***Commission*** is an allowance made to a factor or other agent, for the transaction of business. The commission allowed a broker is called *Brokerage.*

**231.** Commission is computed at a certain per cent. of the amount of property bought or sold, or of the amount of business transacted. The rate per cent. is called the *Rate of Commission*; and the amount of business transacted is the *Base.*

The rate of commission varies with the amount and nature of the business. A broker's commission is usually less than a factor's.

**232.** The ***Net Proceeds*** of a sale or collection are the proceeds less the commission and other charges.

### MENTAL PROBLEMS.

1. An auctioneer sold $300 worth of furniture, and charged a commission of 5%: how much did he receive?
2. A peddler bought $500 worth of rags, at a commission of 10%: what was his commission?
3. An agent sold $1200 worth of school furniture, at a commission of $16\frac{2}{3}$%: how much did he receive?
4. An attorney collected bad debts to the amount of $800, and charged 20% commission: what was his commission?
5. A society paid a lad $6 for collecting membership dues, to the amount of $100: what rate of commission did he receive?
6. A bookseller received $30 for selling $150 worth of maps: what was his rate of commission?

7. A real-estate broker received \$40 for selling a house and lot, at 5% commission: for how much was the property sold?

8. An attorney received \$60 for collecting a note, at 10% commission: what was the amount collected?

9. An agent received \$108, with which to buy peaches, after deducting his commission at 8%: how much did he expend for peaches?

10. A factor received \$309, with which to buy flour, after deducting his commission, at 3%: what was the cost of the flour?

11. A lawyer collected a bill at 25% commission, and remitted \$7.50 as net proceeds: what was the amount collected? What was the lawyer's commission?

12. A bookseller sold a lot of books on commission, at 20%, and remitted \$160 as net proceeds: for how much were the books sold?

## WRITTEN PROBLEMS.

13. A commission merchant sold 540 barrels of flour, at \$6.37½ a barrel: what was his commission at 3%?

14. A real-estate broker sold 325 acres of land, at \$24.50 an acre, and charged a commission of 2½%: what was his commission?

15. An auctioneer sold \$5160.50 worth of dry goods, and \$715.25 worth of furniture: what was his commission, at 1½%?

16. A lawyer collected 65% of a note of \$950, and charged 6¼% commission: what was his commission? What was the amount paid over?

17. A factor in New Orleans purchased \$75000 worth of cotton for a Lowell manufacturer, at 1¾% commission: what was his bill for cotton and commission?

18. An architect charged ¼% for plans and specifications, and 1⅜% for superintending the erection of a building, costing \$120000: how much was his fee?

19. An agent furnished a school-house for \$4500, and received \$540 commission: what was the rate?

20. An attorney charged \$75 for collecting rents to the amount of \$1125: what was the rate of commission?

21. A commission merchant charged $2\frac{1}{2}\%$ for buying produce, and his commission was \$750: how much produce did he purchase?

22. A wool agent received $5\%$ for buying wool, and his commission was \$208.50: how much wool did he buy?

23. My agent has bought 3300 barrels of apples, at \$1.75 a barrel, and I allow him $3\%$ commission: how much money must I remit to pay both the cost of the apples and the commission?

24. A Boston merchant sent his factor in Cincinnati \$3529.20, to be invested in bacon, after deducting his commission at $2\%$: how much did he expend for bacon, and what was his commission?

25. A cotton broker in Charleston received \$11774, with which to purchase cotton, after deducting his commission of $1\frac{1}{2}\%$: how much did he expend for cotton, and what was his commission?

26. A merchant paid a broker $\frac{3}{8}\%$ for a draft of \$1280 on New York: how much was the brokerage?

27. A broker bought \$15600 worth of stocks, and charged $\frac{1}{4}\%$: what was his fee?

28. A real estate broker sold a section of land (640 A.) at \$7.50 an acre, and invested the proceeds in railroad stock, receiving $1\frac{1}{2}\%$ for selling the land and $\frac{7}{8}\%$ for buying the stock: what was his brokerage?

29. What will be the total cost of 750 yards of carpeting, at \$1.75 a yard, if a merchant pays $2\frac{1}{4}\%$ commission for purchasing, $\frac{1}{4}\%$ for a draft covering cost and agent's commission, and \$12.50 for freight?

30. A grain dealer in Chicago received \$5000 with directions to purchase wheat, at \$1.10 a bushel, after deducting his commission at $2\frac{1}{2}\%$: how many bushels of wheat did he purchase?

31. An agent sold 45 sewing machines at $75 apiece, and 9 at $125 apiece, and, deducting his commission, remitted $3375 to the manufacturer as proceeds: what was his rate of commission?

32. A factor sold $15000 worth of goods, at 10 % commission, and invested the proceeds in cotton, first deducting 5 % commission for buying: how much money did he invest in cotton?

33. Smith & Jones sell for C. Bell & Co. 3040 pounds of butter, at 22 cts. a pound, and 10560 pounds of cheese, at 15 cts. a pound, and invest the proceeds in dry goods, first deducting their commission of 5 % for selling and 3 % for buying: how much did they invest in dry goods? What was their entire commission?

34. A commission merchant sold 1300 barrels of flour, at $5.75 a barrel, receiving a commission of $3\frac{1}{2}$ %, and invested the net proceeds in coffee, at 28 cts. a pound, first deducting 2 % commission: how many pounds of coffee did he purchase? What was his entire commission?

## FORMULAS AND RULES.

**233.** Formulas.—1. *Com. or brok. = base × rate %.*
2. *Rate % = com. or brok. ÷ base.*
3. *Base = com. or brok. ÷ rate %.*
4. *Base = (base + com. or brok.) ÷ (1 + rate %.)*

**234.** Rules.—1. To find commission or brokerage, *Multiply the sum of money denoting the amount of business transacted, by the rate per cent. expressed decimally.* (Form. 1.)

2. To find the sum to be invested when the amount given includes both the sum to be invested and the commission or brokerage, *Divide the given amount by* 1 *plus the rate per cent., and the quotient will be the sum to be invested.* (Form. 4.)

Note.—These two rules cover all the ordinary business transactions in commission or brokerage, but the pupil should be required to form rules embodying each of the four formulas.

## CAPITAL AND STOCK.

**235.** ***Capital*** is property invested in trade, manufactures, or other business.

**236.** The ***Par Value*** of capital is its original or nominal value.

The ***Market Value*** of capital is its real value, or the sum for which it will sell.

When the market value of capital equals its par value, it is said to be *at par;* when the market value is more than the par value, it is *above par*, or *at a premium;* when the market value is less than the par value, it is *below par*, or *at a discount.*

**237.** ***Premium*** is the amount which the market value of capital exceeds its par value.

**238.** ***Discount*** is the amount which the market value of capital is less than its par value.

**239.** Premium and discount are computed at a given per cent. of the par value. The rate per cent. is called the *Rate of Premium*, or the *Rate of Discount.*

**240.** A ***Company*** is an association of persons united for the transaction of business.

The association of several persons in business as partners, bound by articles of agreement, is called a *Partnership*, and the company is commonly called a *Firm* or *House.* (Art. 380.)

**241.** An ***Incorporated Company*** is a company organized and regulated by law. It is called a *Corporation*, and the law regulating it is called a *Charter*.

**242.** The capital of a corporation is called *Stock*, and is divided into equal parts, usually $100 each, called *Shares*. The owners of these shares are called *Stockholders*.

**243.** ***Certificates of Stock*** are official statements of the size and number of shares owned by each stockholder. They are called *Scrip*, and are bought and sold like other property.

Stocks are *at par*, *above par*, or *below par*, according as their market value equals, exceeds, or is less than their par value or face.

The market value of stocks is quoted at a certain per cent. of the par value. Stocks quoted at 108 are worth 108 % of their face, that is, are 8% *above par;* stocks quoted at 92 are worth 92% of their face. that is, are 8% *below par.*

The business of buying and selling stocks is called *Stock Jobbing*, and persons engaged in such business are called *Stock Jobbers, or Stock Brokers.*

**244.** The ***Gross Earnings*** of a company are the total receipts from its business; and the *Net Earnings* are the net profits, found by deducting all expenses and losses from the gross earnings.

**245.** A ***Dividend*** is the part of the earnings of a company distributed among the stockholders.

Dividends are usually declared annually or semi-annually, and they are computed as a per cent. of the par value of the stock. The rate per cent. is called the *Rate of Dividend.*

**246.** An ***Assessment*** is a sum levied upon the stockholders to meet the losses or expenses of the business.

The business of incorporated companies is usually managed by directors, who are elected by the stockholders, each being entitled to as many votes as he owns shares.

NOTE.—When a business corporation wishes to raise money in addition to that derived from its capital stock, it issues notes or bonds, payable at a specified time with interest, and secured by mortgage on the property of the corporation. These notes are called *Mortgage Bonds*, and their owners are called *Bondholders.* These bonds are negotiable and are called Stocks (Art. 330), but they should be carefully distinguished from *Capital Stock.*

### MENTAL PROBLEMS.

1. When stock is 6% premium, what is the market value of $1? Of $100?

2. When stock is 12% discount, what is the market value of $1? Of $100?

3. How much will 5 shares of telegraph stock cost, at 4% premium? At 4% discount?

NOTE.—A share is $100, if no other value is named.

4. How is stock quoted when it is 15% premium? When it is 15% discount?

5. When stock is quoted at 107½, what is the value of $1? Of $100?

6. When stock is quoted at 87, what is the value of $1? Of $100?

7. How much will 10 shares of mining stock cost when quoted at 104? At 85?

8. When bank stock is quoted at 105, how many shares can be bought for $525? For $840?

9. A company declares a dividend of 3%: how much will a stockholder, owning 15 shares, receive?

10. A manufacturing company made an assessment of 5%, to repair damages caused by a freshet: how much must a stockholder, owning 20 shares, pay?

### WRITTEN PROBLEMS.

11. A man bought 75 shares of railroad stock at 7½% discount: how much did they cost?

12. Bought 100 shares of Little Miami stock at 109½, and sold them at 112½: how much did I gain in the transaction?

13. A broker bought 70 shares of insurance stock at 6½% premium, and sold them at ¾% discount: how much did he lose?

14. A man bought 52 shares of Illinois Central at 127; and sold 36 shares at 135, and the rest at $137\frac{1}{2}$: how much did he gain?

15. A man exchanged 52 shares of railroad stock at 80, for insurance stock at 104: how many shares of insurance stock did he receive?

16. A broker bought 84 shares, $50 each, of telegraph stock at 94, and sold them at $100\frac{3}{7}$: how much did he gain?

17. The Cincinnati Gas Co. declares a dividend of $16\frac{2}{3}$%: how much will a man holding 36 shares receive?

18. The capital of an insurance company is $500000, and it declares a dividend of $4\frac{2}{5}$%: how much money is distributed among the stockholders?

19. A company with a capital of $125000 declares a dividend of 4%, with $3500 surplus: what were the net earnings of the company?

NOTE.—The surplus is a part of the net earnings set apart to meet future demands.

20. The entire capital stock of the railroads in Ohio for 1869 was $106686116, and their net earnings for the year were $9051998: what was the average rate of dividend?

21. The net earnings of a gas company are $22425, and the capital stock is $215000: what rate of dividend can be declared, no surplus being reserved? What will be the dividend on 45 shares?

22. The capital of a mining company is $450000; the gross receipts are $70680; and the expenses are $40325: what rate of dividend can it declare, reserving a surplus of $6505?

23. How many shares of bank stock at 4% premium, can be bought for $8320?

24. How much railroad stock, at $12\frac{1}{2}$ discount, can be bought for $8750?

25. When N. Y. Central is quoted at $95\frac{5}{8}$, how much stock can be bought for $6894, brokerage $\frac{1}{8}$%?

26. A broker bought 84 shares of coal stock, at $108\frac{1}{2}$, received a dividend of $5\frac{1}{2}\%$, and then sold the stock for 106: how much did he gain?

27. A broker bought stock at 4 % discount, and, selling the same at 5 % premium, gained $450: how many shares did he purchase?

28. A man bought Michigan Central at 120, and sold at 124: what per cent. of the investment did he gain?

## FORMULAS AND RULES.

**247.** FORMULAS.—1. $\textit{Dividend or assessment} = \textit{stock} \times \textit{rate}\,\%.$

2. $\textit{Rate}\,\% = \textit{divid. or assess.} \div \textit{stock}.$

3. $\textit{Stock} = \textit{divid. or assess.} \div \textit{rate}\,\%.$

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4. $\textit{Prem. or dis.} = \textit{par value} \times \textit{rate}\,\%.$

5. $\textit{Rate}\,\% = \textit{prem. or dis.} \div \textit{par value}.$

6. $\textit{Par value} = \textit{prem. or dis.} \div \textit{rate}\,\%.$

7. $\textit{Par val.} = \textit{market val.} \div (1 \pm \textit{rate}\,\%).$

8. $\textit{Market val.} = \textit{par val.} \times (1 \pm \textit{rate}\,\%).$

9. $\textit{Market val.} = \textit{par val.} + \textit{prem. or} - \textit{dis.}$

NOTE.—Formulas 4, 7, and 8 cover all ordinary transactions in stock jobbing. The sign $\pm$, used in 7 and 8, is read *plus* or *minus*.

**248.** RULES.—1. To find the dividend or assessment when the stock and rate per cent. are given, *Multiply the stock by the rate per cent. expressed decimally.* (Form. 1.)

2. To find the rate per cent. when the dividend or assessment and total stock are given, *Divide the dividend or assessment by the amount of stock, and the quotient, expressed in hundredths, will be the rate per cent.* (Form. 2.)

3. To find the stock when the dividend or assessment and the rate per cent. are given, *Divide the dividend or assessment by the rate per cent. expressed decimally.* (Form. 3.)

4. To find the premium or discount on a given amount of stock, *Multiply the amount of stock by the rate per cent. expressed decimally.* (Form. 4.)

5. To find the cost or market value of a given amount of stock, *Multiply the amount of stock* (1) *by* 1 *plus or minus the rate per cent.* (Form. 8); or (2) *by the quoted price expressed as hundredths.*

NOTE.—The cost may also be found by adding the premium to, or subtracting the discount from, the par value. (Form. 9.)

6. To find the amount of stock which can be bought for a given amount of money, *Divide the amount of money to be invested* (1) *by* 1 *plus or minus the rate per cent.* (Form. 7); or (2) *by the quoted price expressed as hundredths.*

NOTE.—When brokerage is paid, the rate of brokerage must be subtracted from the quoted price before dividing.

## INSURANCE.

**249.** ***Insurance*** is a guaranteed indemnity for loss.

**250.** ***Fire Insurance*** is a guaranteed indemnity for loss of property by fire.

**251.** ***Marine Insurance*** is a guaranteed indemnity for loss of property while transported by water.

Insurance on the property transported is called *Cargo Insurance;* that on the vessel is called *Hull Insurance.*

**252.** ***Life Insurance*** is a guaranteed indemnity for loss of life.

*Health Insurance* guarantees the insured a certain sum of money if sick; and *Accident Insurance* pledges a like indemnity if the insured is injured by accident.

**253.** The ***Policy*** is the written contract between the insurer and the insured.

The insurer is called an *Underwriter*, and the insured a *Policy Holder*.

**254.** The ***Premium*** is the sum paid by the insured to obtain the insurance. It is a specified per cent. of the amount insured.

The act of insuring is called *taking a risk*. When property is insured, the valuation or amount is usually made less than the real value of the property.

The insurance business is chiefly carried on by corporations, called *Insurance Companies*. In *Joint Stock Companies* the profits and losses are shared by the stockholders, but in *Mutual Companies* they are divided among the policy holders.

### MENTAL PROBLEMS.

1. A house was insured for $2500, at 1 %: what was the premium?

2. A stock of goods was insured for $8000, at $\frac{3}{4}$ %: what was the premium?

3. A hotel worth $6000 is insured for $\frac{2}{3}$ of its value, at $1\frac{1}{4}$ %: what is the premium?

4. What will it cost to get a house insured for $4000, for 10 years, at $\frac{1}{4}$ % a year?

5. The premium paid for insuring a library for $500, is $5: what is the rate of insurance?

6. An insurance company insures a school house for $10000, and charges $50 premium: what is the rate?

7. The premium for insuring a cargo of goods, at 2 %, was $240: what was the amount of goods insured?

### WRITTEN PROBLEMS.

8. A factory worth $75000 is insured for $\frac{2}{3}$ of its value, at $1\frac{1}{4}$ %: how much is the premium?

9. A merchant has his store insured for $7850, at $\frac{3}{4}$ %, and his goods for $12400, at $\frac{1}{2}$ %: what premium does he pay?

10. A house worth $5400 was insured for $\frac{3}{4}$ of its value, at $\frac{7}{8}$%, and the cost of the survey and policy was $1.50: what was the cost of the insurance?

NOTE.—When a new risk is taken, a small fee is usually charged for examining the property, called the *Survey*, and for issuing the policy.

11. The owners of a vessel paid $561 for a hull insurance of $25500: what was the rate of insurance?

12. A merchant paid $100 for an insurance of $12500 on a stock of goods: what was the rate of insurance?

13. A school house is insured at $\frac{3}{5}$ %, and the premium was $93.60: for how much is the house insured?

14. A grain shipper paid $525 for the insurance of a cargo of wheat, at $1\frac{1}{2}$ %: for how much was the wheat insured?

15. A company insured a block of buildings for $150000, at $\frac{3}{4}$ %, but, the risk being too great, it re-insured $40000 in another company, at $\frac{7}{8}$ %, and $35000 in another company, at $\frac{4}{5}$ %. How much premium did it receive more than it paid?

16. A block of buildings worth $135000 is insured for $\frac{4}{5}$ of its value by three companies, the first taking $\frac{1}{3}$ of the risk at $\frac{3}{4}$ %; the second taking $\frac{2}{5}$ of it at $\frac{7}{8}$ %; and the third taking the remainder at $\frac{2}{3}$ %. What was the total premium?

17. Suppose the above block should be damaged by fire to the amount of $60000, how much of the damage would each company be obliged to pay?

18. A house which has been insured for $3500 for 10 years, at $\frac{3}{5}$ % a year, was destroyed by fire: how much did the money received from the company exceed the cost of premiums?

19. A steamer, burned in 1869, had been insured by a single company 20 years, for $40000, at $2\frac{1}{2}$ % a year: what was the actual loss to the company, no allowance being made for interest?

20. A grain dealer had a cargo of wheat, valued at

$31360, insured at 2 %, so as to cover both the value of the wheat and the cost of the premium: for how much was the wheat insured?

SUGGESTION.—Since the premium was 2 % of the amount insured, the value of the wheat was 100 %—2 %, or 98 % of the amount insured. Hence, $31360 = $\frac{98}{100}$ of the amount insured. (Case IV.)

21. For how much must a cargo of lumber, worth $21825, be insured, at 3 %, to cover both the value of the lumber and the cost of the premium?

22. For how much must property, worth $11859.40, be insured, at $1\frac{1}{2}$ %, to cover both property and premium?

23. For what must a cargo of goods, valued at $11520, be insured, at 4 %, to cover both goods and premium?

24. What amount must be insured to cover property worth $2587, and premium at $\frac{1}{2}$ %?

25. To cover both goods and premium, at 1 %, a merchant had a cargo of goods insured for $35000: what was the value of the goods?

26. A merchant shipped a cargo of flour from New York to Liverpool, and, to cover both the flour and the premium, he took out a policy for $50400, at $3\frac{1}{2}$ %: what was the value of the flour?

## FORMULAS AND RULES.

**255.** FORMULAS.—1. *Premium = amount insured × rate %.*
2. *Rate % = prem. ÷ amount insured.*
3. *Amount insured = prem. ÷ rate %.*
4. *Property and premium = property ÷ (1 — rate %).*

**256.** RULES.—1. To find the premium, *Multiply the amount insured by the rate per cent. expressed decimally.* (Form. 1.)

2. To find for what sum property must be insured to cover both property and premium, *Divide the value of the property insured by* 1 *less the rate per cent. expressed decimally.* (Form. 4.)

## LIFE INSURANCE.

**257.** In ***Life Insurance*** the insurer agrees to pay to the heirs of the insured, or to some person named in the policy, a stipulated sum on the death of the insured, or at a specified time, should his death not occur before.

**258.** When the policy matures at the death of the insured, it is called a *Life Policy;* when it matures in a specified number of years, it is called a *Term Policy* or an *Endowment Policy.*

The premium in life insurance may be paid in a single payment; or it may be paid annually during the term of the policy; or it may be paid annually for a specified number of years, usually for ten years.

A *Non-forfeiting Policy* guarantees the insured an equitable part of the sum insured in case he should fail to pay his annual premiums after a specified number of payments have been made.

**259.** The premium is computed at a certain sum or rate per $1000 insured, the rate varying with the age of the insured at the time the policy is issued.

The basis on which the rate of life insurance is determined is the *expectation of life*, or the average extension of life beyond the given age, as shown by life statistics. Tables have been formed showing the expectation of life for every year of man's age. (See appendix.)

### WRITTEN PROBLEMS.

27. A man 45 years of age has his life insured for $3000, at $37.30 per $1000: what annual premium does he pay?

28. A man 30 years of age has his life insured for $6000, at $23.60 per $1000: what is his annual premium?

29. A man 38 years of age is insured for $5000 on the ten year plan, at $44.50 per $1000. what will be the sum of his premiums should they all be paid?

30. A man 35 years of age took out a life policy for $4000, at the rate of $27.50 per $1000; he died at the age

of 60: how much greater was the amount insured than the sum of the annual payments?

31. A man 27 years of age took out a life policy for $8000, for the benefit of his wife, at the rate of $21.70 per $1000, and his death occurred at the age of 33: how much did the widow receive more than had been paid in annual premiums?

## TAXES.

**260.** A ***Tax*** is a sum of money assessed on the person, property, income, or business of a citizen for the support of government or other public purposes.

**261.** A tax on the person of a citizen is called a *Poll Tax*, or *Capitation Tax*.

**262.** A tax on property is called a *Property Tax*. It is assessed either at a given rate per cent. of the valuation, or at the rate of a given number of mills on the dollar.

Property is classified as *Real Estate* and *Personal Property*, the former including all fixed property, as houses and lands; and the latter, all movable property. The taxable value of real estate is appraised by officers called *Appraisers* or *Assessors*, and the value of personal property is fixed by the owner, under oath, or by the assessor.

**263.** A tax on the annual income of a citizen is called an *Income Tax*; and a tax on his business is called an *Excise Tax*.

Income taxes are assessed at a given rate per cent. of a citizen's net income, less specified exemptions and deductions. Excise taxes consist of fees for business licenses, revenue stamps for business papers, taxes on manufactured products, etc.

The *Internal Revenue* of the United States is chiefly derived from income and excise taxes, assessed and collected by United States officers, called *Assessors* and *Collectors*.

NOTE.—Taxes are classified as *direct* and *indirect*. Property and poll taxes are direct; and excise taxes and duties are indirect, since they are paid *indirectly* by the consumer.

## WRITTEN PROBLEMS.

1. The valuation of the taxable property of a village was $632000, and a tax of $9480 was assessed to build a school house: what was the rate of tax?

PROCESS.

$9480 ÷ $632000 = .015

Rate = $1\frac{1}{2}$%, or 15 mills.

Since the tax was .015 of the property, the rate was 1.5%, or 15 mills on the dollar.

2. The tax levied in a certain city, for all purposes, was $259776, and the taxable property was listed at $21648000: what was the rate of tax in mills?

3. The amount of tax to be assessed in a certain township is $19340.20; the taxable property is $1425400; and the number of polls, assessed at $1.50 each, is 540: what rate of tax must be assessed on property?

4. The cost of the public schools of a certain city for the next school year, is estimated at $36848: what amount of school tax must be assessed, the cost of collecting being 2%, and allowing 6% of the assessed tax to be uncollectible?

PROCESS.

.98 ) $36848

.94 ) $37600, *Tax collected.*

$40000, *Tax assessed.*

Since 2% of tax collected is paid for collection, $36848 is 98% or .98 of the tax to be collected. $36848 is .98 of $37600. (Case IV.) Since 6% of the tax assessed is not collectible, the collectible tax, or $37600, is .94 of the tax to be assessed. $37600 is .94 of $40000. Hence, $40000 is the amount to be assessed.

NOTE.—Since the amount of uncollectible tax can only be estimated, the amount to be assessed may be found, with sufficient accuracy for all practical purposes, by adding the percentages for collection and for uncollectible taxes to the amount of money to be raised for the given purpose.

5. The net proceeds of a certain tax assessment, after deducting $1\frac{1}{2}$% for collection, was $11703.84$\frac{1}{2}$, and $7\frac{1}{2}$% of the tax was not collected: what was the amount of tax assessed?

6. The amount of tax assessed on the property of a cer-

tain city was $145850; the treasurer was allowed a fee of $\frac{3}{4}$% for collection, and 10% of the tax was uncollectible: what were the net proceeds of the assessment?

7. The taxable property of a certain city is valued at $87045060, and the rate of tax for school purposes is $5\frac{1}{2}$ mills on the dollar: what is the amount of school tax assessed?

SUGGESTION.—Since $5\frac{1}{2}$ mills = $.005\frac{1}{2}$ of a dollar, the tax assessed = $.005\frac{1}{2}$ of the property.

8. The valuation of taxable property in a certain county is $35460850, and the rate of tax levied is 25 mills: what will be the net proceeds of the tax, the cost of collection being 3%, and 8% of the tax being uncollectible?

9. When the rate of taxation is 15 mills, what is the amount of tax on A's property, listed at $13560? On B's, listed at $9850.60? On C's, listed at $50060?

10. A man's net income is $2750, of which $1354 is by law exempt from taxation: what is his income tax at 5%? At 3%?

11. A man's income is $3570, and the deductions allowed by law amount to $1650: what is his income tax at 5%?

12. A man pays a tax of $12\frac{1}{4}$ mills on his property, listed at $9850, and an income tax of 5% on a net income of $2750: what is his total annual tax?

## FORMULAS AND RULES.

**264.** FORMULAS.—1. *Tax = property × rate %.*

2. *Rate % = tax ÷ property.*

3. *Tax collected = net proceeds ÷ (1 — rate % for collection).*

4. *Tax assessed = tax collected ÷ (1 — rate % of tax uncollected).*

**265.** RULES.—1. To find the amount of tax, *Multiply the amount of taxable property by the rate of tax, expressed decimally.*

2. To find the rate of tax in mills, *Divide the amount of tax by the amount of property, and express the quotient as thousandths. The number of thousandths will be the number of mills.*

## TAX TABLES.

**266.** The labor of making out a tax list may be much lessened by using tables giving the tax on convenient amounts of property, at the given rate.

*Table for a Rate of* 15 *mills.*

| PROP. | TAX. | PROP. | TAX. | PROP. | TAX. | PROP. | TAX. | PROP. | TAX. |
|---|---|---|---|---|---|---|---|---|---|
| $1 | $.015 | $10 | $0.15 | $100 | $1.50 | $1000 | $15. | $10000 | $150. |
| 2 | .03 | 20 | .30 | 200 | 3.00 | 2000 | 30. | 20000 | 300. |
| 3 | .045 | 30 | .45 | 300 | 4.50 | 3000 | 45. | 30000 | 450. |
| 4 | .06 | 40 | .60 | 400 | 6.00 | 4000 | 60. | 40000 | 600. |
| 5 | .075 | 50 | .75 | 500 | 7.50 | 5000 | 75. | 50000 | 750. |
| 6 | .09 | 60 | .90 | 600 | 9.00 | 6000 | 90. | 60000 | 900. |
| 7 | .105 | 70 | 1.05 | 700 | 10.50 | 7000 | 105. | 70000 | 1050. |
| 8 | .12 | 80 | 1.20 | 800 | 12.00 | 8000 | 120. | 80000 | 1200. |
| 9 | .135 | 90 | 1.35 | 900 | 13.50 | 9000 | 135. | 90000 | 1350. |

13. Find by the above table the tax on $875.64, at the rate of 15 mills.

PROCESS.

$875.64 = $800 + $70 + $5 + $.60 + $.04.

$$\text{Tax on}\left\{\begin{array}{rcr} \$800. & = & \$12.00 \\ 70. & = & 1.05 \\ 5. & = & .075 \\ .60 & = & .009 \\ .04 & = & .0006 \end{array}\right.$$

Tax on $875.64 = $13.1346

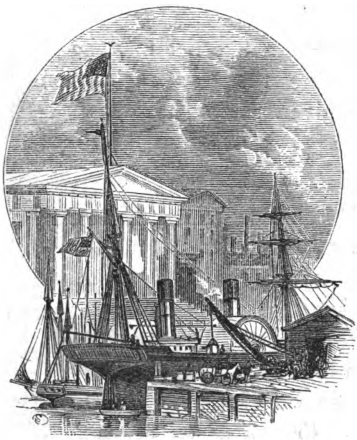
Since $60 are 100 times 60 cts., the tax on 60 cts. is found by dividing the tax on $60 by 100, which is done by removing the decimal point two places to the *left*. The tax on 4 cts. is found, in like manner, from the tax on $4.

Find by the above table the tax of

14. Mr. A on $708.
15. Mr. B on $960.
16. Mr. C on $85.80.
17. Mr. D on $3405.
18. Mr. E on $860.50.
19. Mr. F on $5408.
20. Mr. G on $85600.
21. Mr. H on $90908.
22. Mr. I on $150340.
23. Mr. J on $225350.

## CUSTOMS OR DUTIES.

**267.** ***Customs*** are taxes levied by the national government on imported goods and the tonnage of vessels. Customs are also called *Duties.*



*Ports of Entry* for foreign goods are established by law, and at each port of entry there is a *Custom House,* where customs or duties are collected. The officer in charge of the custom house is called the *Collector of Customs,* and a list of the rates of duties to be collected, is called a *Tariff.*

Duties are *Specific* or *Ad Valorem.*

**268.** ***Specific Duties*** are customs assessed on the quantity of goods imported, without reference to their value.

In assessing specific duties an allowance is made (1) for waste, called *Draft;* (2) for the weight of box, cask, etc., called *Tare* or *Tret;* (3) for waste of liquids, called *Leakage;* and (4) for the breaking of bottles, called *Breakage.* The weight of goods before allowances are made is called *Gross Weight,* and the weight after all allowances are made is called *Net Weight.*

**269.** ***Ad Valorem Duties*** are customs assessed on the cost of goods in the country from which they are imported.

The cost of imported goods is shown by an *Invoice* or *Manifest,* and when the currency of the country from which goods are imported has a depreciated value, the amount of depreciation is stated in a consular certificate, attached to the invoice. When the owner or consignee can not exhibit an invoice of goods at the custom house, their value is determined by appraisement.

### WRITTEN PROBLEMS.

1. What is the duty, at 5 cts. a pound, on 65 casks of raisins, gross weight 115 lb. each, tare 12%?

2. What is the duty, at 25 cts. a pound, on 1240 chests of tea, gross weight 120 lb. each, tare 10%?

3. What is the duty, at 5 cts. a pound, on 340 sacks of coffee, 250 lb. gross each, tare 5%?

4. What is the duty, at $1\frac{1}{2}$ cts. a pound, on 240 tons of bar iron, draft 5%?

NOTE.—In custom house computations a cwt. = 112 lb.

5. A merchant imported from Havana 225 hogsheads of sugar, 475 gross each, tare $12\frac{1}{2}$%; and 120 hogsheads of molasses, 126 gal. each, leakage 2%: what was the duty, at 3 cts. a lb. for sugar, and 8 cts. a gal. for molasses?

6. A merchant imported a lot of silks, invoiced at $45360: what was the duty, at 50% ad valorem?

7. A merchant imported 1450 yards of broadcloth, invoiced at $2.15 a yd.; 3240 yards Brussels carpeting, invoiced at $1.60 a yd.; and 480 yards of silk, invoiced at $2.85: how much was the duty, at 35% for the woolen goods, and 50% for the silk?

8. The duty on 1250 yards of silk, at 40% ad valorem, was $1100: what was the invoice price a yard? For how much a yard must the importer sell the silk to clear 20%?

### FORMULAS AND RULES.

**270.** FORMULAS.—1. *Specific duty = net quantity × rate %.*
2. *Ad val. duty = net inv. price × rate %.*
3. *Net inv. price = ad val. duty ÷ rate %.*

**271.** RULES.—1. To find specific duty, *Multiply the number, denoting the net quantity of the goods, by the rate per cent., or by the duty on one.*

2. To find ad valorem duties, *Multiply the invoice price less deductions allowed, by the rate per cent. expressed decimally.*

## BANKRUPTCY.

**272.** A ***Bankrupt*** is a person who fails in business and has not property enough to pay all his debts. A bankrupt is also called an *Insolvent.*

NOTE.—The term bankrupt is strictly applicable only to a trader, while the term insolvent applies to any person who is unable to pay his debts.

**273.** ***Bankruptcy*** is a failure in business, with inability to pay all debts.

**274.** An ***Assignment*** is the transfer of the property of a bankrupt to certain persons, called *Assignees,* in whom it is vested for the benefit of the creditors.

NOTE.—It is the duty of assignees to convert the property into money and divide the proceeds, after deducting expenses, among the creditors.

**275.** The property of a bankrupt or insolvent is called his *Assets,* and the amount of his indebtedness is called his *Liabilities.* The assets less the expense of settling are the *Net Proceeds.*

### WRITTEN PROBLEMS.

1. A merchant failed in business, owing \$15750, and his assets amount to \$10515: what per cent. of his liabilities can he pay, allowing \$750 for expense of settling.

PROCESS.

\$10515 — \$750 = \$9765, *net proceeds.*
\$9765 ÷ \$15750 = .62, or 62 %

Since the net proceeds of his assets are but .62 of his liabilities, he can pay but 62%, or 62 cts. on a dollar.

2. In the above case of bankruptcy there are four creditors, whose claims are respectively, \$3580, \$4635, \$5300, and \$2235: how much will each receive?

3. Smith, Jones & Co. have become insolvent, owing A \$3500, B \$1250, C \$3750, D \$1000, and E \$2500; their assets amount to \$7150, and the expense of settling will be \$550: what per cent. of their liabilities can they pay? What will each creditor receive?

4. A dry goods merchant failed, with liabilities amounting to $25000; his assets are: goods $9500, building and lot $5400, and bills collectible $2100; and the expense of settling will be 5 % of the assets. How many cents on the dollar can he pay?

### FORMULAS AND RULES.

**276.** FORMULAS.—1. *Rate % = net proceeds ÷ liabilities.*
2. *Dividend = claim × rate %.*

**277.** RULES.—1. To find what per cent. of his liabilities a bankrupt can pay, *Divide the net proceeds of his assets by the amount of his liabilities, and the quotient expressed in hundredths will be the rate per cent.*

2. To find the dividends of creditors in a case of bankruptcy, *Multiply the several claims of creditors by the rate per cent. which the net proceeds of the assets will pay.*

NOTE.—It is more common to find how many cents on the dollar the net proceeds will pay; the process is the same.

# INTEREST.

## PRELIMINARY DEFINITIONS.

**278.** ***Interest*** is the premium paid for the use of money.

**279.** The ***Principal*** is the sum of money for the use of which interest is paid.

**280.** The ***Amount*** is the sum of the principal and interest.

**281.** The ***Rate of Interest*** is the number of hundredths of the principal paid for its use *one year.*

**282.** The rate of interest fixed by law is called the *legal rate;* and any rate of interest higher than the legal rate is *usury.*

The legal rate of interest in most of the states, and on debts due the United States, is 6 %. In several states a rate higher than the legal rate is allowed, when so stipulated in the contract. (Art. 438.)

**283.** ***Simple Interest*** **is interest on the principal only.**

Interest considers the element of *time*, in which respect it differs from the previous applications of percentage. For periods of time greater or less than one year, the interest is proportionally greater or less than the interest for one year.

## GENERAL METHOD OF COMPUTING INTEREST.

### MENTAL PROBLEMS.

1. When money is loaned at 6 % a year, what part of the principal equals the interest for one year?

*Ans.*—Since 6 % = .06, the interest for one year equals .06 of the principal.

2. What part of the principal equals the interest, when money is loaned at 5 %? At 8 %? At 10 %?

3. What part of the principal equals the interest, when money is loaned at 4 %? At $5\frac{1}{2}$ %? At $7\frac{1}{2}$ %?

4. What is the interest of $50 for one year at 6 %?

SOLUTION.—Since the interest for one year, at 6 %, equals .06 of the principal, the interest of $50 for one year equals .06 of $50, which is $3.

5. What is the interest of $400 for one year at 7 %? At 8 %? At 9 %? At 10 %?

6. What is the interest of $650 for one year, at 4 %? At 6 %? At 8 %?

7. What is the interest of $120 for one year, at 5 %? At $5\frac{1}{2}$ %? At 10 %?

8. What is the interest of $250 for 3 years, at 6 %?

SOLUTION.—The interest of $250 for 1 year, at 6%, is $15, and since the interest for 1 year is $15, the interest for 3 years is 3 times $15, which is $45.

9. At 7 %, what is the interest of $300 for 4 years? For 5 years? 10 years?

10. At 8 %, what is the interest of $150 for 2 years? $4\frac{1}{2}$ years? $5\frac{1}{3}$ years? 8 years?

11. At $4\frac{1}{2}\%$, what is the interest of \$200 for 3 years? $4\frac{1}{5}$ years? $6\frac{1}{2}$ years?

12. At 10 %, what is the interest of \$25 for 6 years? 12 years? $8\frac{2}{5}$ years?

13. What is the interest of \$70 for 2 years and 4 months, at 5 %?

SUGGESTION.—4 months $= \frac{1}{3}$ of a year, and 2 yr. 4 mo. $= 2\frac{1}{3}$ yrs.

14. At 4 %, what is the interest of \$15 for 3 years 3 months? For 5 years 6 months?

15. At 7 %, what is the interest of \$30 for 2 years 4 months? 3 years 2 months?

16. At 6 %, what is the interest of \$50 for 4 years 2 months? 6 years 10 months?

17. What is the interest of \$10 for 4 years 6 months, at 4 %? At 6 %? At 9 %? At 10 %?

18. What is the interest of \$500 for 3 years 2 months, at 5 %? At 8 %? At 12 %?

## WRITTEN PROBLEMS.

19. What is the interest of \$145.60 for 5 years 10 months, at 5 %?

PROCESS.

$$
\begin{array}{rl}
\$145.60 & \\
.05 & \\
\hline
\$7.2800 & = \textit{Int. for } 1 \textit{ year.} \\
5\frac{5}{6} & \\
\hline
36.40 & \\
6.07 & \\
\hline
\$42.47 & = \textit{Int. for } 5 \textit{ yr. } 10 \textit{ mo.}
\end{array}
$$

20. What is the interest of \$273.45 for 8 years 3 months, at $7\frac{1}{2}\%$? At 10 %?

What is the interest of

21. \$65.30 for 1 yr. 3 mo., at 6 %? At 8 %?

22. \$640.58 for 4 yr. 11 mo., at 5 %? At 10 %?

23. \$1000 for 1 yr. 1 mo., at $3\frac{1}{2}\%$? At 7.3 %?

24. \$85, at 7 %, for 3 yr. 7 mo.? For 9 months?

25. \$38.10, at 9 %, for 6 yr. 5 mo.? For 3 yr. 10 mo.?
26. \$84.75 for 2 yr. 5 mo. 21 da., at 8 %?

SUGGESTION.—Reduce the 5 mo. 21 da. to the decimal of a year. (Art. 171, N. 2.) 21 da. = .7 mo., and 5.7 mo. = .475 yr. Hence, 2 yr. 5 mo. 21 da. = 2.475 yr.

27. \$208.44 for 7 yr. 8 mo. 15 da., at 5 %?
28. \$356.75 for 5 yr. 10 mo. 24 da., at $6\frac{1}{2}$ %?
29. \$184.80 for 1 yr. 1 mo. 10 da. ($1\frac{1}{9}$ yr.), at 9 %?
30. \$321.70 for 4 yr. 3mo. 27 da., at $12\frac{1}{2}$ %?

What is the amount of
31. \$60.85 for 10 yr. 10 mo. 10 da., at 10 %?
32. \$740.10 for 1 yr. 1 mo. 18 da., at 8 %?
33. \$1.40 for 7 yr. 11 mo. 21 da., at $7\frac{1}{2}$ %?
34. \$121.75 for 3 yr. 18 da., at 12 %?
35. \$80.65 for 1 yr. 6 mo. 12 da., at $10\frac{1}{2}$ %?
36. What is the interest of \$356.50 for 3 yr. 9 mo. 25 da., at 8 %?

PROCESS BY ALIQUOT PARTS FOR DAYS.

| | | | |
|---|---|---|---|
| | \$356.50 | | |
| | .08 | | |
| 12 ) | \$28.5200 × 3 = | \$85.560 | *Int. for* 3 *yrs.* |
| (*Int for* 1 *mo.*) | \$2.3766 × 9 = | 21.389 | " " 9 *mo.* |
| 15 da. = $\frac{1}{2}$ mo. | | 1.188 | " " 15 *da.* |
| 10 da. = $\frac{1}{3}$ mo. | | .792 | " " 10 *da.* |
| | | \$108.929 | *Int. for* 3 *yr.* 9 *mo.* 25 *da.* |

What is the interest of
37. \$84.66 for 5 yr. 7 mo. 20 da., at 5 %?
38. \$4000 for 10 yr. 10 mo. 10 da., at 15 %?
39. \$1262.70 for 11 mo. 27 da., at $7\frac{1}{4}$ %?
40. \$504.08 for 3 yr. 1 mo. 1 da., at 10 %?
41. \$3084.90 for 7 mo. 22 da., at 12 %?
42. \$2016.05 for 1 yr. 1 mo. 29 da., at 8 %?
43. What is the amount of \$262.75 for 1 yr. 5 mo. 19 da., at 6 %? At 7 %? At 9 %?
44. What is the amount of \$192.60 for 2 yr. 2 mo. 2 da., at 5 %? At 10 %? At 12 %?

45. A man borrowed \$60 May 10, 1864, and paid it March 4, 1866, with interest at 6 %: what amount did he pay?

SUGGESTION.—Find the difference of time by compound subtraction.

46. What is the interest of \$15.80, from Oct. 23, 1855, to Apr. 12, 1859, at 8 %?

47. A note of \$565.80, dated June 3, 1864, was paid Nov. 28, 1869, with interest at 8 %: what was the amount paid?

### PRINCIPLE, FORMULA, AND RULE.

**284.** PRINCIPLE.—*The principal multiplied by the rate per cent. equals the interest for one year.*

**285.** FORMULA.—*Interest = principal × rate % × time.*

**286.** RULE.—To find the interest of any sum of money for any time, at any rate per cent., 1. *Multiply the principal by the rate per cent., expressed decimally, and multiply this product by the time in years and the fraction of a year.* Or,

2. *Multiply the interest for one year by the number of years, and $\frac{1}{12}$ of it by the number of months, and find the interest for days by aliquot parts. The sum of the several results will be the interest for the given time.*

NOTE.—In solving the majority of problems in interest, the reduction of the months and days to the decimal of a year will be found as brief as the method by aliquot parts. Those who prefer to use aliquot parts, will find the method given above briefer than the one generally used.

## SIX PER CENT. METHOD.

### MENTAL PROBLEMS.

1. What is the interest of \$1 for 2 years, at 6 %?

SOLUTION.—The interest of \$1 for 1 year at 6% is .06 of \$1, which is 6 cents, and the interest for 2 years is 2 times 6 cents, which is 12 cents.

2. What is the interest of \$1, at 6 %, for 3 years? For 8 years? 12 years? 15 years?

3. What is the interest of \$1, at 6 %, for 5 years? For $10\frac{1}{2}$ years? $16\frac{2}{3}$ years?

4. What is the interest of \$1 for 1 month, at 6 %? For 3 months?

SOLUTION.—Since the interest of \$1 for 1 year, at 6%, is 6 cents, the interest for 1 month is $\frac{1}{12}$ of 6 cents, which is 5 *mills*, and the interest for 3 months is 3 times 5 mills, which is 15 mills.

5. What is the interest of \$1, at 6 %, for 4 months? 6 months? 8 months? 10 months?

6. What is the interest of \$1, at 6 %, for 5 months? 7 months? 9 months? 11 months?

At 6 %, what is the interest of

7. \$1 for 1 year 2 months? 2 yr. 4 mo.? 3 yr. 6 mo.?
8. \$1 for 4 years 5 months? 6 yr. 7 mo.? 5 yr. 9 mo.?
9. \$1 for 2 yr. 11 mo.? 3 yr. 9 mo.? 10 yr. 10 mo.?

10. What is the interest of \$1, at 6 %, for 1 day? For 6 days?

SOLUTION.—Since the interest of \$1 for 30 days, at 6%, is 5 mills, the interest for 1 day, is $\frac{1}{30}$ of 5 mills, which is $\frac{1}{6}$ of 1 mill, and the interest for 6 days is 6 times $\frac{1}{6}$ of 1 mill, which is 1 *mill*.

11. What is the interest of \$1, at 6 %, for 12 days? For 18 days? For 24 days?

12. What is the interest of \$1, at 6 %, for 9 days? 15 days? 21 days? 27 days?

13. What is the interest of \$1, at 6 %, for 10 days? 20 days? 14 days? 22 days?

14. What is the interest of \$1, at 6 %, for 7 days? 11 days? 17 days? 23 days? 25 days?

15. What is the interest of \$1, at 6 %, for 2 months 12 days? 4 mo. 18 da.? 5 mo. 6 da.?

At 6 %, what is the interest of

16. \$1 for 6 mo. 9 da? 8 mo. 15 da.? 10 mo. 21 da.?
17. \$1 for 5 mo. 13 da.? 3 mo. 22 da.? 7 mo. 25 da.?

18. \$1 for 4 mo. 16 da.? 6 mo. 29 da.? 10 mo. 10 da.?
19. \$1 for 6 mo. 16 da.? 9 mo. 28 da.? 11 mo. 11 da.?
20. \$1 for 2 yr. 8 mo. 12 da.? 4 yr. 5 mo. 18 da.?
21. \$1 for 3 yr. 3 mo. 24 da.? 5 yr. 6 mo. 6 da.?
22. At 6%, what is the interest of \$1 for 1 year? For 1 month? 2 months? For 1 day? 6 days?
23. If the interest of a certain principal is \$12, at 6%, what would be the interest of it at 7%? At 8%?

SUGGESTION.—7% is $\frac{1}{6}$ more than 6%; and 8% is $\frac{1}{3}$ more than 6%.

If the interest of a certain principal, at 6%, is \$36, what would be the interest of it

24. At 12%? At 15%? At 18%? At 21%?
25. At 3%? At 4%? At $4\frac{1}{2}$%? At 5%?
26. At 10%? At 11%? At 13%? At 14%?

## WRITTEN PROBLEMS.

27. What is the interest of \$245.60 for 2 yr. 7 mo. 21 da., at 6%?

PROCESS.

Int. of \$1 = \$.158½

$$\begin{array}{r} \$245.60 \\ .158\frac{1}{2} \\ \hline 196480 \\ 122800\phantom{0} \\ 24560\phantom{00} \\ 12280 \\ \hline \$38.9276, \textit{ Int.} \end{array}$$

Since the interest of \$1 for 2 yr. 7 mo. 21 da., at 6%, is \$.158½, or .158½ of \$1, the interest of \$245.60 will be .158½ of \$245.60, which is \$38.928. The interest is as many thousandths of the principal, as the interest of \$1 is thousandths of \$1.

28. What is the interest of \$245.60 for 2 yr. 7 mo. 21 da., at 9%? At 11%?

PROCESS.

| | |
|---|---|
| \$38.928, | *Int. at* 6%. |
| 19.464, | " " 3%. |
| \$58.392, | " " 9%. |

PROCESS.

| | |
|---|---|
| \$38.928, | *Int. at* 6%. |
| 19.464, | " " 3%. |
| 12.976, | " " 2%. |
| \$71.368, | " " 11%. |

29. What is the interest of \$508.09 for 3 yr. 3 mo. 15 da., at 6 %? At 5 %? At 7 %?

What is the interest of

30. \$540 for 10 mo. 24 da., at 6 %? At 8 %?
31. \$327.50 for 1 yr. 3 mo. 6 da., at 7 %? At 10 %?
32. \$142.64 for 2 yr. 15 da., at 4 %? At $4\frac{1}{2}$ %?
33. \$3008.75 for 4 yr. 1 mo. 20 da., at 5 %? At 9 %?
34. \$622.40 for 9 mo. 29 da., at 12 %? At 15 %?

What is the amount of

35. \$804.25 for 1 yr. 5 mo. 10 da., at 8 %? At $7\frac{1}{2}$ %?
36. \$112.40 for 11 mo. 21 da., at $5\frac{1}{2}$ %? At $6\frac{1}{2}$ %?
37. \$2000 for 1 yr. 1 mo. 1 da., at 8 %? At 11 %?
38. \$5.90 for 3 yr. 3 mo. 3 da., at 3 %? At 12 %?
39. \$16.50 for 2 yr. 2 mo. 2 da., at 6 %? At $7\frac{1}{2}$ %?
40. \$50.30 for 3 yr. 3 mo. 3 da., at 8 %? At 5 %?
41. \$200 for 4 yr. 4 mo. 4 da., at 4 %? At 10 %?

42. What is the interest of \$108.60, from Sept. 12, 1866, to May 6, 1870, at 6 %? At 8 %?

43. A debt of \$40.50 was paid May 21, 1870, with interest, at 6 %, from Nov. 9, 1864: what was the amount paid?

44. A note of \$350, dated Oct. 17, 1865, was paid Apr. 11, 1868, with interest at 7 %: what was the amount paid?

45. A note of \$150.75, dated June 15, 1867, was paid Jan. 1, 1870, with interest at 5 %: what was the amount paid?

46. A note of \$1250, dated July 5, 1868, was paid June 1, 1870, with interest at 8 %: what was the amount paid?

47. A note of \$87.50, dated Aug. 8, 1867, and bearing interest at 10 %, was paid March 25, 1869: what was the amount paid?

48. A note of \$65.80, dated Feb. 20, 1868, and bearing interest at 7 %, was paid June 25, 1870: what was the amount paid?

### FORMULA AND RULES.

**287.** FORMULA.—*Int. at 6% = principal × int. of $1 at 6%.*

**288.** RULES.—1. To compute interest at 6%, *Find the interest of $1 for the given time, by taking six times as many cents as there are years, one-half as many cents as there are months, and one-sixth as many mills as there are days; and then multiply the principal by the abstract decimal which corresponds to the interest of $1 thus found.*

2. To compute interest at any other rate than 6%, *Find the interest at 6%, and then increase or diminish this interest by such a part of itself as will give the interest at the given rate.*

## METHOD BY DAYS.

**289.** When the time is short, it is the custom of bankers and other business men to compute interest for the actual number of days included in the time, each day being considered as $\frac{1}{360}$ of a year.

### WRITTEN PROBLEMS.

1. What is the interest of $80.60 from March 15th to June 10th, at 6%?

PROCESS:

| | | | |
|---|---|---|---|
| In | March | 16 days. | $80.60 |
| " | Apr. | 30 " | .014½ |
| " | May | 31 " | 32240 |
| " | June | 10 " | 8060 |
| | 6 ) | 87 days. | 4030 |
| | | 14½ | $1.16870 |

Allowing 360 days to a year, the interest for 87 days is $\frac{87}{360}$ of the interest for one year, and the interest for 1 year, at 6%, is $\frac{6}{100}$ of the principal. Hence, the interest for 87 days is $\frac{87}{360}$ of $\frac{6}{100}$ of the principal, which is $\frac{87}{6000}$ of the principal. But $\frac{87}{6000} = \frac{1}{6}$ of $\frac{87}{1000}$. Hence, the interest, at 6%, equals *one-sixth as many thousandths of the principal as there are days in the time.*

NOTE.—This explanation may be preferred: Allowing 360 days to the year, the interest of $1 for 1 day, at 6%, is $\frac{1}{360}$ of 6 cents, or 60 mills, *which is* $\frac{1}{6}$ *of a mill,* and, hence, the interest of $1 for any number of days *is* $\frac{1}{6}$ *of as many mills as there are days.* Having found the interest of $1, the interest of any principal is found as in the preceding article.

2. What is the interest of $125.80 from July 5th to Oct. 23d, at 6 %? At 8 %?

3. What is the amount of $25.25 from Oct. 30, 1869, to Feb. 1, 1870, at 6 %? At $7\frac{1}{2}$ %?

4. What is the amount of $65.80 from Dec. 28, 1867, to Mch. 15, 1868, at 5 %? At 10 %?

5. What is the amount of $75.40 from Jan. 13, 1869, to June 15, 1869, at 6 %? At 7 %?

6. What is the amount of $120 from Mch. 15, 1870, to July 4, 1870, at 7 %? At 9 %?

7. A note of $420, dated Jan. 25, 1860, was paid Apr. 16, 1860, with interest at 8 %: what was the amount?

8. A man borrowed $150, June 6th, and paid it, with interest at 7 %, Sept. 24th: how much did he pay?

9. A note of $80, dated Jan. 15, 1868, was paid June 21, 1868, with interest at 8 %: what was the amount?

10. A note of $150, dated Mch. 30, 1870, was paid July 4, 1870, with interest at 6 %: what was the amount?

11. A note of $500, dated May 12, 1869, and bearing interest at 7 %, was paid July 24, 1869: what was the amount?

## FORMULA AND RULES.

**290.** FORMULA.—*Interest at 6 % = principal × days ÷ 6000.*

**291.** RULES.—1. To compute interest for days at 6 %, *Multiply the principal by one sixth of as many thousandths as there are days in the time.*

2. To compute interest for days at any %, *Find the interest at 6 %, and then increase or diminish this interest by such a part of itself as the given rate is greater or less than 6.*

NOTES.—1. Since the common year consists of 365 days, instead of 360, the true interest for 360 days is $\frac{360}{365}$ or $\frac{72}{73}$ of the interest for a year; whereas, by the above method, the interest for 360 days equals the interest for a year. Hence, the true interest for any number of days in a common year is $\frac{1}{73}$ less than the interest found by the above rule; in leap year the true interest is $\frac{1}{61}$ less than the interest thus found. An *accurate* rule for computing interest for days is, *to take*

*as many 365ths, and in leap year as many 366ths, of the interest for one year as there are days in the time.*

2. In Great Britain, a day's interest is made $\frac{1}{365}$ of a year's interest, and the same rule is adopted by the United States Government in computing interest upon bonds, etc. The convenience of the method which allows 360 days to the year has secured its very general adoption by the business men of the country, and in several states it is sanctioned by law.

3. There are three methods of finding the time between two dates, to-wit: 1. *By compound subtraction, allowing 30 days to the month.* 2. *By finding the number of calendar months from the first date to the corresponding day of the month of the second date, and then counting the actual number of days left.* 3. *By counting the actual number of days between the two dates.* The third method is strictly accurate, and is generally used in finding the time of "short paper." The number of days may be found from "Time Tables," which give the exact number of days between any two dates less than a year apart.

## PARTIAL PAYMENTS.

**292.** When partial payments have been made on notes and other obligations, the interest is computed by the following rule, which, having been adopted by the Supreme Court of the United States, is called the

### UNITED STATES RULE.

*When partial payments have been made, apply the payment, in the first place, to the discharge of the interest then due. If the payment* EXCEEDS *the interest, the surplus goes toward discharging the principal, and the subsequent interest is to be computed on the balance of principal remaining due.*

*If the payment be* LESS *than the interest, the surplus of interest must not be taken to augment the principal, but interest continues on the former principal until the period when the payments, taken together, exceed the interest due, and then the surplus is to be applied toward discharging the principal, and the interest is to be computed on the balance, as aforesaid.*

**293.** This rule requires, first, that payments be applied to the discharge of interest then due; and, secondly, that no unpaid interest be added to the principal to draw interest. Interest accrues only on the unpaid *principal.*

### PROBLEMS.

1. A note of $650, dated May 20, 1866, and drawing interest at 6 %, had payments indorsed upon it as follows:

Sept. 2, 1866, $25. March 2, 1867, $150.
Dec. 20, 1866, $10. July 8, 1867, $200.

What was the amount due Nov. 11, 1867?

PROCESS.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | $650 | |
| | | | | | | .017 | |
| | | | | | | 4550 | |
| 1866 | 9 | 2 | | | | 650 | |
| 1866 | 5 | 20 | | | | $11.050 | *1st interest.* |
| | | | 3 mo. 12 da. | .017 | | 650. | |
| | | | | | | 661.05 | |
| | | | $25. | | | 25.00 | *1st payment.* |
| 1866 | 12 | 20 | | | | $636.05 | *2d principal.* |
| 1866 | 9 | 2 | | | | .018 | |
| | | | 3 mo. 18 da. | .018 | | 508840 | |
| | | | | | | 63605 | |
| | | | $10. | | $10 | $11.44890 | *2d interest.* |
| 1867 | 3 | 2 | | | | $636.05 | *3d principal.* |
| 1866 | 12 | 20 | | | | .012 | |
| | | | 2 mo. 12 da. | .012 | $150 | $7.63260 | *3d interest.* |
| | | | $150. | | | 11.4489 | *2d interest.* |
| | | | | | | 636.05 | |
| | | | | | | $655.1315 | |
| 1867 | 7 | 8 | | | | 160.00. | *2d + 3d payment.* |
| 1867 | 3 | 2 | | | | $495.1315 | *4th principal.* |
| | | | 4 mo. 6 da. | .021 | | .021 | |
| | | | $200. | | | 4951315 | |
| | | | | | | 9902630 | |
| 1867 | 11 | 11 | | | | $10.3977615 | *4th interest.* |
| 1867 | 7 | 8 | | | | 495.1315 | |
| | | | 4 mo. 3 da. | .0205 | | $505.5293 | |
| | | | | | | 200.00 | *4th payment.* |
| | | | | | | $305.5293 | *5th principal.* |
| | | | | | | .0205 | |
| | | | | | | 15276465 | |
| | | | | | | 6110586 | |
| | | | | | | $6.26335065 | *5th interest.* |
| | | | | | | 305.5293 | |
| | | | | | | $311.7926, | *Amount due Nov. 11, 1867.* |

The first step is to find the difference of time between each two consecutive dates, and form the corresponding decimal multipliers by the six per cent. method (Art. 288). The payments may be written below. This preparation will lessen the liability of error in the calculation.

Since the 1st payment is greater than the 1st interest, form the amount and subtract therefrom the payment. The difference is the 2d principal. Find the 2d interest.

Since the 2d payment is less than the 2d interest, let the interest stand, drawing a double line beneath it, and bringing down the 2d principal for a 3d principal. Find the 3d interest.

Since the sum of the 2d and 3d payments is greater than the sum of the 2d and 3d interests, form the amount and subtract therefrom the sum of the 2d and 3d payments. The difference is the 4th principal. Find the 4th interest.

Since the 4th payment is greater than the 4th interest, form the amount and subtract therefrom the 4th payment. Compute the interest on the difference, the 5th principal, to the last date, and form the amount, which is the sum then due.

NOTES.—1. Sometimes an estimate of the interest may be made *mentally* with sufficient accuracy to determine whether it is greater or less than the payment. If greater, the *sum* of the two or more decimal multipliers, can be used for a multiplier. Instead of multiplying by .018 and .012 above, their sum, or .03, might have been used. When the rate is other than 6%, the several interests should be increased or diminished, as the rate may require, before forming the amounts.

2. The above rule is generally used when the time between the date of the note and its payment exceeds *one year*.

2. A note of $600, dated June 10, 1867, had indorsements as follows: Dec. 4, 1867, $50; Mch. 25, 1868, $12; July 9, 1868, $75. How much was due Oct. 15, 1868, at 6 % interest?

3. A note of $1000, dated Apr. 10, 1864, was indorsed as follows: Nov. 10, 1865, $80.50; July 5, 1866, $100; Jan. 10, 1867, $450.80; Oct. 1, 1869, $500. What was due Jan. 1, 1870, at 7 % interest?

4. A note of $450, dated July 4, 1868, was indorsed as follows: Jan. 20, 1869, $15; June 9, 1869, $200; Oct. 20, 1869, $10. What was due Jan. 10, 1870, at 10 % interest?

5. A note of $850, dated March 4, 1865, had indorse-

ments as follows: Sept. 1, 1865, $12; May 4, 1866, $10; Sept. 15, 1866, $250; Jan. 20, 1867, $400. What was due July 1, 1868, at 6 % interest?

6. A note of $520, dated Apr. 12, 1867, had three indorsements as follows: Dec. 6, 1867, $120; July 9, 1868, $12; Nov. 30, 1868, $9. What was due May 1, 1869, at 9 % interest?

7. $1250. CINCINNATI, JULY 1, 1868.

On demand, I promise to pay Peter Smith, or order, twelve hundred and fifty dollars, with interest at $7\frac{1}{2}$ %, for value received. JOHN COONS.

*Indorsements:* Sept. 14, 1868, $300; Jan. 20, 1869, $12; Oct. 20, 1869, $20; Nov, 8, 1869, $500.

What was due on the above note Jan. 1, 1870?

8. $1000. SAN FRANCISCO, APR. 10, 1867.

For value received, I promise to pay to Wm. Penn, Jr., or order, thirty days after date, one thousand dollars, with interest at 10 %. GOULD DIVES.

*Indorsements:* July 28, 1867, $500; Dec. 13, 1867, $8; Feb. 25, 1868, $12; July 7, 1868, $125; Oct. 3, 1868, $200; Mch. 15, 1869, $50.

What was due on the above note June 3, 1869.

**294.** When partial payments are made on mercantile accounts, past due, and on notes running a *year or less*, the interest is often computed by

## THE MERCHANT'S RULE.

*Compute the interest on the principal from the time it begins to draw interest to the time of settlement, and also on each payment from the time it was made to the time of settlement.*

*From the sum of the principal and its interest, subtract the sum of the payments and their interests, and the difference will be the balance due.*

9. A note of \$800, dated March 12, 1869, and drawing interest at 8 %, was indorsed as follows: May 15, 1869, \$200; Aug. 10, 1869, \$75; Oct. 20, 1869, \$125. What was due Dec. 30, 1869?

10. Payments were made on a debt of \$350, due Feb. 1, 1868, as follows: March 20, 1868, \$45; May 1, 1868, \$60; July 5, 1868, \$80; Oct. 1, 1868, \$50. What was due Nov. 1, 1868, at 6 % interest?

NOTES.—1. There are several other rules for computing interest when partial payments are made, but they are not in general use, and hence are omitted. The old rules, called the "Vermont Rule" and the "Connecticut Rule," and found in most arithmetics, have been modified by recent legislation.

2. The chief aim of legislative enactments on this subject has been to protect the debtor from paying *interest on interest*, but there is no essential difference between applying payments to the discharge of interest instead of principal, and paying interest on such accrued interest. The debtor loses the use of so much of every payment as is applied to interest, and the creditor gains the use of it. The "Merchant's Rule" is the only one that does not practically allow interest on interest.

## FIVE PROBLEMS IN INTEREST.

**295.** Five quantities are considered in interest, and such is the relation between them that, if any three are given, the other two may be found. These quantities are the *Principal*, *Rate Per Cent.*, *Time*, *Interest*, and *Amount*. There are five classes of problems of practical importance.

### Problem I.

**Principal, Rate Per Cent., and Time given, to find the Interest and Amount.**

NOTE.—This problem has already been considered. The following problems may be solved by the pupil by either of the preceding methods, but the time in the last three problems should be found by the method by days.

1. What is the interest of \$12.50 for 3 yr. 1 mo. 15 da., at 6 %? At 10 %?

2. What is the interest of \$160.80 for 2 yr. 3 mo. 3 da., at 7 %? At 9 %?

3. What is the interest of \$56.40 for 21 days, at 10 %? What is the amount?

4. What is the interest of \$1000 from May 13, 1867, to July 8, 1868, at 6 %? At 7½ %?

5. What is the amount of \$204.50 from Jan. 21, 1869, to Feb. 3, 1870, at 9 %? At 12 %?

6. What is the interest of \$80.25 from May 15, 1869, to Sept. 24, 1869, at 6 %? At 8 %?

7. A note of \$920, dated Nov. 12, 1869, was paid Apr. 3, 1870: what was the amount, at 9 %?

8. A note of \$7.50, dated Apr. 20, 1870, was due Oct. 12, 1870, with interest at 8 %: what was the amount?

**296.** FORMULAS.—1. *Interest = principal × rate % × time.*
2. *Amount = principal + interest.*

## Problem II.

### Principal, Interest, and Time given, to find the Rate Per Cent.

9. The interest on \$540 for 8 mo. 18 da. was \$27.09: what was the rate per cent.?

PROCESS.

$$
\begin{array}{rl}
 & \$540 \\
 & \underline{.043} \\
 & 1620 \\
 & \underline{2160\phantom{0}} \\
6\,) & \underline{\$23.220} \quad \textit{Int. at } 6\%. \\
 & \$3.87 \quad \text{“ “ } 1\%. \\
\end{array}
$$

$\$27.09 \div \$3.87 = 7.$

Since the interest on \$540 for 8 mo. 18 da., at 1%, is \$3.87, the rate % which produced \$27.09 interest, was as many times 1% as \$3.87 is contained times in \$27.09, which is 7. Hence, the interest accrued at 7%.

NOTE.—The interest at 1% may be found by multiplying \$540 by $\frac{1}{6}$ of .043, which is .007$\frac{1}{6}$.

10. The interest of \$456 for 3 yr. 5 mo. 18 da., is \$79.04: what is the rate per cent.?

11. The interest of \$216 for 5 yr. 7 mo. 27 da., is \$122.22: what is the rate per cent.?

12. The interest of \$560 for 2 yr. 4 mo. 15 da., was \$106.40: what was the rate per cent.?

13. The interest of \$95.40 for 3 yr. 9 mo., is \$28.62: what is the rate per cent.?

14. The interest of \$240 from Feb. 15, 1868, to Apr. 27, 1869, was \$23.04: what was the rate per cent.?

15. The interest of \$252 from Aug. 2, 1867, to March 9, 1868, was \$12.152: what was the rate per cent.?

16. A note of \$345.60, dated Feb. 5, 1863, was paid Aug. 20, 1865, and the amount was \$407.088: what was the rate per cent.?

### FORMULA AND RULE.

**297.** FORMULA.—*Rate* % = *interest* ÷ (*prin.* × 1 % × *time*).

**298.** RULE.—To find the rate per cent., *Divide the given interest by the interest of the principal for the given time, at* 1 *per cent.*

## Problem III.

### Principal, Interest, and Rate Per Cent. given, to find the Time.

17. The interest of \$300, at 9 %, is \$60.75: what was the time?

PROCESS.

\$300
.09
\$27.00, *Int. for* 1 *yr.*

\$60.75 ÷ \$27 = 2.25
2.25 yr. = 2 yr. 3 mo.

Since the interest of \$300 for 1 year, at 9%, is \$27, \$300 must be on interest as many years to produce \$60.75 interest, as \$27 are contained times in \$60.75, which is 2.25. Hence the time is 2.25 years, or 2 yr. 3 mo. (Art. 170).

18. The interest of \$908, at $3\frac{1}{2}$ %, was \$79.45: what was the time?

19. The interest of \$56.78 for a certain time, at 10 %, was \$22.24: what was the time?

20. How long must a note of \$300 run to give an amount of \$347.25, at 6 %?

21. In what time will the interest of \$150, at 4 %, be \$9? \$11?

22. In what time will \$2040 produce \$334.05 interest, at 5 %?

23. In what time will any principal double itself at 4 % interest? At 6 %? At 10 %?

24. In what time will any principal double itself at 5 % interest? At 7 %? At 12 %?

25. In what time will any principal *treble* itself, at 5 % interest? At 10 %? At 6 %?

### FORMULA AND RULE.

**299.** FORMULA.—*Time = interest ÷ (principal × rate %).*

**300.** RULE.—To find the time, *Divide the given interest by the interest of the principal for* 1 *year, at the given rate per cent.*

NOTE.—Reduce the fraction of a year to months and days. If preferred, the interest may be divided by the interest for 1 month, at the given rate per cent.

## Problem IV.

### Interest, Rate Per Cent., and Time given, to find the Principal.

26. What principal will produce \$49.20 of interest in 1 yr. 4 mo. 12 da., at 6 %?

1ST PROCESS.

\$.082 = Int. of \$1.

\$.082 ) \$49.20 ( 600
492

\$1 × 600 = \$600. *Ans.*

Since \$1 of principal produces \$.082 of interest, it will take as many times \$1 of principal to produce \$49.20 of interest as \$.082 is contained times in \$49.20, which is 600. 600 times \$1 = \$600, the required principal. Or,

2D PROCESS.

.082 ) \$49.200 ( \$600
492

Since the interest of \$1 is .082 of itself, \$49.20 is .082 of the required principal. \$49.20 ÷ .082 = \$600.

What principal will produce

27. \$15.24 interest in 7 mo. 6 da., at 8 %?

28. \$1000 interest in 5 yr. 6 mo. 20 da., at 5%?

29. \$519 interest in 5 mo. 23 da., at 12 %?

30. What sum invested, at 7 %, will produce \$378 interest annually?

31. What sum invested, at $4\frac{1}{2}$ %, will yield an annual income of \$900?

32. What principal will produce \$220 interest, from Oct. 25, 1871, to March 7, 1872, at 8 %?

33. What principal will produce \$17.78 interest, from Jan. 10, 1872, to March 13, 1872, computed *by days*, at 4 %?

### FORMULA AND RULES.

**301.** FORMULA.—*Principal* = *interest* ÷ (*rate* % × *time*).

**302.** RULES.—To find the principal when the interest, rate per cent., and time are given, 1. *Divide the given interest by the interest of* \$1 *for the given time and rate per cent., and multiply* \$1 *by the quotient.* Or,

2. *Divide the given interest by the decimal corresponding to the interest of* \$1 *for the given time and rate per cent.*

## Problem V.

### Amount, Rate Per Cent., and Time given, to find the Principal.

34. What principal, on interest at 8 %, for 1 yr. 6 mo. 18 da., will give an amount of \$730.60?

1ST PROCESS.

\$1.124 = amount of \$1.

```
$1.124 ) $730.600 ( 650.
          6744
           5620
           5620
              0
```

The interest of \$1 for 1 yr. 6 mo. 18 da. is \$.124, and the amount is \$1.124. If the amount of \$1 is \$1.124, it will take as many times \$1 to yield an amount of \$730.60 as \$1.124 is contained times in \$730.60, which is 650. 650 times \$1 is \$650, the required principal. Or,

2D PROCESS.

```
1.124 ) $730.600 ( $650.
         6744
          5620
          5620
```

Since the amount of \$1 is 1.124 of itself, \$730.60 is 1.124 of the required principal. \$730.60 ÷ 1.124 = \$650.

35. What principal on interest, at 5 %, for 1 yr. 10 mo. 12 da., will amount to \$70.24?

36. What sum of money put at interest, at 7 %, for 8 mo. 18 da., will amount to \$567.09?

37. What sum of money put at interest, at 8 %, for 2 yr. 1 mo. 15 da., will amount to \$421.20?

38. What sum of money put at interest March 15, 1870, at 6 %, will amount to \$2600.40, Aug. 6, 1871?

### FORMULA AND RULE.

**303.** FORMULA.—*Principal = amt. ÷ [1 + (rate % × time)].*

**304.** RULES.—1. To find the principal when the amount, rate per cent., and time are given, 1. *Divide the given amount by the amount of \$1 for the given time and rate per cent., and multiply \$1 by the quotient.* Or,

2. *Divide the given amount by the decimal corresponding to the amount of \$1 for the given time and rate per cent.*

NOTE.—The principal thus found is the *present worth* of the amount. See Art. 306.

## REVIEW OF THE FIVE PROBLEMS.

**305.** The formulas for the five problems in interest are here presented together:

FORMULAS.—1. *Interest = principal × rate % × time.*
2. *Rate % = int. ÷ (principal × 1 % × time)*
3. *Time = interest ÷ (principal × rate %).*
4. *Principal = interest ÷ (rate % × time).*
5. *Principal = amt. ÷ [1 + (rate % × time)].*

### WRITTEN PROBLEMS.

1. What is the interest of $205 for 2 yr. 5 mo. 24 da., at 7 %?

2. What is the amount of $160, from Jan. 12, 1869, to July 3, 1870, at 8 %?

3. At what rate per cent. will $512.60 yield $25.72 interest in 8 mo. 18 da.?

4. The principal is $126.75, the interest $20.97, and the time 2 yr. 24 da.: what is the rate?

5. How long will it take $5000 to produce $1125 interest, at 8 %?

6. The principal is $326.50, the interest $2.76, and the rate 8 %: what is the time?

7. The amount is $1563.75, the interest $63.75, and the rate $7\frac{1}{2}$ %: what is the time?

8. What principal will yield $1.36 interest in 20 days, at 6 %?

9. The interest on a certain principal from Nov. 11, 1857, to Dec. 15, 1859, at 6 %, was $4.474: what was the principal?

10. What principal, at 7 %, will amount to $659.40 in 8 months?

11. The interest is $12.78, the time 1 yr. 2 mo. 6 da., and the rate 6 %: what is the amount?

12. What principal will amount to $609.20 in 4 mo. 18 da., at 4 %?

13. What principal will amount to $288.85 in 1 yr. 6 mo., at 6 %?

14. What principal will produce $21.757 interest, from Jan. 1. to Oct. 20, 1869, at 7 %?

15. How long will it take any principal to double itself at 6 %? At 4 %? At 10 %?

16. What is the amount of $420, from June 10, 1869, to Jan. 21, 1870, at 10 %?

17. What sum, bearing interest at 7 %, will yield an annual income of $1000?

## PRESENT WORTH AND DISCOUNT.

**306.** The most common application of Problem V is in computing *Present Worth* and *Discount.*

**307.** The ***Present Worth*** of a debt due at a future time, without interest, is the sum or principal which, at the current rate of interest, will amount to the debt when it becomes due.

**308.** ***Discount*** is the amount deducted from a debt for its payment before it is due.

**309.** ***True Discount*** is the difference between a debt, not bearing interest, and its present worth.

True discount is the interest on the *present worth* of a debt, while simple interest is computed on the *debt itself.* The difference is the interest on the true discount for the time.

### WRITTEN PROBLEMS.

1. What is the present worth of a note of $212, due 1 year hence, without interest, the current rate of interest being 6%? What is the true discount?

PROCESS.

$$\$1.06\ )\ \$212.00\ (\ 200$$
$$\underline{212}$$

$1 × 200 = $200, *Present worth.*
$212 — 200 = $12, *True discount.*

The amount of $1 for 1 year, at 6%, is $1.06, and hence the present worth of $1.06, due 1 year hence, is $1. If the present worth of $1.06 is $1, the present worth of $212. is as many times $1 as $1.06 is contained times in $212, which is 200. Or, since $1.06 is 1.06 of $1, $212 is 1.06 of its present worth. $212 ÷ 1.06 = $200.

NOTE.—This is only an application of Problem V., the *debt* being the *amount,* the *present worth* the *principal,* and the *true discount* the *interest.*

2. What is the present worth of a bill of $260 due in 8 months, without interest, the current rate of interest being 6%? What is the true discount?

Find the present worth and the true discount of

3. $220 due in 1 yr. 6 mo., without interest, current rate 7 %.

4. $145.60 due in 8 mo. 12 da., without interest, current rate 8 %.

5. $305.75 due in 9 mo. 6 da., without interest, current rate 9 %.

6. $1250 due in 1 yr. 7 mo. 21 da., without interest, current rate 5 %.

7. $1508 due in 90 days, without interest, current rate 7 %.

8. $2040.50 due in 36 days, without interest, current rate 10 %.

9. $884.125 due in 1 yr. 2 mo. 10 da., without interest, current rate 6 %.

10. What is the difference between the true discount and the interest of $216 for 2 years, at 8 %?

What is the difference between the true discount and the interest of

11. $199.80 for 1 yr. 10 mo., at 6 %? At 12 %?
12. $666.40 for 2 yr. 4 mo. 15 da., at 6 %? At 8 %?
13. $534 for 1 yr. 1 mo. 18 da., at 6 %? At 5 %?
14. $175.20 for 1 yr. 10 mo. 12 da., at 9 %?
15. $1250.60 for 1 yr. 7 mo. 24 da., at 5 %?
16. $884.12 for 96 days, at $7\frac{1}{2}$ %? At 10 %?

17. How large a note, due in 1 yr. 6 mo., with interest at 7 %, will cancel a debt of $442, due in 1 yr. 6 mo., without interest?

18. What is the difference in the present value of a cash payment of $345, and a note of $371 due in 9 months, without interest, the use of money being worth 8 %?

## FORMULAS AND RULES.

**310.** Formulas.—1. *Present worth = debt ÷ [1 + (rate % × time)].*

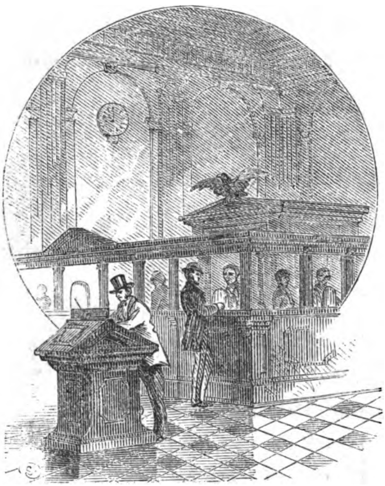
2. *True discount = debt — present worth.*

**311.** RULES.—1. To find the present worth of a debt due at a future time, without interest, 1. *Divide the debt by the amount of $1 for the given time, at the current rate of interest, and multiply $1 by the quotient.* Or,

2. *Divide the debt by the decimal corresponding to the amount of $1 for the given time, and at the current rate of interest.*

## BANK DISCOUNT.

**312.** ***Bank Discount*** is the interest on a note for the number of days from the time it is discounted to the time it is legally due.



**313.** ***The Proceeds*** of a note are its face, or the sum discounted, less the discount. The proceeds are also called *Avails* and *Cash Value.*

**314.** ***Days of Grace*** are the three days allowed for the payment of a note after the specified time has expired.

NOTE.—A note is payable, or *nominally* due, at the expiration of the specified time, but it does not *mature*, or become *legally* due, until the last day of grace, or the day preceding, when the last day of grace falls on Sunday or a legal holiday. The date of expiration and the date of maturity are usually written with a line between them; thus, Jan. 9/12.

**315.** When a bank loans money, the borrower gives his note payable at a specified time, *without interest.* This note is then discounted by the bank for the actual number of days in the time plus the three days of grace, and the *proceeds* are paid to the borrower.

**316.** When a note *drawing interest* is discounted by a

bank, the discount is computed on the *amount* of the note at the time of its *maturity.*

Business men generally discount notes and bills, not drawing interest, by deducting the interest for the time, with or without grace as per agreement. This is sometimes called *Business Discount.* The rate of interest allowed is usually greater than the current rate.

Bills due in three, four, or six months are often discounted by deducting 5% or more of their face, without regard to time. This is called *Per Cent. Off.*

### WRITTEN PROBLEMS.

1. What is the bank discount of a note of $350, payable in 60 days, discounted at 10%? What are the proceeds?

PROCESS.

$$\begin{array}{r} \$350. \\ .0105 \\ \hline 1750 \\ 350\phantom{0} \\ \hline 6\,)\,\$3.6750 \\ \hline \end{array} \qquad \text{Time} = 60 \text{ da.} + 3 \text{ da.} = 63 \text{ da.}$$

$\$.6125 \times 10 = \$6.125$, *Bank discount.*

$\$350 - \$6.125 = \$343.875$, *Proceeds.*

2. What is the bank discount of a note of $250, payable in 90 days, discounted at 8%? What are the proceeds?

Find the bank discount and the proceeds of a note of

3. $145, payable in 60 days, discounted at 7%.
4. $80.50, payable in 30 days, discounted at 8%.
5. $1000, payable in 90 days, discounted at 7½%.
6. $750, payable in 45 days, discounted at 9%.
7. $1250, payable in 100 days, discounted at 6%.
8. $56, dated Jan. 1, 1870, payable May 1, 1870, discount 6%.
9. $120, dated Apr. 3, 1869, payable June 15, 1869, discount 8%.
10. $500, dated Dec. 15, 1870, payable Feb. 18, 1871, discount 9%.
11. $8.75, dated Nov. 21, 1870, payable Mch. 12, 1871, discount 9%.

12. $400, dated Mch. 4, 1871, payable July 24, 1871, discount 5%?

13. What is the difference between the bank discount and the true discount of $1319.50, due in 90 days, discounted at 6%?

14. What is the difference between the bank discount and the true discount of $768, due in 108 days, discount 8%?

15. What is the difference between the bank discount and the true discount of $3330.80, due in 45 days, discount 7%?

16. A note, dated Apr. 10, 1871, is payable in 90 days: what is the time of its maturity?

NOTE.—The date of maturity is found by counting forward the number of days plus three days, when the time is expressed in days; and the number of calendar months plus three days, when the time is expressed in months.

17. A note, dated Feb. 6, 1868, was payable in 60 days: what was the date of its maturity?

18. A note, dated Aug. 9, 1870, is payable 4 months from date: what is the date of its maturity?

19. A note of $460, dated Apr. 3, 1870, and payable in 90 days, with interest at 6%, was discounted May 10, 1870, at 8%: what were the proceeds?

SUGGESTION.—Find the amount of $460 for 93 days, at 6%, and then discount this amount for 56 days, at 8%.

20. A note of $125, dated May 21, 1870, and payable in 60 days, with interest at 6%, was discounted May 25, 1870, at 10%: what were the proceeds?

21. A note of $1000, dated Aug. 15, 1869, and payable in 6 months, with interest at 7%, was discounted Nov. 27, 1869, at 9%: what was the bank discount?

SUGGESTION.—Compute the interest for 6 mo. 3 da., and the discount for 83 days.

22. A note of $90, dated Apr. 12, 1870, and payable in

4 months, with interest at 5 %, was discounted June 1, 1870, at 7 %: what were the proceeds?

23. A note of $650, dated Mch. 2, 1869, payable Apr. 1, 1870, and indorsed $300 Oct. 1, 1869, was discounted Feb. 3, 1870: what were the proceeds at 6 % ?

24. For what sum must a note, payable in 60 days, be drawn to produce $493, when discounted at 8 %?

PROCESS.

$1 — $.014 = $.986, *Proceeds of* $1.
$493 ÷ $.986 = 500
$1 × 500 = $500, *Face of note.*

Since the proceeds of $1 are $.986, it will require as many times $1 to produce $493 as $.986 is contained times in $493, which is 500; and 500 times $1 = $500.

25. For what sum must a note, payable in 90 days, be drawn to produce $1969, when discounted at 6 %?

26. What must be the face of a note, dated July 5, 1871, and payable in 4 months, to produce $811, when discounted at 9 %?

27. What must be the face of a note, dated Jan. 10, 1870, and payable in 3 months, to produce $1938, when discounted at 12 %?

28. A merchant discounted a bill of $750, payable in 4 months, by deducting the interest for the time without grace, at 10 %: what were the cash proceeds of the bill?

29. A note of $340, due in 9 months, without interest, was discounted by deducting the interest for the time, at 8 %: what was the cash value of the note?

30. A merchant having sold a bill of goods amounting to $1030, on three months' time, allowed 5 % off for cash: what were the cash proceeds of the sale?

31. A retail dealer having bought $950 worth of goods, on 6 months' time, cashed the bill for $7\frac{1}{2}$ % off: what were the cash proceeds?

32. A merchant bought, March 20, 1870, a bill of goods amounting to $3540, on three months' time, but, being offered 5 % off for cash, he borrowed the money at a bank

for the time, at 10%, and cashed the bill. How much did he gain by the transaction?

### FORMULAS AND RULES.

**317.** FORMULAS.—1. *Bank discount = sum discounted × (int. of \$1 for the days + 3 days).*

2. *Business discount = sum discounted × int. of \$1 for the time.*

3. *Proceeds = sum discounted — discount.*

**318.** RULES.—1. To compute bank discount, *Find the interest on the sum discounted at the given rate per cent. and for the actual number of days in the time plus three days.*

2. To compute business discount, *Find the interest on the sum discounted, at the given rate per cent. and for the given time.*

3. To find the proceeds, *Subtract the discount from the sum discounted.*

4. To find the face of a note to yield given proceeds, *Divide the given proceeds by \$1 minus the interest of \$1 for the given time with grace, and multiply \$1 by the quotient.*

NOTES.—1. In discounting a note bearing interest, the interest is computed by months or by days, according as the time is expressed in the note, but the discount is usually computed by days.

2. Business discount is computed by months or days, according as the time is expressed in the paper discounted, and with or without grace. In these respects it differs from bank discount.

3. Bank discount is not only interest paid *in advance*, but the interest is computed on both the proceeds *and the discount.* The borrower pays interest on more money than he receives.

## PROMISSORY NOTES AND DRAFTS.

### I. PROMISSORY NOTES.

**319.** A ***Promissory Note*** is a written agreement by one party to pay to another a specified sum at a specified time. The sum whose payment is promised, is called the *Face* of the note.

The person who signs a note is called its *Maker;* the person to whom it is payable is the *Payee;* and its owner is the *Holder.*

**320.** A ***Joint Note*** is a note signed by two or more persons who are jointly liable for its payment.

**321.** A ***Joint and Several Note*** is a note signed by two or more persons who are both jointly and singly liable for its payment.

**322.** An ***Indorser*** is a person who signs his name on the back of a note as security for its payment.

**323.** The following are the more common forms of promissory notes:

FORM I.—DEMAND NOTE.

$95\frac{50}{100}$. NASHVILLE, TENN., May 1, 1870.

*For value received, I promise to pay to John Wilson, on demand, Ninety-five $\frac{50}{100}$ Dollars.*

HENRY SMITH.

[STAMP.]

FORM II.—TIME NOTE.

$95\frac{50}{100}$. ST. LOUIS, MO., May 1, 1870.

*Ninety days after date, I promise to pay to John Wilson, or bearer, Ninety-five $\frac{50}{100}$ Dollars, with interest, for value received.*

HENRY SMITH.

[STAMP.]

FORM III.—JOINT AND SEVERAL NOTE.

$95\frac{50}{100}$. LOUISVILLE, KY., March 12, 1870.

*Four months after date, we jointly and severally promise to pay Henry Cooke, or order, Ninety-five $\frac{50}{100}$ Dollars, with interest at 8 %, for value received.*

THOMAS HUGHES,
CHARLES G. KNIGHT.

[STAMP.]

FORM IV.—NOTE PAYABLE AT A BANK.

$500. BALTIMORE, MD., Apr. 10, 1870.

*Sixty days after date, we promise to pay to Wilson, Hinkle & Co., or order, at the First National Bank, Five Hundred Dollars, for value received.*

CHARLES COOKE & Co.

[STAMP.]

REMARKS.—1. A note should contain the words "value received," otherwise the holder may be required to prove that the maker received its value.

2. When the time for the payment of a note is not specified, it is due on demand. If the place of payment is not mentioned, it is payable at the maker's residence or place of business.

3. When a note contains the words "with interest," and no rate is specified (Form II.), interest accrues at the legal rate. If the words "with interest," are omitted (Form IV.), no interest accrues until after maturity, when the note draws interest, at the legal rate, until paid.

**324.** A ***Negotiable Note*** is one which may be bought and sold.

A note is negotiable when it is made payable "to the bearer," or to the payee "or bearer," or to the payee "or order," or "to the order of" the payee. A note drawn as in Form I. is not negotiable.

A note made payable to the *bearer* is negotiable without indorsement. U. S. treasury notes and bank notes, used as money, are payable to the bearer, and are transferred by delivery.

A note payable to *order* must be *indorsed* by the payee before it is negotiable.

When the payee indorses a note by simply writing his name on the back, it is called an *indorsement in blank*, and the note is payable to the holder. When the indorser orders the payment to be made to a particular person, as: "Pay to Charles Williams," it is called a *special indorsement.*

**325.** If the maker of a note fails to pay it at maturity, a written notice of the fact, made by a notary public, is served on the indorsers, who are responsible for the payment of the note. Such a notice is called a *Protest.*

NOTE.—A protest must be made out on the day a note matures, and it must be sent on that day or the next, otherwise the indorsers are not responsible.

## II. DRAFTS, OR BILLS OF EXCHANGE.

**326.** A ***Draft*** is an order made by one person upon another to pay a specified sum to a third person named. It is also called a *Bill of Exchange.*

The person who makes the order, is called the *Drawer;* the person to whom it is addressed, is called the *Drawee;* and the person to whom the money is payable, is the *Payee.*

**327.** The following are the common forms of drafts:

### FORM I.—SIGHT DRAFT.

$100. CINCINNATI, O., Oct. 1, 1870.

*Pay to the order of Bartlit & Smith, One Hundred Dollars, and place to the account of*

CHARLES S. KELLEY.

*To* GEORGE BROWN, Esq., New York.

[STAMP.]

### FORM II.—TIME DRAFT.

$100. CINCINNATI, O., Oct. 1, 1870.

*Thirty days after sight* [*or date*], *pay to the order of Bartlit & Smith, One Hundred Dollars, and place to the account of*

CHARLES S. KELLEY.

*To* GEORGE BROWN, Esq., New York.

[STAMP.]

When the drawee accepts a draft, he writes the word "Accepted," with the date, across the face, and signs his name, thus: "Accepted, Oct. 3, 1870—George Brown." The draft is then called an *Acceptance,* and the acceptor is responsible for its payment.

A draft made payable to *bearer* or *order* is negotiable, like a promissory note, and is subject to *protest,* in case payment or acceptance is refused.

NOTES.—1. In most of the states both time and sight drafts are entitled to three days of grace. In New York no grace is allowed on sight drafts.

2. When a draft is drawn, "acceptance waived," it is not subject to protest until maturity; and when an indorser writes over his name, "demand and notice waived," a protest in his case is not necessary.

3. The liability of an indorser of a note or draft may be avoided by his writing over his indorsement, "without recourse."

**328.** A ***Domestic*** or ***Inland Bill*** is a draft which is payable in the country where it is drawn.

A ***Foreign Bill*** is a draft which is drawn in one country and is payable in another.

**329.** ***Exchange*** is the process of making payments at distant places by the remittance of drafts, instead of money.

When a draft can be bought for its face, it is said to be *at par;* when the cost is less than the face, it is *below par,* or at a *discount;* and when the cost is more than the face, it is *above par,* or at a *premium.* The rate per cent., which the cost of a draft is more or less than its face, is called the *Rate of Exchange.*

NOTES.—1. The rate of exchange between two places depends chiefly on their relative trade. If Cincinnati owes New York, drafts on New York are at a premium in Cincinnati; if New York owes Cincinnati, drafts on New York are at a discount; if the trade of the two cities with each other is equal, exchange is at par.

2. In foreign exchange, drafts are expressed in the currency of the country on which they are drawn. The comparative value of the money of two countries is called the *Par of Exchange.*

## WRITTEN PROBLEMS.

1. What is the cost of a draft on New York for $800, exchange being $\frac{3}{4}$% premium?

PROCESS: $800 × .00$\frac{3}{4}$ = $6, *Prem.* $800 + $6 = $806, *Cost.*

2. What is the cost of a draft on New Orleans for $1250, at $\frac{1}{2}$% discount?

3. What is the cost of a draft on Philadelphia for $1050, at $\frac{1}{4}$% premium?

4. A merchant in St. Louis wishes to remit $2500 by draft to New York: what will be the cost of the draft, exchange being $1\frac{1}{2}$% premium?

5. What will be the cost of a draft for \$500, payable in 30 days after sight, exchange being 1 % premium, and interest 6 %?

PROCESS.

| | | |
|---|---|---|
| | \$500. | |
| \$500 × .0055 = | \$2.75 | *Discount at 6 % for 33 days.* |
| | \$497.25 | *Proceeds of Draft* (cost at par). |
| \$500 × .01 = | 5.00 | *Premium at 1 %.* |
| | \$502.25 | *Cost of Draft.* |

NOTE.—If preferred, the face may be multiplied by the cost of \$1, which is \$1 — \$.0055 + \$.01, or \$1.0045. \$500 × 1.0045 = \$502.25.

6. What will be the cost of a draft for \$650, payable in 60 days after sight, exchange being $\frac{1}{2}$ % premium, and interest 8 %?

7. What will be the cost of a draft for \$320, payable in 45 days after sight, exchange being $\frac{3}{4}$ % discount, and interest 7 %?

8. How large a sight draft can be bought for \$259.52, exchange being $1\frac{3}{8}$ % premium?

PROCESS: \$259.52 ÷ $1.01\frac{3}{8}$ = \$256, *Face of Draft.* (Case IV.)

9. How large a sight draft can be bought for \$962.85, exchange being $1\frac{3}{4}$ % discount?

10. A sight draft, bought at $\frac{1}{2}$ % premium, cost \$1256.25: what was its face?

11. How large a draft, payable 30 days after sight, can be bought for \$502.25, exchange being 1 %, and interest 6 %?

PROCESS.

\$1 — \$.0055 = \$.9945 *Proceeds of \$1 discounted for 33 days.*
\$.9945 + \$.01 = \$1.0045 *Cost of \$1.*
\$502.25 ÷ \$1.0045 = \$500 *Face of Draft.*

12. How large a draft, payable 60 days after sight, can be bought for \$798.80, exchange being $1\frac{1}{4}$ % premium, and interest 8 %?

13. A draft, payable in 30 days after sight, was bought for \$352.62, exchange being $1\frac{1}{2}$ % discount, and interest 6 %: what was its face?

## BONDS.

**330.** The interest-bearing notes issued by nations, states, cities, railroad companies, and other corporations, as a means of borrowing money, are called *Bonds*.

Bonds are issued in denominations of convenient size, with interest usually payable annually or semi-annually, and they are made negotiable like certificates of capital stock. (Art. 243.)

The *Coupons* attached to bonds are due-bills for the interest, which, as the interest becomes due, are cut off and presented for payment.

**331.** The several classes of bonds issued by the United States Government, are called *United States Securities*, or *Government Securities*, the principal of which are known as *Sixes of* 1881, *Five-Twenties*, and *Ten-Forties*.

The 6's *of* '81, bear interest at 6 % in gold, payable semi-annually, and are payable in 1881.

The 5-20's bear interest at 6% in gold, payable semi-annually, and are redeemable, at the option of the Government, after *five* years from date, and are payable in *twenty* years. The several series were issued in 1862 (Old 5-20's), 1864, 1865, 1867, and 1868.

The 10-40's, issued in 1864, bear interest at 5% in gold, payable semi-annually (except the $50 and $100 bonds bearing annual interest), and are redeemable, at the option of the Government, in *ten* years, and are payable in *forty* years.

**332.** The market value of United States bonds is quoted at a certain per cent. of their par value or face. Bonds quoted at 110 are worth in currency 110 % of their face; that is, are 10 % above par. The quotation includes accrued interest.

### WRITTEN PROBLEMS.

1. When U. S. 5-20's are quoted at 109½, what will three $500 bonds cost?

2. When U. S. 6's of '81 are worth 114, what will $1250 in bonds cost?

3. A widow invested $4725 in U. S. 10-40's, at 105: what amount in bonds did she receive?

4. A broker invested $26250 in 5-20's at $106\frac{1}{2}$, and sold them at 109: how much did he gain?

5. When gold is at 115, what amount in currency can be bought for $8500 in gold?

6. At 112, what amount in gold can be bought for $1400 in treasury notes?

7. When gold is at 115, and U. S. 10-40's at 105, what is a $500 bond worth in gold?

8. When gold is at 120 and U. S. 5-20's at 115, what is the gold value of a $1000 bond?

9. When gold is at 110 and U. S. 10-40's at 108, what is the gold value of a $500 bond?

10. When gold is worth $112\frac{1}{2}$, what is the gold value of a dollar treasury note?

11. When gold is at 115, what is the semi-annual interest in currency on $9500 in 10-40's?

12. When gold is at 120, what rate per cent. in currency is the interest on 5-20's?

13. A college invested its endowment fund in U. S. 5-20's, at 106: what rate of interest in currency will it receive when gold is at 120? At 110? At par?

## ANNUAL INTEREST.

**333.** When a note reads "with interest payable annually," the interest on the face of the note, due at the close of each year, is called ***Annual Interest.***

**334.** When annual interest is not paid at the close of the year, when due, it draws *simple interest* until paid.

### WRITTEN PROBLEMS.

1. A note of $500, dated May 10, 1870, is due in 4 years, with interest at 6 %, payable annually: if both interest and principal remain unpaid, what will be the amount due on the note at maturity?

PROCESS.

$\$500 \times .06 = \$30$, *Interest on principal due annually.*
$\$30 \times 4 =$ $120, *Total annual interest.*

| | | |
|---|---|---|
| $\$30 \times .06 \times 3 =$ | 5.40, | *Simple interest on 1st annual interest for 3 yrs.* |
| $\$30 \times .06 \times 2 =$ | 3.60, | " " " *2d* " " " 2 " |
| $\$30 \times .06 \times 1 =$ | 1.80, | " " " *3d* " " " 1 " |
| | $130.80, | *Total interest due at maturity of note.* |
| | 500.00 | |
| | $630.80, | *Amount due at maturity of note.* |

The $30 annual interest, due at the close of the 1st year, being unpaid, draws simple interest until paid, or for 3 years; the $30 annual interest due at the close of the 2d year, draws interest for 2 years; and the $30 annual interest due at the close of the 3d year, draws interest for 1 year. The fourth annual interest is paid when due. Hence, the total interest due at the maturity of the note, consists of (1) the *annual interest* for 1 year ($30) multiplied by 4, the number of years; and (2) the *simple interest* on the $30 annual interest for 3 years, 2 years, and 1 year, or for 6 years. The amount due is $\$500 + \$30 \times 4 + \$1.80 \times 6$.

2. A note of $750, with interest payable annually, at 8%, was paid 3 yr. 3 mo. 18 da. after date, and no interest had been previously paid: what was the amount due?

PROCESS.

$\$750 \times .08 =$ $60. *Interest on face due annually.*
$\$60 \times 3.3 = \$198.00$, *Total annual int. for 3 yr. 3 mo. 18 da. (3.3 yr.)*
$\$60 \times .312 = \$18.72$, *Simple interest on $60 for 3 yr. 10 mo. 24 da.*
$750.
$966.72, *Amount due.*

The first annual interest draws simple interest for 2 yr. 3 mo. 18 da.; the second, for 1 yr. 3 mo. 18 da.; and the third, for 3 mo. 18 da.; and hence, the simple interest on the several annual interests equals the interest of $60 for 2 yr. 3 mo. 18 da. + 1 yr. 3 mo. 18 da. + 3 mo. 18 da., or for 3 yr. 10 mo. 24 da. The amount due consists of (1) *the principal;* (2) *the total annual interest;* and (3) *the simple interest on the annual interest.*

3. A note of $1000, with annual interest at 6%, is due 4 yr. 6 mo. after date: no interest having been paid, what will be due at maturity?

4. A man bought a farm for $3500, to be paid in 4 years, with annual interest at 6%, but failed to pay the interest: what was due at the close of the 4th year?

5. $650. NEW YORK, July 1, 1869.

On the first day of January, 1872, for value received, I promise to pay John Black, or order, six hundred and fifty dollars, with annual interest at 7%.

CHARLES CHURCH.

If no interest be paid on the above note, what will be due at maturity?

6. If the above note and interest be not paid until Sept. 13, 1872, what will be the amount due?

7. A note of $800, dated March 18, 1867, and due in 3 years, with interest at 6%, payable annually, has the following indorsements: Oct. 24, 1868, $150: Nov. 12, 1869, $240. What was the amount due March 18, 1870?

PROCESS.

| | |
|---|---|
| $800 × .06 = $48, | *First annual interest.* |
| $48 × 2 = $96, | *Annual interest due Mch.* 18, 1869. |
| $48 × .06 = 2.88, | *Interest on* 1*st annual interest.* |
| $98.88, | *Interest due Mch.* 18, 1869. |
| 800 | |
| $898.88, | *Amount due Mch.* 18, 1869. |
| Payment, $150. Int. on same, 3.60. } = $153.60, | *Amount of* $150, Mch. 18, 1869. |
| $745.28, | *New principal.* |
| $745.28 × .06 = $44.716, | *Interest due Mch.* 18, 1870. |
| $789.996, | *Amount due Mch.* 18, 1870. |
| Payment, $240. Int. on same, 5.04. } = $245.04, | *Amount* $240, *Mch.* 18, 1870. |
| $544.956, | *Amount due at maturity.* |

NOTE.—The annual interest and the interest on the same are computed to the close of the year in which the first payment is made, and the interest on the payment is computed to the same date. The difference between the amount of the face of the note and the amount of the payment is the new principal for the third year.

8. A note of $500, dated Jan. 15, 1865, and due in 2 years, with interest at 10%, payable annually, is indorsed

as follows: May 21, 1866, \$100; Mch. 9, 1867, \$200. What was the amount due July 15, 1867?

**335.** RULES.—1. To compute unpaid annual interest, *Compute the interest on the principal for the entire time it is on interest, and the interest on each year's interest for the time it is unpaid. The sum of the principal, the interest on the principal, and the interest on the unpaid interest will be the amount due.*

NOTE.—Instead of computing the interest on the several annual interests separately, *simple interest may be computed on one year's interest for a time equal to the sum of the periods of time the several annual interests are unpaid.*

2. To compute annual interest when partial payments have been made, 1. *Compute the interest on the principal to the end of the first year in which any payment is made, and also the interest on the unpaid annual interest to the same date, and form the amount.*

2. *Compute the interest on the payment or payments to the end of the year, and form the amount.*

3. *Subtract the amount of the payment or payments from the amount of the principal and interest, and taking the difference for a new principal, proceed as before with succeeding payments, making the date of settlement the last date.*

NOTE.—1. This rule is based on the principle that the payments with added interest should be applied first to the discharge of the accrued interest at the end of a year, and then to the discharge of the principal. When the amount of the payment or payments will not cancel all the interest due, the unpaid interest draws *simple interest* to the end of the next year in which a payment is made.

## COMPOUND INTEREST.

**336.** ***Compound Interest*** is interest on the principal and also on the interest which, at regular intervals of time, is added to the principal. It is generally compounded annually, semi-annually, or quarterly.

### WRITTEN PROBLEMS.

1. A note of $450 is due in 3 years with interest at 6 %, compounded annually. What will be the amount due at maturity? What will be the compound interest due?

PROCESS.

| | |
|---|---|
| $450 | |
| .06 | |
| $27.00 | *1st year's interest.* |
| 450. | |
| $477 | *2d principal.* |
| .06 | |
| $28.62 | *2d year's interest.* |
| 477. | |
| $505.62 | *3d principal.* |
| .06 | |
| $30.3372 | *3d year's interest.* |
| 505.62 | |
| $535.9572 | *Amount due at the end of the third year.* |
| 450. | |
| $85.9572 | *Compound interest at the end of the third year.* |

2. What is the amount of $600 for 4 years at 5 %, compounded annually? What is the compound interest?

3. What is the compound interest of $1500 for 3 years at 10 %? At 8 %?

4. What is the amount of $800 for 2 years at 6 %, compounded semi-annually?

SUGGESTION.—Compute the interest at 3 %.

5. What is the compound interest of $650 for 3 yr. 4 mo. 12 d. at 6 % per annum?

**337.** RULE.—To compute compound interest, *Find the amount of the given principal for one interval of time; then, taking this amount as a new principal, find the amount for the second interval, and so continue for the entire time. The difference between the last amount and the principal is the compound interest for the time.*

NOTES.—1. When the interest is compounded semi-annually, the rate per cent. is one half the yearly rate, and when compounded quarterly, it is one fourth the yearly rate.

2. When the time contains years, months, and days, the amount is found for the number of whole intervals in the time, and then the interest is computed on this amount for the remaining months and days.

**338.** Compound interest is usually computed by the aid of a table giving the amount of $1 at several different rates per cent., and for any number of years which may be included.

## 339. A TABLE

*Showing the amount of* $1 *at compound interest, at* 3, 4, 5, 6, 7, *or* 8 *per cent., for any number of years from* 1 *to* 25.

| YRS. | 3 PER CENT. | 4 PER CENT. | 5 PER CENT. | 6 PER CENT. | 7 PER CENT. | 8 PER CENT. |
|---|---|---|---|---|---|---|
| 1 | 1.03 | 1.04 | 1.05 | 1.06 | 1.07 | 1.08 |
| 2 | 1.0609 | 1.0816 | 1.1025 | 1.1236 | 1.1449 | 1.1664 |
| 3 | 1.092727 | 1.124864 | 1.157625 | 1.191016 | 1.225043 | 1.259712 |
| 4 | 1.125509 | 1.169859 | 1.215506 | 1.262477 | 1.310796 | 1.360489 |
| 5 | 1.159274 | 1.216653 | 1.276282 | 1.338226 | 1.402552 | 1.469328 |
| 6 | 1.194052 | 1.265319 | 1.340096 | 1.418519 | 1.500730 | 1.586874 |
| 7 | 1.229874 | 1.315932 | 1.407100 | 1.503630 | 1.605781 | 1.713824 |
| 8 | 1.266770 | 1.368569 | 1.477455 | 1.593848 | 1.718186 | 1.850930 |
| 9 | 1.304773 | 1.423312 | 1.551328 | 1.689479 | 1.838459 | 1.999005 |
| 10 | 1.343916 | 1.480244 | 1.628895 | 1.790848 | 1.967151 | 2.158925 |
| 11 | 1.384234 | 1.539454 | 1.710339 | 1.898299 | 2.104852 | 2.331639 |
| 12 | 1.425761 | 1.601032 | 1.795856 | 2.012196 | 2.252192 | 2.518170 |
| 13 | 1.468534 | 1.665074 | 1.885649 | 2.132928 | 2.409845 | 2.719624 |
| 14 | 1.512590 | 1.731676 | 1.979932 | 2.260904 | 2.578534 | 2.937194 |
| 15 | 1.557967 | 1.800944 | 2.078928 | 2.396558 | 2.759032 | 3.172169 |
| 16 | 1.604706 | 1.872981 | 2.182875 | 2.540352 | 2.952164 | 3.425943 |
| 17 | 1.652848 | 1.947900 | 2.292018 | 2.692773 | 3.158815 | 3.700018 |
| 18 | 1.702433 | 2.025817 | 2.406619 | 2.854339 | 3.379932 | 3.996019 |
| 19 | 1.753506 | 2.106849 | 2.526950 | 3.025600 | 3.616528 | 4.315701 |
| 20 | 1.806111 | 2.191123 | 2.653298 | 3.207135 | 3.869684 | 4.660957 |
| 21 | 1.860295 | 2.278768 | 2.785963 | 3.399564 | 4.140562 | 5.033834 |
| 22 | 1.916103 | 2.369919 | 2.925261 | 3.603537 | 4.430402 | 5.436540 |
| 23 | 1.973587 | 2.464716 | 3.071524 | 3.819750 | 4.740530 | 5.871464 |
| 24 | 2.032794 | 2.563304 | 3.225100 | 4.048935 | 5.072367 | 6.341181 |
| 25 | 2.093778 | 2.665836 | 3.386355 | 4.291871 | 5.427433 | 6.848475 |

**340.** *The amount of $1 for the given time and rate, multiplied by the given principal, gives its amount for the same time and rate.*

NOTE.—When the interest is compounded semi-annually, it is computed from the table by taking one half the rate and twice the number of years.

6. What is the compound interest of $750 for 15 years at 6 %? What is the amount?

7. What is the amount of $500 for 6 years at 8 %, compounded semi-annually?

8. What is the amount of $1250 for 10 yr. 4 mo. 15 da. at 5 %, compound interest?

## EQUATION OF PAYMENTS.

**341.** ***Equation of Payments*** is the process of finding an equitable time for the payment of several debts, due at different times, without interest. It is also called the *Average of Payments.*

The equitable time sought is called the *Average Time,* or the *Equated Time.*

### PROBLEMS.

1. A owes B $300, of which $200 is due in 3 months, and $100 in 6 months: when will the payment of $300 equitably discharge the debt?

PROCESS.

$200 × 3 = $600
$100 × 6 = $600
$300 ) $1200
4
*Ans.*, 4 mos.

A is entitled to the use of $200 for 3 months, which equals the use of $600 for 1 month, and to the use of $100 for 6 months, which equals the use of $600 for 1 month; and, hence, he is entitled to the use of $300 until it equals the use of $600 + $600, or $1200, for 1 month. It will take $300 as many months to equal the use of $1200 for 1 month, as $300 is contained times in $1200, which is 4. Hence, the payment of $300 in 4 months will equitably discharge the debt.

**Proof.**—In paying the $200 in 4 months, A *gains* the use of $200 for 1 month, and in paying the $100 in 4 months, he *loses* the use of $100 for 2 months, which equals the use of $200 for 1 month. Hence, his gain and loss are *equal*.

2. A owes a merchant $200 due in 4 months, and $600 due in 8 months: what is the equated time for the payment of both debts?

3. A owes B $1200, of which $300 are due in 4 months, $400 in 6 months, and the remainder in 12 months: what is the equated time for the payment of the whole?

4. A owes B $800, of which $\frac{1}{8}$ is due in 2 months, $\frac{1}{2}$ in 3 months, and the remainder in 6 months: what is the equated time for the payment of the whole?

5. A man owes $300 due in 4 months, $600 due in 5 months, and $700 due in 10 months: what is the equated time for the payment of the whole?

6. Smith & Jones bought $500 worth of goods on 4 months' credit, $700 worth on 6 months' credit, and $1000 worth on 5 months' credit: what is the equated time for the payment of the whole?

7. A bought $2000 worth of goods, $\frac{1}{4}$ of which was to be paid down, $\frac{1}{5}$ in 3 months, $\frac{1}{4}$ in 4 months, and the remainder in 8 months: what is the equated time for the payment of the whole?

8. What is the equated time for the payment of $220, due in 30 days; $300, due in 40 days; $250, due in 60 days; and $100, due in 90 days?

9. What is the equated time for the payment of $300, due in 30 days; $250, due in 45 days; and $350, due in 60 days?

**Process by Interest.**

| | | | |
|---|---|---|---|
| Int. of | $300 | for 30 days, at 6% = | $1.50 |
| Int. of | $250 | for 45 " " 6% = | 1.875 |
| Int. of | $350 | for 60 " " 6% = | 3.50 |
| | $900 | | $6.875 |

$9.00 = Int. of $900 for 60 days.
.15 = " " $900 for 1 day.

$6.875 ÷ $.15 = 45.9. *Ans.*, 46 days.

The debtor is entitled to the use (1) of \$300 for 30 days, which, at 6%, equals \$1.50 interest; (2) of \$250 for 45 days, which equals \$1.875 interest; (3) of \$350 for 60 days, which equals \$3.50 interest. Hence, he is entitled to the use of \$900, the sum of the debts, until the interest thereon, at 6%, equals the sum of \$1.50 + \$1.875 + \$3.50, which is \$6.875. The interest of \$900 for 1 day is \$.15; and since \$6.875 ÷ \$.15 = 45.9, it will take 45.9 days for \$900 to yield \$6.875 interest. The equated time for payment is 46 days.

NOTE.—When the fraction of a day in the equated time is more than ½, it is counted as a day; when it is less than ½, it is disregarded.

10. What is the equated time for the payment of \$520, due in 45 days; \$340, due in 60 days; and \$640, due in 90 days?

11. What is the equated time for the payment of \$375, due now; \$425, due in 30 days; \$500, due in 60 days; and \$600, due in 75 days?

12. What is the equated time for the payment of \$340, due May 10, 1870; \$450, due June 10; \$560, due July 15; and \$650, due Aug. 10?

NOTE.—Begin with the first date (May 10), and find the exact number of days between it and each succeeding date. The equated time is counted forward from the first date.

13. What is the equated time for the payment of \$1000, due June 1, 1870; \$850, due July 1; \$750, due Sept. 1; and \$900, due Oct. 1?

14. What is the equated time for the payment of \$75, due May 6, 1870; \$115, due May 26; \$220, due June 25; \$315, due July 16; and \$350, due July 30?

## PRINCIPLES AND RULES.

**342.** The time between the contraction of a debt and its payment is called the *Term of Credit*, or *Time of Credit.*

**343.** PRINCIPLES.—1. *The payment of a sum of money* BEFORE *it is due is offset by keeping an equal sum of money an equal time* AFTER *it is due.*

2. *The use of any sum of money is measured by its interest for the time.*

**344.** Rules.—To equate the time of several debts or payments,

1. *Multiply each debt or payment by its time of credit, and divide the sum of the products by the sum of the debts or payments.* Or,

2. *Compute the interest of each debt or payment for its time of credit, and divide the sum of the interests by the interest of the sum of the debts or payments for one month or one day.*

Notes.—1. As the result will be the same at any rate, the interest may be computed at that rate which is most convenient.

2. The correctness of each of the above methods has been called in question by a number of authors, who commend the following as "the only accurate rule":

"*Find the present worth of each of the given amounts due; then find in what time the sum of these present worths will amount to the sum of all the payments.*

The *inaccuracy* of this so-called "accurate rule" is easily shown. The methods given above are both strictly accurate, and they are in general use. (See appendix.)

**345.** When partial payments are made on a debt before it is due, the time for the payment of the balance of the debt is proportionately extended.

15. A owes a merchant $200, due in 12 months, without interest; in 4 months he pays $50 on the debt, and in 8 months, $50: when in equity should he pay the balance?

Process.

$$\$50 \times 8 = \$400$$
$$\underline{\$50 \times 4 = \$200}$$
$$\$200 - \$100 = \$100\,)\,\underline{\$600}$$
$$6$$

In paying $50 in 4 months, A loses its use for 8 months, and in paying $50 in 8 months, he loses its use for 4 months, and hence he loses the use of $400 + $200, or $600, for 1 month. To offset this loss, he is entitled to keep the balance ($100) 6 months *after* its maturity.

16. A owes B $300, due in 8 months: if he pay $200 in 5 months, when should he pay the balance?

17. A man bought a horse, agreeing to pay $150 in 6

months, without interest: if he pay $50 down, when should he pay the balance?

18. A owes B $600, payable in 6 months, but, at the close of 3 months, he proposes to make a payment sufficiently large to extend the time for the payment of the balance 6 months. How large a payment must he make?

19. A owed B $1500, due in 12 months, but in 4 months paid him $400, and in 6 months $500: when in equity ought the balance to be paid?

20. Clark and Brown bought March 10, 1870, a bill of goods amounting to $2500, on 4 months' credit; but they paid $650 Apr. 7; $500 Apr. 30; and $350 May 20. When ought they to pay the balance?

**346.** RULE.—*Multiply each payment by the time it was paid before it was due, and divide the sum of the products by the balance unpaid.*

## EQUATION OF ACCOUNTS.

**347.** ***Equation of Accounts*** is the process of finding the equated time for the payment of the balance of an account; or the time when the balance was due.

### Case I.

#### Accounts containing only Debit Items.

#### PROBLEMS.

1. A bookseller bought of Wilson, Hinkle & Co. the following bills of goods, on 4 months' credit:

| | | | | |
|---|---|---|---|---|
| Feb. | 3, | 1870, | a bill of | $450. |
| " | 24, | " | " | 500. |
| Mch. | 25, | " | " | 750. |
| Apr. | 20, | " | " | 600. |

What is the equated time of maturity?

PROCESS.

| | | | | |
|---|---|---|---|---|
| Due June | 3, 1870, | $450 × 00 = | | |
| " " | 24, " | 500 × 21 = | 10500 | |
| " July | 25, " | 750 × 52 = | 39000 | |
| " Aug. | 20, " | 600 × 78 = | 46800 | |
| | | $2300 | ) $96300 ( | 41.8 days. |

The equated date of maturity of the above bills is 42 days from June 3, 1870, which is July 15, 1870.

NOTES.—1. The date of maturity of each bill is found by counting forward 4 months from the date of purchase. The same result would be obtained by finding the average or equated date of purchase, and counting forward 4 months.

2. The equated time of maturity may also be found by beginning at the last date, and taking the exact number of days between each preceding date and the last date for a multiplier. The equated date is then found by counting back from the last date.

2. Murray & Co. bought of Smith & Moore goods as follows:

| | | | |
|---|---|---|---|
| Apr. 15, 1869, | a bill of | $400, | on 3 mo. credit. |
| May 20, " | " | 245, | on 4 " " |
| June 25, " | " | 375, | on 4 " " |
| Sept. 15, " | " | 625, | on 3 " " |

What is the equated time of maturity?

3. A merchant has the following charges against a customer:

| | | |
|---|---|---|
| May 9, 1870, | $340, | on 4 mo. credit. |
| June 6, " | 530, | on 4 " " |
| July 8, " | 213, | on 3 " " |
| Aug. 30, " | 150, | on 4 " " |

What is the equated time of maturity?

4. J. O. Bates & Co. bought of Smith & Brown several bills of goods, as follows:

| | | | |
|---|---|---|---|
| March 3, 1868, | a bill of | $250, | on 3 mo. credit. |
| April 15, " | " | 180, | on 4 " " |
| June 20, " | " | 325, | on 3 " " |
| Aug. 10, " | " | 80, | on 3 " " |
| Sept. 1, " | " | 100, | on 4 " " |

What is the equated date of maturity? How much would pay the account Dec. 1, 1868?

**348.** RULE.—To find the equated time of maturity for the debit items of an account, *First find the maturity of each item or bill, and then, counting from the first date for the time of credit, find the equated time as in the equation of payments. The date of the equated time is found by counting forward from the first date.*

NOTES.—1. The equated time may be found by interest, as in the Equation of Payments. (Art. 344, Rule 2.)

2. The sum of the debit items draws interest from the equated date of maturity to the date of payment.

## Case II.

### Accounts containing both Debits and Credits.

5. What is the equated date of maturity of each side of the following account?

*Dr.* JOHN SMITH *in account with* JOHN JONES. *Cr.*

| 1868. | | | *Time of Cred.* | 1868. | | |
|---|---|---|---|---|---|---|
| Apr. 3, | To Mdse. | $220 | 3 mo. | July 1, | By Cash | $200 |
| June 1, | " | 125 | 4 " | Oct. 3, | " | 150 |
| July 15, | " | 200 | 4 " | Dec. 20, | " | 300 |
| Aug. 24, | " | 140 | 6 " | | | |
| Oct. 1, | " | 190 | 6 " | | | |

PROCESS.

*Debits.*

| Due | | | | |
|---|---|---|---|---|
| July 3, 1868, | $220 × | 00 = | | |
| Oct. 1, " | 125 × | 90 = | 11250 | |
| Nov. 15, " | 200 × | 135 = | 27000 | |
| Feb. 24, 1869, | 140 × | 236 = | 33040 | |
| Apr. 1, " | 190 × | 272 = | 51680 | |
| | $875 | ) | $122970 | |
| | | | 141 | |

Debits are due 141 days from July 3, 1868, which is Nov. 21.

*Credits.*

| Due | | | |
|---|---|---|---|
| July 1, 1868, | $200 × | 00 = | |
| Oct. 3, " | 150 × | 94 = | 14100 |
| Dec. 20, " | 300 × | 172 = | 51600 |
| | $650 | ) | $65700 |
| | | | 101 |

Credits are due 101 days from July 1, which is Oct. 10.

NOTE.—Each side of the account may be equated without reference to the other, as is done above, or the first or last date of the account may be made a common *starting-point* for both sides.

6. The above account, as equated, stands thus:

| | *Dr.* | | *Cr.* |
|---|---|---|---|
| Due Nov. 21, 1868 . . | $875 | Due Oct. 10, 1868 . . | $650 |

When is the *balance* of the account due?

PROCESS.

| | | | |
|---|---|---|---|
| Debits . . . . . . . . . . | $875 | | $650 |
| Credits . . . . . . . . . . | 650 | | 42 |
| Balance . . . . . . . . | $225 | $225 ) | $27300 |
| Difference in time, 42 days. | | | 121 |

Balance is due 121 days from Nov. 21, 1868, which is March 22, 1869.

Suppose the account settled Nov. 21, the *later* date. Since the credit side of the account has been due since Oct. 10, it has been drawing interest for 42 days. To increase the debit side of the account by an equal amount of interest, the balance must remain unpaid 121 days. Counting *forward* 121 days from Nov. 21, the balance is found to be due March 22, 1869.

7. Suppose that the debit and credit sides of an account when equated stand as follows:

| | *Dr.* | | *Cr.* |
|---|---|---|---|
| Due Nov. 21, 1868 . . | $650 | Due Oct. 10, 1868 . . | $875 |

What would be the equated time of payment for the balance?

PROCESS.

| | | | |
|---|---|---|---|
| Credits . . . . . . . . . . | $875 | | $875 |
| Debits . . . . . . . . . . | 650 | | 42 |
| Balance . . . . . . . . | $225 | $225 ) | $36750 |
| Difference in time, 42 days. | | | 163 |

Balance is due 163 days *prior* to Nov. 21, 1868, which is June 11, 1868.

Suppose the account settled Nov. 21, as before. The credit side, having been due since Oct. 10, has been drawing interest for 42 days. That the debit side of the account may be increased by an equal amount of interest, the balance must be regarded as due 163 days prior to Nov. 21.

8. The debit and credit sides of an account when equated stand as follows:

| *Dr.* | | *Cr.* | |
|---|---|---|---|
| Due June 5, 1870 . . | $1285 | Due July 1, 1870 . . | $1000 |

What is the equated time of payment for the balance?

9. At what time did the balance of the following equated account begin to draw interest:

| *Dr.* | | *Cr.* | |
|---|---|---|---|
| Due July 12, 1870 . . | $450 | Due Sept. 1, 1870 . . | $800 |

10. When will the balance of the following account begin to draw interest, the debit items having a credit of 3 months?

*Dr.* R. HILL & Co., *in account with* O. COOKE. *Cr.*

| | | | | | |
|---|---|---|---|---|---|
| 1870. | | | 1870. | | |
| July 10 | To Mdse. | $120 | Nov. 20 | By Cash | $350 |
| " 30 | " | 450 | Dec. 25 | " Mdse. | 250 |
| Aug. 30 | " | 380 | 1871. | | |
| Sept. 9 | " | 560 | | | |
| " 30 | " | 400 | Jan. 1 | " Cash | 750 |

**349.** RULE.—To find the equated time for the payment of the balance of an account,

1. *Find the equated time for each side of the account.*

2. *Multiply the side of the account which falls due* FIRST *by the number of days between the dates of the equated time of the two sides, and divide the product by the balance of the account.*

3. *The quotient will be the number of days to the maturity of the balance, to be counted* FORWARD *from the later equated date when the* SMALLER *side of the account falls due first, and* BACKWARD *when the* LARGER *side falls due first.*

NOTES.—1. When an account is settled by *cash*, each side of the account is increased by its interest from maturity to the date of settlement, and the difference between the two sides thus increased by interest, is called the *Cash Balance.* Instead of adding the accrued interest to each side, the *balance of interest* may be found and added

to or subtracted from the balance of items, according as the two balances fall upon the same or upon opposite sides of the account. Thus, in problem 6 above, the balance of interest, which is the interest of $650 for 42 days, falls on the credit side, and the balance of items on the debit side. The cash balance is $225 — $3.90, which is $221.10.

2. The cash balance may be found directly, without equating the account, by finding the interest of each item from its maturity to the date of settlement, and taking the difference between the sums of the debit interests and credit interests for *the balance of interest.* When the balance of interest and the balance of items fall on the *same* side, the cash balance is their *sum;* when they fall on opposite sides, the cash balance is their *difference.*

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# SECTION XV.

# RATIO AND PROPORTION.

## RATIO.

**350.** The relation between two numbers expressed by their quotient, is called their *Ratio.* The ratio of 6 to 2 is $6 \div 2$, or 3; and the ratio of 2 to 6 is $2 \div 6$, or $\frac{1}{3}$.

### MENTAL EXERCISES.

1. What is the ratio of 8 to 4? 24 to 8? 45 to 15?
2. What is the ratio of 6 to 12? 12 to 36? 16 to 64?
3. What is the ratio of 42 to 14? 14 to 42? 12 to 30?
4. What is the ratio of 50 to 15? 15 to 50? 80 to 25?
5. What is the ratio of 36 to 16? 60 to 25? 70 to 40?
6. What is the ratio of 45 to 60? 18 to 45? 75 to 45?
7. What is the ratio of $33 to $11? $20 to $50? $45 to $36? $50 to $150?
8. What is the ratio of 16 lb. to 40 lb.? 28 lb. to 13 lb.?
9. What is the ratio of $\frac{9}{10}$ to $\frac{3}{10}$? $\frac{3}{10}$ to $\frac{9}{10}$? $\frac{14}{15}$ to $\frac{4}{15}$?
10. What is the ratio of $\frac{1}{2}$ to $\frac{1}{4}$? $\frac{1}{3}$ to $\frac{1}{6}$? $\frac{1}{5}$ to $\frac{1}{4}$?
11. What is the ratio of $\frac{2}{3}$ to $\frac{3}{4}$? $\frac{3}{5}$ to $\frac{5}{8}$? $\frac{5}{6}$ to $\frac{2}{3}$?
12. What is the ratio of 5 to $\frac{1}{3}$? $\frac{1}{3}$ to 4? $\frac{1}{4}$ to $2\frac{1}{2}$?

## WRITTEN EXERCISES.

**351.** The ratio of two numbers is expressed by placing a colon between them. The ratio of 4 to 10 is denoted by $4:10$, and the ratio of $\frac{1}{2}$ to $\frac{5}{6}$ by $\frac{1}{2}:\frac{5}{6}$. The expression $4:10$ is read *the ratio of* 4 *to* 10.

13. Express the ratio of 7 to 15. 12 to 35. 35 to 17.
14. Express the ratio of 2.5 to 7.5. 3.4 to .62.
15. Express the ratio of $\frac{3}{4}$ to $\frac{5}{8}$. $\frac{3}{8}$ to $5\frac{1}{2}$. $2\frac{1}{3}$ to $\frac{3}{5}$.
16. What is the value of the ratio of 112 to 35?

PROCESS: $112 : 35 = 112 \div 35 = 3\frac{1}{5}$, *Ans.*

What is the value of

17. $216 : 81$?
18. $129 : 215$?
19. $14.3 : 6.5$?
20. $1.44 : 3.2$?
21. $\frac{4}{16} : \frac{5}{8}$?
22. $150 : 16\frac{2}{3}$?
23. $12\frac{1}{2} : 30\frac{1}{4}$?
24. $34\frac{1}{2} : 5\frac{3}{4}$?
25. 6 qt. : 3 pk.?
26. 5 lb. 12 oz. : 17 lb. 4 oz.?
27. 2 ft. 6 in. : 12 ft. 6 in.?
28. 15 pk. : 12 bu. 2 pk.?

29. Reduce 24 : 60 to its lowest terms.

PROCESS: $24 : 60 = \frac{24}{60} = \frac{2}{5} = 2 : 5$, *Ans.*

Reduce the following ratios to their lowest terms:

30. $35 : 84$.
31. $63 : 108$.
32. $121 : 220$.
33. $105 : 140$.
34. $81 : 189$.
35. $105 : 195$.
36. $169 : 65$.
37. $256 : 112$.
38. $225 : 120$.

39. Reduce $\frac{5}{6} : \frac{3}{4}$ to an equal ratio with integral terms.

PROCESS: $\frac{5}{6} : \frac{3}{4} = \frac{10}{12} : \frac{9}{12} = 10 : 9$.

Reduce the following ratios to equal ratios with integral terms:

40. $\frac{2}{5} : \frac{3}{4}$.
41. $\frac{5}{7} : \frac{11}{14}$.
42. $\frac{7}{12} : \frac{5}{8}$.
43. $\frac{7}{9} : \frac{5}{6}$.
44. $\frac{11}{16} : \frac{7}{12}$.
45. $\frac{13}{15} : \frac{17}{20}$.
46. $\frac{7}{8} : 10$.
47. $2\frac{1}{3} : \frac{5}{6}$.
48. $14 : 5\frac{1}{4}$.

49. Multiply 10 : 21 by 14 : 15.

PROCESS: $\begin{cases} 10 : 21 = \frac{10}{21}. \quad 14 : 15 = \frac{14}{15}. \\ \text{Hence, } (10 : 21) \times (14 : 15) = \frac{10}{21} \times \frac{14}{15} = \frac{4}{9}, \textit{ Ans.} \end{cases}$

50. What is the product of 9 : 10 and 24 : 33?
51. What is the product of 7 : 15, 25 : 14, and 24 : 35?
52. What is the product of 12 : 25, 15 : 24, and 16 : 21?

## DEFINITIONS, PRINCIPLES, AND RULES.

**352.** ***Ratio*** is the relation between two numbers of the same kind expressed by their quotient.

**353.** The two numbers compared are called the ***Terms*** of the ratio.

The first term is the *Antecedent*, and the second term the *Consequent*. The two terms form a *Couplet*.

**354.** The value of a ratio is the quotient obtained by dividing the antecedent by the consequent.

When the antecedent is greater than the consequent, the value of the ratio is greater than 1; when the antecedent is less than the consequent, the value is less than 1.

**355.** The ratio of two numbers is expressed by placing a colon (:) between them; as, 5 : 12. The colon is called the *Sign of Ratio*.

NOTE.—The sign of ratio is the sign of division with the horizontal line omitted.

**356.** A ratio is also expressed in the form of a fraction, the antecedent being made the numerator and the consequent the denominator. Thus, $5 : 12 = \frac{5}{12}$.

NOTE.—Several American authors divide the consequent by the antecedent, thus reversing the positions of dividend and divisor, as indicated by the sign of division. The great majority of mathematical writers divide the antecedent by the consequent.

Ratios are either *Simple* or *Compound*.

**357.** A ***Simple Ratio*** is the ratio of two numbers; as 5 : 8, or $\frac{3}{5} : \frac{4}{5}$.

NOTE.—A simple ratio, having one or both of its terms fractional, is called by several authors a *Complex Ratio*.

**358.** A ***Compound Ratio*** is the product of two or more simple ratios; as, $(5 : 6) \times (\frac{3}{5} : 10)$.

It may be expressed in three ways, as follows:

$$(5 : 6) \times (8 : 9) \times (\tfrac{3}{5} : 10); \text{ or } \frac{5}{6} \times \frac{8}{9} \times \frac{\frac{3}{5}}{10}; \text{ or } \begin{matrix} 5 : 6. \\ 8 : 9. \\ \frac{3}{5} : 10. \end{matrix}$$

**359.** An ***Inverse Ratio*** is a ratio resulting from an inversion of the terms of a given ratio. Thus, 5 : 7 is the *inverse* of 7 : 5. It is also called a *Reciprocal Ratio.*

**360.** PRINCIPLES.—1. *The two terms of a ratio must be like numbers.*

2. *The antecedent equals the consequent multiplied by the ratio.*

3. *The consequent equals the antecedent divided by the ratio.*

4. *If the product of the two terms of a ratio be divided by either term, the quotient will be the other term.*

5. *A ratio is multiplied by multiplying the antecedent or dividing the consequent by a number greater than* 1.

6. *A ratio is divided by dividing the antecedent or multiplying the consequent by a number greater than* 1.

7. *A ratio is not changed by multiplying or dividing both of its terms by the same number.*

8. *The product of two or more ratios equals the ratio of their products.*

**361.** RULES.—1. To reduce a simple ratio to its lowest terms, *Divide both terms by their greatest common divisor.* (Pr. 7.)

2. To reduce a simple ratio with fractional terms to one with integral terms, *Multiply both terms by the least common multiple of the denominators of the fractions.* (Pr. 7.)

3. To find the product of two or more simple ratios, *Multiply the antecedents together for an antecedent and the consequents together for a consequent.* (Pr. 8.)

NOTE.—The process may be shortened by cancellation.

## PROPORTION.

### MENTAL EXERCISES.

The ratio of 12 to 6 is equal to the ratio of 14 to 7. since the value of each ratio is 2.

1. What two numbers have a ratio to each other equal to the ratio of 15 to 5? 24 to 12?
2. What two numbers have a ratio to each other equal to 6 : 24? 7 : 21? 11 : 44?
3. What two numbers have a ratio to each other equal to 45 : 15? 12 : 60? 72 : 24?
4. To what number has 10 a ratio equal to the ratio of 30 to 15? 14 to 28?
5. To what number has 16 a ratio equal to 11 : 33?
6. To what number has 12 a ratio equal to 6 : 30? 24 : 16? 20 to 15?
7. 12 is to 60 as 5 is to what number?
8. 13 is to 39 as 15 is to what number?
9. 14 is to 42 as 25 is to what number?
10. 56 is to 8 as 63 is to what number?

**362.** The equality of two ratios is expressed by placing a double colon (: :) between them. Thus, 5 : 10 = 7 : 14 is written 5 : 10 : : 7 : 14, and is read 5 *is to* 10 *as* 7 *is to* 14.

11. Read 8 : 40 : : 12 : 60, and show that the two ratios are equal.
12. Read 27 : 9 : : 63 : 21, and show that the two ratios are equal.
13. Read 5 : $2\frac{1}{2}$ : : 25 : $12\frac{1}{2}$, and show that the two ratios are equal.

### DEFINITIONS AND PRINCIPLES.

**363.** A ***Proportion*** is an equality of ratios.

**364.** The first ratio of a proportion is called the *First Couplet,* and the second ratio the *Second Couplet.*

**365.** The first and third terms of a proportion are the *Antecedents*, and the second and fourth terms, the *Consequents.*

NOTE.—The antecedents of a proportion are the antecedents of its ratios, and the consequents are the consequents of its ratios.

**366.** The first and fourth terms of a proportion are the *Extremes*, and the second and third terms, the *Means.*

The four terms of a proportion are called *Proportionals*, and the last is the *fourth* proportional to the other three in their order.

**367.** Three numbers are in proportion when the ratio of the first to the second equals the ratio of the second to the third; as, 8 : 12 :: 12 : 18. The second number is called a *mean proportional.*

**368.** Proportions are either *Simple* or *Compound.*

A ***Simple Proportion*** is an equality of two simple ratios.

A ***Compound Proportion*** is an equality of two ratios, one or both of which are compound.

## SIMPLE PROPORTION.

### Case I.

#### Any Term found, when the other Three Terms are given.

**369.** The proportion 4 : 8 :: 6 : 12 may be written 4 : 8 = 6 : 12, or $\frac{4}{8} = \frac{6}{12}$ (Art. 356); and multiplying the two equal fractions by 12 and 8, their denominators, we have $4 \times 12 = 6 \times 8$. Hence, the following

PRINCIPLES.—1. *The product of the extremes of a proportion equals the product of the means.* Hence,

2. *If the product of the extremes of a proportion be divided by either mean, the quotient will be the other mean.*

3. *If the product of the two means of a proportion be divided by either extreme, the quotient will be the other extreme.*

### WRITTEN PROBLEMS.

Find the missing term in the following proportions:

14. 21 : 7 :: 36 : —
15. 15 : 40 :: 18 : —
16. — : 24 :: 8 : 32
17. — : 9 :: 60 : 18
18. 45 : 30 :: — : 24
19. 2.5 : 62.5 :: — : 3.25
20. 7.2 : — :: 4.7 : 9.4
21. .25 : — :: 2.5 : 7.5
22. $\frac{2}{3} : \frac{3}{4} :: \frac{5}{6}$ : —
23. $\frac{3}{5} : \frac{2}{3}$ :: — : $\frac{7}{8}$
24. — : $2\frac{1}{3} :: \frac{1}{4} : \frac{3}{7}$
25. $\frac{1}{3}$ : — :: $\frac{1}{5} : \frac{1}{6}$
26. \$5 : \$45 :: 6 lb. : —
27. \$.75 : \$3 :: — : 56 oz.
28. 16 men : 96 men :: 15 days : —
29. 8 horses : 14 horses :: $\frac{4}{7}$ : —

**370.** RULES.—1. To find either extreme of a simple proportion, *Divide the product of the two means by the other extreme.*

2. To find either mean of a simple proportion, *Divide the product of the two extremes by the other mean.*

## Case II.

### The Solution of Problems by Simple Proportion.

**371.** The solution of a problem by proportion consists of two parts, viz.:

1. The arranging of the three given terms, called the *Statement.*

2. The finding of the fourth term by Case I.

**372.** If the required answer be made the *fourth* term of a proportion, the given number of the problem, which is of the same kind as the answer, will be the *third* term, since the two terms of a ratio must be *like numbers.* (Art. 360.)

**373.** Of the two remaining numbers given in the problem, the *greater* will be the second term when the answer is to be *greater* than the third term, and the *less* will be the second term when the answer is to be *less* than the third term, *otherwise the two ratios can not be equal.*

## WRITTEN PROBLEMS.

30. If 15 yards of cloth cost \$24, what will 40 yards cost?

STATEMENT.

15 yd. : 40 yd. :: \$24 : *Ans.*

$$\begin{array}{r} 40 \\ \hline 15)\ \$960 \\ \hline \$64,\ \textit{Ans.} \end{array}$$

Since the cost of 40 yards is to be the answer, make \$24, the cost of 15 yards, the third term of a proportion; and since 40 yards will cost more than 15 yards, the fourth term is to be *greater* than the third, and hence the second term must be *greater* than the first. Make 40 yards the second term and 15 yards the first, giving the proportion 15 yd. : 40 yd. :: \$24 : *cost* of 40 yards, which, by Case I, is found to be \$64.

31. If 45 sheep cost \$565, what will 140 sheep cost?

32. If 13 tons of hay cost \$97.50, what will $7\frac{1}{2}$ tons cost?

33. If 70 acres of land cost \$1875, what will 320 acres cost?

34. If 120 acres of land cost \$3000, how many acres can be bought for \$4500?

35. If 4 lb. 6 oz. of butter cost \$1.75, what will $17\frac{1}{4}$ pounds cost?

36. If a man's pulse beat 75 times in a minute, how many times will it beat in 8 hours?

37. If a clock ticks 120 times in a minute, how many times does it tick in $9\frac{1}{4}$ hours?

38. If a comet move 4° 20′ in 15 hours, how far will it move in 5 days?

39. If a garrison of 160 men consume 24 barrels of flour in 6 weeks, how many barrels will supply it one year?

40. If 24 barrels of flour will supply 160 men 6 weeks, how many barrels will supply 360 men the same time?

41. If a vertical staff 3 feet high casts a shadow 5 feet long, how long a shadow will a pole 120 feet high cast at the same time?

42. If a pole 20 feet high casts a shadow 12 feet long, how high is the tree whose shadow, at the same time, is 90 feet long?

43. If $\frac{5}{9}$ of a farm is worth \$4500, what is $\frac{3}{4}$ of it worth?

44. If $\frac{7}{8}$ of a yard of silk cost \$2.10, what will $16\frac{1}{2}$ yards cost?

45. If $6\frac{1}{4}$ tons of hay cost \$58.75, how many tons can be bought for \$173.90?

46. At the rate of 5 peaches for 8 apples, how many apples can be bought for 5 dozen peaches?

47. If 12 men can mow 20 acres of grass in a day, how many acres can 25 men mow?

48. If 9 men can build a wall in 15 days, how long will it take 5 men to build it?

STATEMENT.

$$\begin{array}{r} 5 \text{ men} : 9 \text{ men} :: 15 \text{ days} : \textit{Ans.} \\ 9 \\ \hline 5\,)\,135 \\ \hline 27 \text{ days, } \textit{Ans.} \end{array}$$

The 15 days is the third term, since the answer is to be in days. If it take 9 men 15 days to build a wall, it will take 5 men more than 15 days, and hence the answer, or fourth term, is greater than the third term, and consequently the second term must be greater than the first term. The proportion is 5 men : 9 men :: 15 days : 27 days.

NOTE.—The principle involved in this class of problems may thus be stated: *The greater the cause, the less the time required to produce a given effect; and, conversely, the greater the time, the less the cause required.*

49. If a quantity of provisions will supply a garrison of 90 men 125 days, how long will the same provisions supply 150 men?

50. If 15 men can harvest a field of wheat in 12 days, how many men can harvest it in 5 days?

51. Divide 90 into two parts whose ratio is equal to the ratio of 4 and 5.

$$\text{PROPORTIONS, } \begin{cases} (4+5) : 90 :: 4 : \textit{Smaller part.} \\ (4+5) : 90 :: 5 : \textit{Greater part.} \end{cases}$$

NOTE.—These proportions are based on the principle that when four numbers are in proportion, *the sum of the first and second terms is to the sum of the third and fourth terms as the first term is to the third, or as the second term is to the fourth.*

52. Divide 640 into two parts proportional to 8 and 12. To 9 and 11.

53. An estate worth $9600 was divided between two heirs in proportion to their ages, which were 15 years and 17 years respectively: how much did each receive?

54. Two men, 150 miles apart, are approaching each other, one traveling 2 miles to the other 3: how far will each travel before they meet?

☞ For additional problems see Problems for Analysis. p. 239

## PRINCIPLES AND RULE.

**374.** Principles.—1. *The ratio of two like causes equals the ratio of their effects.* Conversely,

2. *The ratio of two like effects equals the ratio of their causes.*

3. *The ratio of two like causes equals the* INVERSE *ratio of their times.* Conversely,

4. *The ratio of the times of two like causes equals the* INVERSE *ratio of the causes.*

5. *The two terms of each couplet of a simple proportion must be like numbers.* (Art. 360, Pr. 1.)

6. *The fourth term of a proportion equals the product of the second and third terms divided by the first term.* (Art. 370.)

**375.** Rule.—To solve a problem by simple proportion,

1. *Take for the third term the number which is of the same kind as the answer sought, and make the other two numbers the first couplet, placing the* GREATER *for the second term, when the answer is to be* GREATER *than the third term; and the* LESS *for the second term, when the answer is to be* LESS *than the third term.*

2. *Divide the product of the second and third terms by the first term, and the quotient will be the fourth term, or answer.*

Notes.—1. When the terms of the first couplet are denominate numbers, they must be reduced to the same denomination.

2. The process of finding the fourth term may be shortened by cancellation. The proportion 15 : 45 :: 27.5 : — may be completed thus:

$$\frac{\overset{3}{\cancel{45}} \times 27.5}{\cancel{15}} = 82.5. \qquad \textit{Or:} \quad \begin{array}{r|l} \cancel{15} & \cancel{45}^{\,3} \\ & 27.5 \\ \hline \multicolumn{2}{c}{27.5 \times 3 = 82.5} \end{array}$$

3. The process of solving problems by simple proportion is also called "*The Rule of Three.*"

## COMPOUND PROPORTION.

### Case I.

**Reduction of Compound Ratios and Proportions to Simple Ones.**

1. Reduce the compound ratio $\left\{\begin{array}{c} 20:80 \\ 4:3 \\ 6:8 \end{array}\right\}$ to a simple ratio in its lowest terms.

PROCESS.

$$\begin{array}{c} 20:80 \\ 4:3 \\ 6:8 \\ \hline 20\times4\times6:80\times3\times8 \\ 480:1920 \\ 1:4,\ \textit{Ans.} \end{array}$$

*Or:*

$$\begin{array}{c} \cancel{20}:\cancel{80}\ \cancel{4} \\ \cancel{4}:\cancel{3} \\ \cancel{2}\ \cancel{6}:\cancel{8}\ 4 \\ \hline 1:4,\ \textit{Ans.} \end{array}$$

A compound ratio is the product of two or more simple ratios (Art. 358), and the product of two or more simple ratios is found by multiplying the antecedents together for an antecedent, and the consequents for a consequent (Art. 361). Hence, the compound ratio given is equal to $20\times4\times6:80\times3\times8$, or $480:1920$, which, by dividing both terms by 480, is reduced to $1:4$.

The process may be shortened by canceling the factors common to the product of the antecedents and the product of the consequents.

NOTE.—The process may be explained directly by changing each ratio to the fractional form, thus:

$$\left.\begin{array}{c} 20:80 \\ 4:3 \\ 6:8 \end{array}\right\} = \tfrac{20}{80}\times\tfrac{4}{3}\times\tfrac{6}{8} = \frac{\cancel{20}\times\cancel{4}\times\cancel{6}\ \cancel{2}}{\underset{\cancel{4}}{\cancel{80}}\times\cancel{3}\times\underset{4}{\cancel{8}}} = \tfrac{1}{4} = 1:4.$$

Reduce the following compound ratios to simple ratios in their lowest terms:

2. $\left\{\begin{array}{c} 6:8 \\ 10:12 \\ 16:15 \end{array}\right.$

3. $\left\{\begin{array}{c} 6:9 \\ 15:18 \\ 5:3 \end{array}\right.$

4. $\left\{\begin{array}{c} 8:9 \\ 40:32 \end{array}\right.$

5. $\left\{\begin{array}{c} 7:20 \\ 40:21 \\ 12:16 \\ 32:45 \end{array}\right.$

6. Reduce $\left\{\begin{array}{c} 4:5 \\ 8:6 \end{array}\right\}::16:15$ to a simple proportion.

**SUGGESTION.—Reduce the compound ratio to a simple ratio.**

Reduce the following compound proportions to simple proportions:

7. $\left\{\begin{array}{c} 8 : 12 \\ 21 : 10 \end{array}\right\} :: 42 : 30$  9. $\$108 : \$216 :: \left\{\begin{array}{c} 24 : 36 \\ 3 : 4 \end{array}\right.$

8. $\left\{\begin{array}{l} 48 : 336 \\ \ \ 5 : 8 \\ 12 : 5 \end{array}\right\} :: 12 : 56$  10. $\left\{\begin{array}{c} 6 : 9 \\ 12 : 3 \\ 15 : 36 \end{array}\right\} :: \left\{\begin{array}{c} 15 : 27 \\ 12 : 6 \end{array}\right.$

11. What is the fourth term of the compound proportion, $\left\{\begin{array}{c} 5 : 8 \\ 10 : 9 \\ 12 : 25 \end{array}\right\} :: 13 : — ?$

PROCESS.

$$\begin{array}{r}
\cancel{5} : \cancel{8}\,\cancel{4} \qquad\qquad \\
\cancel{2}\,\cancel{10} : \cancel{9}\,3 :: 13 : — \\
\cancel{3}\,\cancel{12} : \cancel{25}\,\cancel{5} \qquad\qquad \\
\hline
1 : 3 :: 13 : — \\
3 \qquad \\
\hline
39,\ \textit{Ans.}
\end{array}
\qquad\qquad
\textit{Or:}\quad
\begin{array}{r|l}
\cancel{5} & \cancel{8}\,\cancel{4} \\
\cancel{2}\,\cancel{10} & \cancel{9}\,3 \\
\cancel{3}\,\cancel{12} & \cancel{25}\,\cancel{5} \\
 & 13 \\
\hline
\multicolumn{2}{c}{3 \times 13 = 39,\ \textit{Ans.}}
\end{array}$$

An inspection of the second process shows that the four numbers on the right of the vertical line are *the factors of the product of the means*, and that the three numbers on the left are *the factors of the first extreme.* By canceling the factors common to dividend and divisor, the fourth term is found directly.

Find the fourth term of these compound proportions:

12. $\left\{\begin{array}{c} 20 : 48 \\ 36 : 15 \\ 10 : 4 \end{array}\right\} :: 25 : —$  14. $\left\{\begin{array}{c} 5 : 9 \\ 2.5 : 7.5 \\ 4 : 10 \end{array}\right\} :: 6 : —$

13. $\left\{\begin{array}{c} 16 : 35 \\ 21 : 8 \\ 9 : 6 \\ 12 : 45 \end{array}\right\} :: 16 : —$  15. $\left\{\begin{array}{c} 2\frac{1}{2} : 7 \\ 25 : 10 \\ 4 : 6\frac{1}{4} \\ 15 : 12 \end{array}\right\} :: 35 : —$

**376.** RULES.—1. To reduce a compound ratio to a simple ratio, *Multiply the antecedents together for an antecedent, and the consequents for a consequent.*

2. To reduce a compound proportion to a simple proportion, *Reduce the compound ratio, or each compound ratio, if there are two, to a simple ratio.*

## Case II.

### The Solution of Problems by Compound Proportion.

16. If 2 men can mow 16 acres of grass in 10 days, working 8 hours a day, how many men can mow 27 acres in 9 days, working 10 hours a day?

STATEMENT.

$$\left.\begin{array}{l} 16 \text{ acres} : 27 \text{ acres} \\ 9 \text{ days} : 10 \text{ days} \\ 10 \text{ hours} : 8 \text{ hours} \end{array}\right\} :: 2 \text{ men} : Ans.$$

$$2 : 3 :: 2 \text{ men} : 3 \text{ men}, \; Ans.$$

*Or:*

| | |
|---|---|
| $\cancel{2}\ \cancel{16}$ | $\cancel{27}\ 3$ |
| $\cancel{9}$ | $\cancel{10}$ |
| $\cancel{10}$ | $\cancel{8}$ |
| | $\cancel{2}$ |
| | 3 |

Since the answer required is to be a number of *men*, make 2 *men* the third term.

If the mowing of 16 acres requires 2 men, the mowing of 27 acres will require *more* than 2 men, and hence the first ratio is 16 acres : 27 acres, the *greater* number being the second term.

If 10 days require 2 men, 9 days will require *more* than 2 men, and hence the second ratio is 9 days : 10 days, the *greater* number being the second term.

If working 8 hours a day requires 2 men, working 10 hours a day will require *less* than 2 men, and hence the third ratio is 10 hours : 8 hours, the *less* number being the second term.

This statement gives 3 men for the fourth term.

NOTE.—In determining which number of each ratio of the compound ratio is to be the second term, *reason* FROM *the number in the* CONDITION.

17. If 12 men can build 50 rods of wall in 15 days, how many men can build 80 rods in 16 days?

18. If it cost $30 to make a walk 10 feet wide and 90 feet long, how much will it cost to make a walk 8 feet wide and 225 feet long? 60

19. If 6 men can excavate 576 cubic feet of earth in 8 days of 9 hours each, how much can 8 men excavate in 9 days of 10 hours each?

20. If 7 horses eat 35 bushels of oats in 25 days, how many bushels will 15 horses eat in 21 days? 63

21. If a man walk 120 miles in 6 days of 10 hours each, how many miles will he walk in 16 days of 8 hours each? 256

22. If 1500 bricks, each 8 in. long and 4 in. wide, will make a walk, how many slabs of stone, each 2 ft. long and 1 ft. 4 in. wide, will be required for the same purpose? 125

23. If the interest of $250 for 9 months is $11.25, what is the interest of $650 for 7 months? 22.75

24. If it cost $84 to carpet a room 36 ft. long and 21 ft. wide, what will it cost to carpet a room 33 ft. long and 27 ft. wide? 99

25. If it cost $120 to build a wall 40 ft. long, 14 ft. high, and 1 ft. 6 in. thick, what will it cost to build a wall 180 ft. long, 21 ft. high, and 1 ft. 3 in. thick?

26. If 4 men can dig a ditch 72 rd. long, 5 ft. wide, and 2 ft. deep in 12 days, how many men can dig a ditch 120 rd. long, 6 ft. wide, and 1 ft. 6 in. deep in 9 days? 8

27. If 16 men can excavate a cellar 50 ft. long, 36 ft. wide, and 8 ft. deep in 10 days of 8 hours each, in how many days of 10 hours each can 6 men excavate a cellar 45 ft. long, 25 ft. wide, and 6 ft. deep? 10

28. If 32 men can dig a ditch 40 rd. long, 6 ft. wide, and 3 ft. deep in 9 days, working 8 hours a day, how many men can dig a ditch 15 rd. long, $4\frac{1}{2}$ ft. wide, and 2 ft. deep in 12 days, working 6 hours a day? 6

NOTE.—For additional problems, see Problems for Analysis.

## PRINCIPLES AND RULE.

**377.** PRINCIPLES.—1. *A compound proportion, used in the solution of a problem, has only one compound ratio.*

2. *The order of the terms of each ratio composing the compound ratio, is determined as in simple proportion.*

3. *The fourth term of a compound proportion is equal to the product of all the factors of the second and third terms, divided by the product of the factors of the first term.*

**378.** RULE.—1. *Take for the third term the number which is of the same kind as the answer sought, and arrange the first and second terms of each ratio composing the compound ratio as in simple proportion.*

2. *Reduce the compound ratio to a simple ratio, and divide the product of the second and third terms of the resulting proportion by the first term. The quotient will be the fourth term, or answer sought.* Or,

*Divide the product of all the factors of the second and third terms of the compound proportion by the product of the factors of the first term, shortening the process by cancellation.*

NOTES.—1. The terms of each ratio composing the compound ratio are arranged precisely as they would be if the answer depended wholly on them and the third term.

2. The process of solving problems by compound proportion is also called "*The Double Rule of Three.*"

# PARTNERSHIP.

**379.** A ***Partnership*** is an association of two or more persons for the transaction of business.

A partnership is organized and regulated by a contract, called articles of agreement. (Art. 240.)

**380.** A partnership association is called a *Company*, *Firm*, or *House*, and the persons associated together are called *Partners*.

**381.** The money or property invested in the business by the partners is called *Capital*, *Joint-stock*, or *Stock in Trade*.

When a partner furnishes capital but does not assist in conducting the business, he is called a *Silent Partner*.

**382.** Partnership is either *Simple* or *Compound*.

In ***Simple Partnership*** the capital of the several partners is invested an *equal* time.

In ***Compound Partnership*** the capital of the several partners is invested an *unequal* time.

## SIMPLE PARTNERSHIP.

### PROBLEMS.

1. A, B, and C entered into partnership in business for 2 years; A put in \$3600, B \$2400, and C \$2000, and their net profits were \$3000. What was each partner's share?

### I. Process by Proportion.

| | | |
|---|---|---|
| \$3600, *A's cap'l.* | \$8000 : \$3600 :: \$3000 : \$1350, | *A's share of profits.* |
| 2400, *B's cap'l.* | \$8000 : \$2400 :: \$3000 : \$900, | *B's* " " |
| 2000, *C's cap'l.* | \$8000 : \$2000 :: \$3000 : \$750, | *C's* " " |
| \$8000, *Entire capital.* | \$3000, | *Entire profits.* |

Since the capital of the several partners was employed an equal time, their shares of the profits are proportional to their capitals. Hence, the entire capital is to each partner's capital as the entire profits are to his share of the profits.

### II. Process by Percentage.

$\$3000 \div \$8000 = .37\frac{1}{2}$
$\$3600 \times .37\frac{1}{2} = \$1350$, *A's share.*
$\$2400 \times .37\frac{1}{2} = \$900$, *B's* "
$\$2000 \times .37\frac{1}{2} = \$750$, *C's* "

Since the profits were equal to $.37\frac{1}{2}$, or $37\frac{1}{2}$ % of the entire capital, each partner's share of the profits was equal to $37\frac{1}{2}$ % of his capital.

*Or:*

$\$3600 \div \$8000 = .45$, *A's per cent. of the capital.*
$\$2400 \div \$8000 = .30$, *B's* " " "
$\$2000 \div \$8000 = .25$, *C's* " " "
$\$3000 \times .45 = \$1350$, *A's share of the profits.*
$\$3000 \times .30 = \$900$, *B's* " " "
$\$3000 \times .25 = \$750$, *C's* " " "

### III. Process by Fractional Parts.

$\$3000 \div \$8000 = \frac{3000}{8000} = \frac{3}{8}$.
$\frac{3}{8}$ of $\$3600 = \$1350$, *A's share.*
$\frac{3}{8}$ of $\$2400 = \$900$, *B's* "
$\frac{3}{8}$ of $\$2000 = \$750$, *C's* "

Since the profits were equal to $\frac{3}{8}$ of the entire capital, each partner's share of the profits was equal to $\frac{3}{8}$ of his capital.

*Or:*

$\$3600 \div \$8000 = \frac{9}{20}$, *A's part of the capital.*
$\$2400 \div \$8000 = \frac{3}{10}$, *B's* " " "
$\$2000 \div \$8000 = \frac{1}{4}$, *C's* " " "
$\frac{9}{20}$ of $\$3000 = \$1350$, *A's share of the profits.*
$\frac{3}{10}$ of $\$3000 = \$900$, *B's* " " "
$\frac{1}{4}$ of $\$3000 = \$750$, *C's* " " "

NOTE.—Let the following problems be solved by proportion and by either of the other methods, which the teacher or pupil may prefer.

2. A and B were partners in business; A put in $5000 and B $4000, and their profits in three years were $4500: what was each partner's share of the profits?

3. A, B, and C formed a partnership in business; A put in $8000, B $4500, and C $3500, and their loss the first year was $3200: what was each partner's share?

4. A, B, and C are partners, and B has invested $\frac{3}{4}$ as much capital as A, and C $\frac{2}{3}$ as much as B: if their profits amount to $6300, what will be each partner's share?

5. The capital of two partners is proportional to 4 and 3; their profits are $10000 and their expenses $2300: what is each partner's share of the net profits?

6. A, B, and C form a partnership, A's capital being $4000, B's $6400, and C's $5600; they make a net gain of $3200, and then sell out for $2000: what is each partner's share of the gain? Of the proceeds of the sale?

## PRINCIPLES AND RULE.

**383.** PRINCIPLES.—1. *The gain or loss of a partnership is shared by the partners in proportion to the* USE *of the capital invested by them, which is its partnership value.*

2. *When the time is equal, the use of the capital of the several partners is in proportion to its amount.* Hence,

3. *In a simple partnership, the gain or loss is shared by the partners in proportion to the amounts of their capital.*

**384.** RULE.—To divide the gain or loss of a simple partnership, *Divide the gain or loss among the several partners in proportion to the amounts of capital invested by them.*

NOTES.—1. The above principles and rule are applicable only when the several partners devote equal time or render equal service in carrying on the business. The division of profits or losses is usually settled by the terms of the contract.

2. The problems in bankruptcy (Art. 272) may also be solved by the above methods.

## COMPOUND PARTNERSHIP.

7. A and B formed a partnership; A put in \$3000, and, at the close of the first year, added \$2000; B put in \$4000, and, at the close of the second year, took out \$2000; at the close of the third year, the profits amounted to \$3450. What was each partner's share?

### I. PROCESS BY PRODUCTS.

\$3000 × 1 = \$3000
\$5000 × 2 = \$10000
\$13000, *A's capital for* 1 *year.*
\$4000 × 2 = \$8000
\$2000 × 1 = \$2000
\$10000, *B's capital for* 1 *year.*
\$13000 + \$10000 = \$23000, *Entire capital for* 1 *year.*
\$23000 : \$13000 :: \$3450 : \$1950, *A's share of profits.*
\$23000 : \$10000 :: \$3450 : \$1500, *B's* " "

Since A had \$3000 invested for 1 year and \$5000 for 2 years, the use of his capital was equivalent to the use of \$13000 for 1 year. Since B had \$4000 invested for 2 years and \$2000 for 1 year, the use of his capital was equivalent to the use of \$10000 for 1 year. Hence the profits, amounting to \$3450, should be shared by them in proportion to \$13000 and \$10000.

### II. PROCESS BY INTEREST.

Int. of \$3000 for 1 yr. = \$180
" " \$5000 for 2 yr. = \$600
\$780, *Int. of A's capital.*
Int. of \$4000 for 2 yr. = \$480
" " \$2000 for 1 yr. = \$120
\$600, *Int. of B's capital.*
\$780 + 600 = \$1380, *Int. of entire capital.*
\$1380 : \$780 :: \$3450 : \$1950, *A's share.*
\$1380 : \$600 :: \$3450 : \$1500, *B's* "

Since the use of capital is represented by its interest for the time, the use of A's capital is represented by $780, and the use of B's by $600. Hence, the profits ($3450) should be shared by them in proportion to $780 and $600.

NOTE.—The ratio of the interests will be the same whatever be the rate per cent.; and hence the interest may be computed at any rate.

8. A and B entered into a partnership for 4 years, A putting in $6000 and B $8000. At the close of the second year, A took out $2000 and B put in $2000; and, at the close of the fourth year, they divided $8890 as net profits. What was the share of each?

9. A and B entered into a partnership in business for 3 years, A's invested capital being $3500 and B's $4500. At the end of the first year they each took out $1000, and C was received as a partner with a capital of $2500. At the end of the third year they dissolved partnership, dividing $5000 as net profits. What was each partner's share?

10. A, B, and C entered into business as partners, each putting in $5000 as capital. At the end of 2 years A took out $1000, B $2000, and C $3000, and, at the end of the fourth year, they closed the business with a loss of $3600. What was the loss of each?

## PRINCIPLE AND RULES.

**385.** PRINCIPLE.—*The value of capital in compound partnership depends jointly on its amount and the time of its investment.*

**386.** RULES.—To divide the gain or loss of a compound partnership, 1. *Multiply the amount of capital invested by each partner by the time of its investment, and taking the product as the partnership value of his capital, proceed as in simple partnership.* Or,

2. *Find the interest of each partner's capital for the time of its investment, at any rate per cent.; and taking the interest thus found as the partnership value of his capital, proceed as in simple partnership.*

# PROBLEMS FOR ANALYSIS.

NOTE.—These problems are here given to afford an additional drill in analysis and, if needed, in proportion. For the latter purpose, the teacher can select as many problems as may be necessary.

## MENTAL PROBLEMS.

1. If 7 pounds of sugar cost 91 cents, what will 20 pounds cost?

2. If 12 yards of muslin cost \$1.02, what will 20 yards cost?

3. If $\frac{7}{8}$ of a yard of silk cost \$$\frac{21}{25}$, what will $\frac{5}{6}$ of a yard cost?

4. If $\frac{3}{4}$ of a barrel of flour cost \$$5\frac{1}{4}$, what will $\frac{5}{7}$ of a barrel cost?

5. If $\frac{3}{4}$ of a pound of coffee cost 15 cents, what will $3\frac{1}{2}$ pounds cost?

6. A man sold a watch for \$120, which was $\frac{4}{5}$ of what it cost him: how much did it cost?

7. If 40 yards of carpeting, $\frac{3}{4}$ of a yard wide, will cover a floor, how many yards of matting, $1\frac{1}{4}$ yards wide, will cover a floor of equal size?

8. Two men, traveling in the same direction, are 60 miles apart; the one in advance travels 5 miles an hour, and the other 7 miles an hour: in how many hours will the latter overtake the former?

9. If a vertical staff 3 feet long casts a shadow 2 feet in length, how long a shadow will a tree 90 feet high cast at the same time of day?

10. If a steeple 200 feet high casts a shadow 150 feet long, what is the height of a pole which, at the same time of day, casts a shadow 80 feet long?

11. If 5 men can do a piece of work in 12 days, how long will it take 6 men to do it?

12. If 8 men can do a piece of work in 15 days, how many men can do the same work in 10 days?

13. If 9 men can do a piece of work in $4\frac{2}{3}$ days, how long will it take 7 men to do it?

14. If 3 pipes will empty a cistern in 30 minutes, how many pipes will empty it in 10 minutes?

15. If a quantity of provisions will supply 15 men 20 days, how long will it supply 50 men?

16. If it require 12 days of 10 hours each to do a piece of work, how many days of 8 hours each will be required to do the same work?

17. If 5 men can do $\frac{3}{8}$ of a piece of work in a day, how long will it take one man to do the entire work?

18. If 8 men can do $\frac{3}{4}$ of a piece of work in 3 days, how long will it take 4 men to do the entire work?

19. If 20 men earn \$120 in 4 days, how much will 5 men earn in 8 days?

20. If 6 men can mow 30 acres of grass in 3 days, how many acres will 9 men mow in 5 days?

21. If 5 horses eat 40 bushels of oats in 3 weeks, how many bushels will supply 12 horses 10 weeks?

22. If 8 men can dig a ditch 40 rods long in 6 days, how long will it take 12 men to dig a ditch 60 rods long?

23. If the interest of \$50 for 9 months is \$6, what would be the interest of \$150 for 1 yr. 6 mo.?

24. A school enrolls 180 pupils, and the number of boys is $\frac{4}{5}$ of the number of girls: how many pupils of each sex are enrolled in the school?

25. A lady paid \$130 for a watch and chain, and the cost of the watch was $\frac{3}{5}$ more than the cost of the chain: what was the cost of each?

26. A tree 120 feet in height was broken into two parts by falling, and $\frac{2}{5}$ of the shorter part equaled $\frac{2}{7}$ of the longer: what was the length of each part?

27. A person giving the time of day, said that $\frac{2}{3}$ of the time past noon equaled the time to midnight: what was the hour of day?

28. A person being asked the time of day, said that $\frac{1}{3}$ of the time past midnight equaled the time to noon: what was the hour of day?

29. What is the hour of day when $\frac{3}{4}$ of the time past noon equals $\frac{3}{8}$ of the time to midnight?

30. What is the time of day when $\frac{2}{3}$ of the time past noon, multiplied by $4\frac{1}{2}$, is equal to the time to midnight?

31. What is the time of day when $\frac{2}{3}$ of the time to noon is equal to $\frac{2}{5}$ of the time past midnight?

32. A man being asked his age said, 10 years ago my age was $\frac{7}{9}$ of my present age: what was his age?

33. A son's age is $\frac{3}{5}$ of the age of his father, and the sum of their ages is 80 years: what is the age of each?

34. Ten years ago the age of A was $\frac{3}{4}$ of the age of B, and 10 years hence the age of A will be $\frac{5}{6}$ of the age of B: what is the age of each?

35. At the time of marriage a wife's age was $\frac{3}{5}$ of the age of her husband, and 10 years after marriage her age was $\frac{7}{10}$ of the age of her husband: how old was each at the time of marriage?

36. $\frac{3}{4}$ of A's age equals $\frac{4}{7}$ of B's, and the difference between their ages is 10 years: how old is each?

37. Twice the age of A is 20 years more than the age of B, and 10 years more than the age of C, and the sum of their ages is 120 years: what is the age of each?

38. A man bought a horse and carriage for $275, and $\frac{1}{5}$ of the cost of the carriage + $33 was equal to $\frac{1}{3}$ of the cost of the horse: what was the cost of each?

39. A man bought a horse, saddle, and bridle for $150; the cost of the saddle was $\frac{1}{6}$ of the cost of the horse, and the cost of the bridle was $\frac{1}{2}$ of the cost of the saddle: what was the cost of each?

40. A man and his two sons earned $140 in a month; the man earned twice as much as the elder son, and the elder son earned twice as much as the younger: how much did each earn?

41. Two men bought a barrel of sirup, one paying $20 and the other $30: what part should each have?

42. Two men hired a pasture for $40, and one put in 3 cows and the other 5 cows: how much ought each to pay?

43. A and B rented a pasture for \$72; A puts in 40 sheep and B 8 cows: if 4 sheep eat as much as one cow, how much ought each to pay?

44. Two men, A and B, agreed to build a wall for \$300; A sent 5 men for 4 days, and B 5 men for 6 days: how much ought each to receive?

45. A and B engage to plow a field for \$81; A furnished 3 teams for 5 days, and B furnished 4 teams for 3 days: how much should each receive?

46. A man can do $\frac{1}{5}$ of a piece of work in a day, and a boy can do $\frac{1}{8}$ of it in a day: in how many days can both of them, working together, do it?

47. A and B together can build a wall in 8 days, and A can build it alone in 12 days: how long will it take B to build it?

48. A can do a piece of work in 6 days, and B in 8 days: if they both work together 3 days, how long will it take B alone to complete the work?

49. John can saw a pile of wood in 6 days, and, with the assistance of Charles, he can saw it in 4 days: how long will it take Charles to saw it alone?

50. A man can do a piece of work in 5 days and a boy in 8 days; the man works 2 days alone and is then assisted by the boy: how long will it take both to complete the work?

51. A and B can do a piece of work in 10 days, and A, B, and C in 8 days: how long will it take C alone to do the work?

52. A and B can do $\frac{1}{8}$ of a piece of work in a day, and A can do twice as much in a day as B: how long will it take B alone to do it?

53. A can make a fence in $\frac{1}{4}$ of a month, B in $\frac{1}{5}$ of a month, and C in $\frac{1}{6}$ of a month: in what time can all three together build it?

54. A can do a piece of work in 4 days, B in 5 days, and C in 6 days: in what time can they together do it?

55. A, B, and C can do a piece of work in 4 days, A

and C in 8 days, and B and C in 6 days: how long will it take A and B together to do it?

56. A and B did a piece of work, and $\frac{2}{5}$ of what A did equaled $\frac{4}{5}$ of what B did: if B received \$18, how much did A receive?

57. A man spent $\frac{3}{4}$ of his money and then earned $\frac{1}{2}$ as much as he had spent, and then had \$21 less than he had at first: how much money did he have at first?

58. At what time between one and two o'clock will the hour and minute hands of a watch be together?

59. At what time between two and three o'clock are the hour and minute hands of a watch together? At what time between four and five o'clock?

60. A man has 2 watches, and a chain worth \$20; if he put the chain on the first watch it will be worth $\frac{2}{3}$ as much as the second watch, but if he put the chain on the second watch it will be worth $2\frac{3}{4}$ times the first watch: what is the value of each watch?

## WRITTEN PROBLEMS.

61. A father bequeathed \$14535 to two sons, giving the younger $\frac{7}{10}$ as much as the elder: what was the share of each?

62. An estate was so divided between two heirs that $\frac{2}{3}$ of the share of the elder was equal to $\frac{3}{4}$ of the share of the younger, and the difference between their shares was \$362: what was the share of each?

63. An estate worth \$27520 was divided between two daughters in proportion to their ages, which were 14 and 18 years respectively: how much did each receive?

64. A man paid \$8100 for 2 farms, and $\frac{3}{5}$ of the cost of the larger farm was equal to $\frac{9}{10}$ of the cost of the smaller: what was the cost of each?

65. A earns \$15.50 as often as B earns \$12.40, and in a certain time they together earn \$697.50: how much did each earn?

66. The fore wheels of a carriage are each $9\frac{1}{2}$ feet in cir-

cumference, and the hind wheels are each $12\frac{1}{2}$ feet in circumference: if each fore wheel revolve 9500 times in going a certain distance, how many times will each hind wheel revolve?

67. If it take 13200 steps of 2 ft. 9 in. each to walk a certain distance, how many steps of 1 ft. 10 in. each will it take to walk the same distance?

68. If \$75 yield \$10.80 interest, what principal will yield \$89.28 interest in the same time?

69. If the interest of \$475 is \$118.75, what would be the interest of \$850 for the same time and at the same rate?

70. If the interest of a certain principal for a certain time at 5 per cent. is \$120.50, what would be the interest of the same principal for the same time at 12 per cent.?

71. A broker sold 90 shares of railroad stock and gained \$315: how much would he have gained if he had sold 245 shares?

72. If a gain of 15 per cent. on a certain investment yields \$2347.50, what would a gain of 24 per cent. on the same investment yield?

73. If the commission for selling 3050 pounds of butter at 30 cents a pound is \$45.75, what would be the commission for selling 7500 pounds at 35 cents a pound?

74. If the annual dividend on \$40325 worth of mining stock is \$3226, what is the dividend on \$70680 of the same stock?

75. If 6 ranks of wood, each 60 ft. long and 6 ft. high, are worth \$337.50, what is the value of 15 ranks of wood, each 45 ft. long and 9 ft. high?

76. If it cost \$110 to dig a cellar 40 ft. long, 27 ft. wide, and 4 ft. deep, how much will it cost to dig a cellar 36 ft. long, 30 ft. wide, and 5 ft. deep?

77. If 45 men can do a piece of work in 15 days, by working 8 hours a day, in how many days can 30 men, working 9 hours a day, do the same work?

78. If 5 men can cut 45 cords of wood in 6 days, how many cords can 8 men cut in 15 days?

79. If 4 men dig a trench in 15 days of 10 hours each, in how many days of 8 hours each can 5 men perform the same work?

80. A and B are partners in business; A's capital is equal to $\frac{5}{8}$ of B's, and their profits are $3250: what is the share of each?

81. A and B are partners; $\frac{3}{5}$ of A's capital is equal to $\frac{5}{6}$ of B's, and their loss in business is $2150: what is the share of each?

82. A, B, and C are partners in business; A's capital is twice B's and three times C's, and their profits in business are $4675: what is the share of each?

83. A and B, trading in partnership 2 years, make a profit of $5460; during the first year A owned $\frac{2}{3}$ of the stock, and during the second year B owned $\frac{3}{4}$ of it: what is each partner's share of the profits?

84. A and B, trading in partnership 2 years, make each year a profit of $1200; A's capital the first year was $2\frac{1}{2}$ times B's, and the second year it was $1\frac{1}{2}$ times B's: what is each partner's share of the profits?

85. A and B traded in partnership 3 years; A's stock the first year was $5000, the second year $6000, and the third year $7000; B's stock the first year was $7000, and the last two years $5000; their loss was $1750. What was the loss of each?

86. A mechanic agreed to work 80 days on the condition that he should receive $1.75 and board for every day that he worked, and that he should pay 75 cents a day for board when he was idle; his net earnings for the time were $80: how many days did he work?

87. A piece of carpeting containing 135 yards was cut into 3 carpets, and $\frac{3}{10}$ of the number of yards in the first carpet was equal to $\frac{1}{3}$ of the number of yards in the second carpet, and to $\frac{3}{8}$ of the number of yards in the third carpet: what was the number of yards in each carpet?

# SECTION XVI.

# INVOLUTION AND EVOLUTION.

## I. INVOLUTION.

**387.** The first power of 4 is 4; the second power of 4 is $4 \times 4$, which is 16; the third power is $4 \times 4 \times 4$, which is 64; the fourth power is $4 \times 4 \times 4 \times 4$, which is 256; etc.

1. What is the second power of 5? Of 6? 8? 10?
2. What is the third power of 3? Of 4? 5? 6? 10?
3. What is the fourth power of 2? Of 3? 4? 10?
4. What is the second power of 1? 2? 3? 4? 5? 6? 7? 8? 9?
5. What is the third power of 1? 2? 3? 4? 5? 6? 7? 8? 9?
6. What is the second power of $\frac{1}{4}$? Of $\frac{3}{4}$? $\frac{1}{5}$? $\frac{3}{5}$? $\frac{1}{6}$? $\frac{5}{6}$?

### WRITTEN PROBLEMS.

7. What is the second power of 406?
8. What is the third power of 42?
9. What is the fourth power of 24?
10. What is the fifth power of 16?
11. What is the second power of 6.5?
12. What is the third power of .42?
13. What is the fifth power of .6?
14. What is the third power of $\frac{18}{25}$.
15. What is the second power of $\frac{25}{32}$?
16. What is the fourth power of $\frac{11}{12}$?

REMARK.—The power to which a number is to be raised may be denoted by a little figure, called an *exponent*, placed at the right of the upper part of the figures expressing the number. Thus, $24^2$ denotes the second power of 24; $16^3$ denotes the third power of 16, etc.

Raise the following numbers to the powers indicated by the exponents:

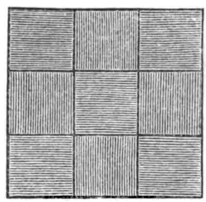
| | | |
|---|---|---|
| 17. $623^2$ | 22. $.045^2$ | 27. $(6\frac{1}{4})^3$ |
| 18. $105^3$ | 23. $(\frac{13}{15})^3$ | 28. $(\frac{16}{33})^2$ |
| 19. $34.6^2$ | 24. $(\frac{7}{9})^4$ | 29. $.005^5$ |
| 20. $.016^3$ | 25. $(16\frac{2}{3})^2$ | 30. $2.04^3$ |
| 21. $1.4^4$ | 26. $(3\frac{1}{3})^4$ | 31. $(\frac{1}{7})^4$ |

## DEFINITIONS AND RULE.

**388.** The ***Power*** of a number is the product obtained by taking the number one or more times as a factor.

**389.** The ***First Power*** of a number is the number itself.

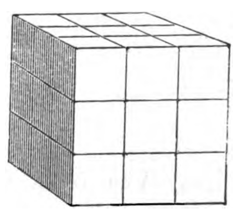
**390.** The ***Second Power*** of a number is the product obtained by taking the number *twice* as a factor.



$3 \times 3 = 9.$

It is also called the *Square* of the number, since the area of a geometrical square is represented by the product obtained by taking the number of linear units in one of its sides twice as a factor.

**391.** The ***Third Power*** of a number is the product obtained by taking the number *three* times as a factor.



$3 \times 3 \times 3 = 27.$

It is also called the *Cube* of the number, since the capacity of a geometrical cube is represented by the product obtained by taking the number of linear units in one of its edges *three* times as a factor.

**392.** The *Exponent* of a power is a small figure placed at the right of the number, to show how many times it is to be taken as a factor. It denotes the *degree* of the power.

The first power contains the number *once* as a factor, and the exponent is $^1$; the second power, or square, contains the number *twice* as a factor, and the exponent is $^2$; the third power, or cube, contains the number *three* times as a factor, and the exponent is $^3$; etc.

**393.** ***Involution*** is the process of finding the powers of numbers.

**394.** RULE.—To raise a number to a given power, *Multiply the number by itself as many times* LESS ONE *as there are units in the exponent of the given power. The last product will be the required power.*

## 395. ANOTHER METHOD OF INVOLUTION.

32. What is the square of 53?

PROCESS.

$$53 = 50 + 3, \text{ and } 53^2 = (50 + 3)^2$$

$$\begin{array}{rl}
50 + 3 & \\
50 + 3 & \\
\hline
50 \times 3 + 3^2 & = (50 + 3) \times 3 \\
50^2 + 50 \times 3 \quad & = (50 + 3) \times 50 \\
\hline
50^2 + 2\,(50 \times 3) + 3^2 & = (50 + 3)^2 = 53^2
\end{array}$$

*Parts added.*

$$\begin{array}{lr}
50^2 & = 2500 \\
2\,(50 \times 3) & = 300 \\
3^2 & = 9 \\
\hline
 & = 2809
\end{array}$$

An inspection of the above process will show that the square of 53 is equal to the square of the 5 tens, plus twice the product of the 5 tens by the 3 units, plus the square of the units.

In like manner, it may be shown that the square of any number, composed of tens and units, is equal to *The square of the tens, plus twice the product of the tens by the units, plus the square of the units.*

33. What is the square of 45?

$$\text{PROCESS: } 45^2 = \left\{\begin{array}{lr}
40^2 & = 1600 \\
2\,(40 \times 5) & = 400 \\
5^2 & = 25 \\
\hline
 & 2025, \textit{ Ans.}
\end{array}\right.$$

34. What is the square of 67? Of 75?

35. What is the square of 82? Of 38?

36. What is the square of 93? Of 125?

SUGGESTION.—$125 = 120 + 5$.

37. What is the square of 115? Of 124?

38. What is the cube of 53?

The cube of 53 $= (50+3)^3 = 50^3 + 3\,(50^2 \times 3) + 3\,(50 \times 3^2) + 3^3$, as may be shown by multiplying $50^2 + 2\,(50 \times 3) + 3^2$ by $50 + 3$.

In like manner, it may be shown that the cube of any number, composed of tens and units, is equal to *The cube of the tens, plus three times the product of the square of the tens by the units, plus three times the product of the tens by the square of the units, plus the cube of the units.*

39. What is the cube of 45?

$$\text{PROCESS: } 45^3 = \left\{ \begin{array}{rr} 40^3 & = 64000 \\ 3\,(40^2 \times 5) & = 24000 \\ 3\,(40 \times 5^2) & = 3000 \\ 5^3 & = \underline{\quad 125} \\ & 91125, \textit{ Ans.} \end{array} \right.$$

40. What is the cube of 23? Of 32?
41. What is the cube of 24? Of 43?
42. What is the cube of 33? Of 54?
43. What is the cube of 51? Of 35?
44. What is the cube of 45? Of 52?
45. What is the cube of 41? 55?
46. What is the cube of 75? 80?

## II. EVOLUTION.

### MENTAL PROBLEMS.

1. What are the two equal factors of 16? Of 25? 49?
2. Of what number is 81 the second power or square?
3. What are the three equal factors of 8? Of 27? 125?
4. Of what number is 125 the third power or cube?

One of the *two* equal factors of a number is called its *second* or *square root;* one of its *three* equal factors, its *third* or *cube root;* one of its *four* equal factors, its *fourth root,* etc.

5. What is the square root of 25? Of 49? 64? 81?
6. What is the cube root of 8? Of 27? 64? 125?
7. What is the cube root of 216? 512? 1000?
8. What is the fourth root of 16? Of 81? 256? 625?
9. What is the square root of 1? 4? 9? 16? 25? 36? 49? 64? 81?

10. What is the cube root of 1? 8? 27? 64? 125? 216? 343? 512? 729?

11. What integers between 1 and 100 are perfect squares?

12. What integers between 1 and 1000 are perfect cubes?

13. Show that the square root of a perfect square expressed by two figures, can not exceed 9.

14. Show that the cube root of a perfect cube expressed by three figures, can not exceed 9.

## DEFINITIONS.

**396.** The ***Root*** of a number is one of the equal factors which will produce it.

**397.** The ***First Root*** is the number itself.

**398.** The ***Second Root*** is one of the two equal factors of the number. It is also called the *Square Root.*

**399.** The ***Third Root*** is one of the three equal factors of the number. It is also called the *Cube Root.*

A number is the second power of its square root; the third power of its cube root; the fourth power of its fourth root; etc.

**400.** A ***Perfect Power*** is the product of equal factors. It has an exact root.

**401.** An ***Imperfect Power*** is a number which is not the product of equal factors. Its root is called a *Surd.*

**402.** The ***Radical Sign*** is a character, $\sqrt{\ }$, placed before a number to show that its root is to be taken.

**403.** A small figure placed above the radical sign is called the *Index* of the root.

Thus, $\sqrt[1]{25}$ denotes the *first* root of 25; $\sqrt[2]{25}$ denotes the *second* root of 25; $\sqrt[3]{25}$, the *third* root of 25, etc.

When the square root is indicated, the index is usually omitted. $\sqrt[2]{16}$ and $\sqrt{16}$ alike denote the square root of 16.

NOTE.—The root of a number may also be indicated by a fractional exponent. Thus, $16^{\frac{1}{2}}$ denotes the square root of 16; $16^{\frac{1}{3}}$, the cube root of 16, etc.; $16^{\frac{2}{3}}$ denotes the cube root of the square of 16.

**404.** ***Evolution*** is the process of finding the roots of numbers.

NOTE.—Evolution is the reverse of involution.

## SQUARE ROOT.

### The Division of the Number into Periods.

**405.** The smallest integer composed of one order of figures is 1, and the greatest is 9; the smallest integer composed of two orders is 10, and the greatest is 99, and so on.

The squares of the smallest and greatest integers composed of one, two, three, and four orders, are as follows:

$$\begin{array}{rlrl} 1^2 = & 1 & 9^2 = & 81 \\ 10^2 = & 100 & 99^2 = & 9801 \\ 100^2 = & 10000 & 999^2 = & 998001 \\ 1000^2 = & 1000000 & 9999^2 = & 99980001 \end{array}$$

A comparison of the above numbers with their squares shows that the square of a number contains twice as many orders as the number, or twice as many orders less one.

**406.** Hence, if a number be separated into periods of two orders each, beginning at the right, *there will be as many orders in its square root as there are periods in the number.*

1. How many orders in the square root of 2809?

SUGGESTION.—First divide the number into periods of two orders each, thus: $\dot{2}8\dot{0}9$.

2. How many orders in the square root of 36864?
3. How many orders in the square root of 345744?

4. How many orders in the square root of 87616?
5. How many orders in the square root of 5308416?
6. How many orders in the square root of 5475600?
7. How many orders in the square root of 14440000?

**407.** The squares of the smallest and greatest number of units, tens, hundreds, and thousands, are as follows:

$$
\begin{array}{rlrl}
1^2 = & 1 & 9^2 = & 81 \\
10^2 = & 100 & 90^2 = & 8100 \\
100^2 = & 10000 & 900^2 = & 810000 \\
1000^2 = & 1000000 & 9000^2 = & 81000000
\end{array}
$$

A comparison of the above numbers with their squares shows that the square of units gives no order higher than tens; that the square of tens gives no order lower than hundreds, nor higher than thousands; that the square of hundreds gives no order lower than ten-thousands, nor higher than hundred-thousands, etc.

**408.** Hence, if a number be separated into periods of two orders each, *the left-hand period will contain the square of the left-hand or first term of the square root; the first two left-hand periods will contain the square of the first two terms of the square root, etc.*

8. What is the tens' term of the square root of 2025?

*Ans.*—The left-hand period of 2025 is 20; the greatest square in 20 is 16, and the square root of 16 is 4. Hence, the tens' figure of the square root of 2025 is 4.

9. What is the hundreds' term of the square root of 87616? Of 345741?
10. What is the left-hand term of the square root of 16129? Of 336400?
11. What is the left-hand term of the square root of 87616?
12. What are the first two terms of the square root of 16129?

## WRITTEN PROBLEMS.

13. What is the square root of 3364?

PROCESS.

$$\begin{array}{rl} & 3364 \mid \underline{58} \\ 5^2 = & \underline{25} \\ 5 \times 2 = 10\ ) & 864 \\ 108 \times 8 = & \underline{864} \end{array}$$

Since 3364 is composed of two periods, its square root will be composed of two orders. (Art. 406.)

The left-hand period 33 contains the square of the tens' term of the root. (Art. 408.) The greatest square in 33 is 25, and the square of 25 is 5. Hence, 5 is the tens' term of the root.

The square of a number composed of tens and units is equal to the square of the tens plus twice the product of the tens by the units, plus the square of the units. (Art. 395.) Hence, the difference between 3364 and the square of the 5 tens of its root, is composed of *twice the product of the tens of the root by the units, plus the square of the units.*

But the product of tens by units contain no order lower than tens, and hence the 86 tens in the 864, the difference, contains *twice the product of the tens by the units.* Hence, if the 86 tens be divided by twice the 5 tens of the root, the quotient, which is 8, will be *the units' term of the root.*

If the 8 units be annexed to the 10 tens, used as a trial divisor, and the result, 108, be multiplied by 8, the product will be twice the product of the tens of the root by the units, plus the square of the units. $108 \times 8 = 2(5 \times 8) + 8^2$.

PROOF.— $58 \times 58 = 3364$.

14. What is the square root of 625? Of 4225?
15. What is the square root of 576? Of 7744?
16. What is the square root of 1444? Of 6241?
17. What is the square root of 3025? Of 7569?
18. What is the square root of 133225?
19. What is the square root of 210681?
20. What is the square root of 419904?
21. What is the square root of 94249? Of 492804?

PROCESS.

$$\begin{array}{rl} & 133225 \mid \underline{365} \\ & \underline{9} \\ 3 \times 2 = 6\ ) & 432 \\ 66 \times 6 = & \underline{396} \\ 36 \times 2 = 72\ ) & 3625 \\ 725 \times 5 = & \underline{3625} \end{array}$$

22. What is the square root of 57600? Of 40960000?

PROCESS.

$$
\begin{array}{rl}
 & \dot{1}\dot{0}.4\dot{9}7\dot{6} \,\underline{|\, 3.24} \\
 & \underline{9} \\
3 \times 2 = 6\,) & 1.49 \\
6.2 \times 2 = & \underline{1.24} \\
3.2 \times 2 = 6.4\,) & .2576 \\
6.44 \times .04 = & \underline{.2576}
\end{array}
$$

23. What is the square root of 10.4976?

24. What is the square root of 176.89?

25. What is the square root of .0625?

26. What is the square root of .451584? Of .008836?

27. What is the square root of 586.7?

SUGGESTION.—Point thus $\dot{5}8\dot{6}.7\dot{0}$, and carry the root to three decimal places by annexing periods of decimal ciphers.

28. What is the square root of 75.364? Of 5.493?
29. What is the square root of 263.85? Of 13467?
30. What is the square root of $\frac{289}{324}$? Of $\frac{625}{2304}$?
31. What is the square root of $272\frac{1}{4}$? Of $1040\frac{1}{16}$?
32. What is the square root of 2? Of 3? Of 5?

## PRINCIPLES AND RULE.

**409.** PRINCIPLES.—1. *The square root of a number contains as many orders as there are periods of two orders each in the number.*

2. *The left-hand period of a number contains the square of the first term of its square root.*

3. *The square of a number, composed of tens and units, is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.*

**410.** RULE.—To extract the square root of a number,

1. *Begin at the units' order and separate the number into periods of two orders each.*

2. *Find the greatest perfect square in the left-hand period, and place its square root at the right for the first or highest term of the root.*

3. *Subtract the square of the term of the root found from the*

*left-hand period, and to the difference annex the second period for a dividend.*

4. *Take twice the term of the root found for a trial divisor, and the dividend, exclusive of its right-hand figure, for a trial dividend. The quotient (or the quotient reduced) will be the next term of the root.*

5. *Annex the second term of the root to the trial divisor, and multiply the result by the second term, and subtract the product from the dividend.*

6. *Annex the third period to the remainder for the next dividend, and divide the same, exclusive of the right-hand figure, by twice the terms of the root found; and continue in like manner until all the periods are used.*

NOTES.—1. The left-hand period may contain but one order.

2. Twice the term or terms of the root, as the case may be, is called a *trial divisor*, since the next term of the root is obtained from the quotient. The term of the root sought is sometimes less than the quotient, since the dividend may contain a part of the square of the next term of the root. The true divisor is the trial divisor with the next term of the root annexed.

3. If the number is not a perfect square, the exact root can not be found. The exact root may be approximated by annexing periods of decimal ciphers. Since the square of no one of the nine digits ends with a cipher, the operation may be continued indefinitely.

4. In pointing off a decimal, or a mixed decimal number, begin with the order of units. If there be an odd number of decimal places, annex a decimal cipher.

5. When both terms of a common fraction are not perfect squares, the exact square root can not be found. An approximate root may be obtained by multiplying both terms of the fraction by the denominator, and extracting the root of the resulting fraction. Thus,

$$\sqrt{\frac{21}{40}}=\sqrt{\frac{21\times 40}{40^2}}=\frac{29}{40}, \text{ nearly.}$$

6. The square root of a perfect square may be found by resolving it into its prime factors, and taking the product of one of every two of those that are equal.

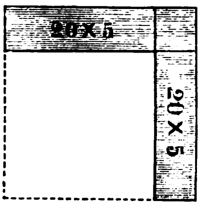
### Geometrical Explanation.

**411.** The area of a square surface is found by squaring the length of one side; and, conversely, the length of the side is found by extracting the square root of the number denoting the area.

Let the annexed diagram represent a square surface whose area is 625. Required the length of one side.

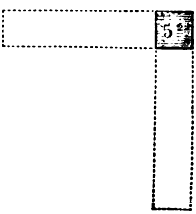
Since the number denoting the area contains two periods, there are two terms in the square root; and since the greatest square in the left-hand period is 4, the tens' term of the root is 2. (Art. 409.) Hence the length of the side of the square is 20 plus the units' term of the root. What is the units' term?

Taking from the given surface a square whose side is 20 and whose area is 400, there remains a surface whose area is 625 — 400, or 225. This surface consists of two equal rectangles, each 20 in length, and a small square, the length of whose side equals the width of each rectangle. What is the width of each rectangle?




Since the two rectangles contain most of the surface whose area is 225, their width may be found by dividing 225 by their joint length, which is twice 20, or 40. The quotient is 5, and hence the width of each rectangle is 5, and their joint area is $40 \times 5$, or 200.

Removing the two rectangles, there remains the small square, whose side is 5 and whose area is 25, the difference between 225 and 200. Hence, 5 is the units' term of the root, and the length of the side of the square is $20 + 5$, or 25.




Adding the area of the several parts, we have $20^2 + 20 \times 5 \times 2 + 5^2 = 400 + 200 + 25 = 625$.

It is seen that the square whose side is 20, represents *the square of the tens of the root;* the two rectangles, *twice the product of the tens by the units;* and the smaller square, *the square of the units.*

NOTE.—The entire length of the surface whose area is 225, is twice the side of the square removed, plus the side of the smaller square ($20 \times 2 + 5 = 45$), and this multiplied by 5 gives an area of 225.

## CUBE ROOT.

### The Division of the Number into Periods.

**412.** The cubes of the smallest, greatest, and an intermediate number, composed of one, two, and three orders, are as follows:

| | | | | | |
|---|---|---|---|---|---|
| $1^3 =$ | 1 | $9^3 =$ | 729 | $4^3 =$ | 64 |
| $10^3 =$ | 1000 | $99^3 =$ | 970299 | $44^3 =$ | 85184 |
| $100^3 =$ | 1000000 | $999^3 =$ | 997002999 | $444^3 =$ | 87528384 |

A comparison of the above numbers with their cubes shows that the cube of a number contains three times as many orders as the number, or three times as many orders less two or less one.

**413.** Hence, if a number be separated into periods of three orders each, *there will be as many orders in its cube root as there are periods in the number.*

1. How many orders in the cube root of 91125?

SUGGESTION.—First point off the number into periods of three orders each; thus, $9\dot{1}12\dot{5}$.

2. How many orders in the cube root of 84604519?
3. How many orders in the cube root of 912673?
4. How many orders in the cube root of 48228544?
5. How many orders in the cube root of 2357947691?

**414.** The cubes of the smallest and greatest number of units, tens, and hundreds are as follows:

| | | | |
|---|---|---|---|
| $1^3 =$ | 1 | $9^3 =$ | 729 |
| $10^3 =$ | 1000 | $90^3 =$ | 729000 |
| $100^3 =$ | 1000000 | $900^3 =$ | 729000000 |

A comparison of the above numbers with their cubes shows that the cube of units gives no order higher than

hundreds; that the cube of tens gives no order lower than thousands nor higher than hundred-thousands; and that the cube of hundreds gives no order lower than millions nor higher than hundred-millions.

Hence, if a number be separated into periods of three orders each, *the left-hand period will contain the cube of the first term of the cube root; the first two left-hand periods will contain the cube of the first two terms of the cube root, etc.*

6. What is the tens' term of the cube root of 91125?

7. What is the tens' term of the cube root of 912673?

8. What is the hundreds' term of the cube root of 48228544?

9. What is the first term of the cube root of 529475129?

10. What is the first term of the cube root of 257259456?

### WRITTEN PROBLEMS.

11. What is the cube root of 262144?

PROCESS.

| | | | |
|---|---|---|---|
| | | 262144 | 64 |
| $6^3$ | $=$ | 216 | 4 |
| $6^2 \times 3 = 108$ | | )461 44 | |
| $64^3$ | $=$ | 262144 | |

Since 262144 is composed of two periods, its cube root will be composed of two orders (Art. 413). The left-hand period, 262, contains the cube of the tens' term of the root (Art. 414). The greatest cube in 262 is 216, the cube root of which is 6; hence, 6 is the tens' term of the root. How is the units' term to be found?

The cube of a number, composed of tens and units, is equal to the cube of the tens, plus three times the product of the square of the tens by the units, plus three times the product of the tens by the square of the units, plus the cube of the units (Art. 395). Hence, the difference between 262144 and the cube of the 6 tens of its cube root, is composed of *three times the product of the square of the tens of its root by the units, plus three times the product of the tens by the square of the units, plus the cube of the units.*

But since the square of tens gives no order lower than hundreds (Art. 407), the 461 hundreds of the difference (46144) contains *three times the product of the square of the tens by the units.* Hence, if the 461 hundreds (rejecting the two right-hand figures) be divided by three

times the square of the 6 tens of the root, the quotient, which is 4, will be *the units' term of the root.* Cube 64, and subtract the result from 262144. There is no remainder, and hence 64 is the cube root sought.

NOTE.—Instead of cubing 64, the parts which compose the difference, 46144, may be formed and added, thus:

$$\begin{array}{r} 60^2 \times 4 \times 3 = 43200 \\ 60 \times 4^2 \times 3 = \phantom{0}2880 \\ 4^3 = \underline{\phantom{000}64} \\ 46144 \end{array}$$

11. What is the cube root of 42875? Of 91125?
12. What is the cube root of 117649? Of 185193?
13. What is the cube root of 274625? Of 405224?
14. What is the cube root of 704969? Of 912673?
15. What is the cube root of 48228544?

PROCESS.

$$\begin{array}{rl} \dot{4}8\dot{2}28\dot{5}44 & \underline{364}, \textit{ Cube root.} \\ 3^3 = \underline{27\phantom{000}} & \phantom{3}74, \textit{ Trial quotients.} \\ 3^2 \times 3 = 27\,)\,\underline{212\phantom{00}} & \\ 36^3 = \underline{46656\phantom{0}} & \\ 36^2 \times 3 = 3888\,)\,\underline{\phantom{0}15725} & \\ 364^3 = \underline{48228544} & \end{array}$$

Since the two right-hand figures of each dividend are rejected, only the first figure of each period need be brought down and annexed to the difference.

The quotient obtained by dividing 212 by 27 is 7, which is too large for the second term of the root, since the cube of 37 is more than 48228, the first two periods.

The second difference is found by subtracting the cube of 36, the first two terms of the root, from 48228, the first two periods of the number.

16. What is the cube root of 3048625? Of 34328125?
17. What is the cube root of 41063625? Of 43614208?
18. What is the cube root of 27270901? Of 515849608?
19. What is the cube root of 185193? 128024064?
20. What is the cube root of 103823? Of 27054036008?
21. What is the cube root of 15.625? Of .074256?
22. What is the cube root of 97.336? Of .015625?

23. What is the cube root of 56.47? Of 12.3456?

SUGGESTION.—Point from units' order, and fill decimal periods, thus: $5\dot{6}.47\dot{0}$, and $1\dot{2}.34\dot{5}60\dot{0}$.

24. What is the cube root of .000042875? Of 67.917312?

25. What is the cube root of 9? Of 31? Of 50?

SUGGESTION.—Annex periods of decimal ciphers and carry the root to three decimal places.

26. What is the cube root of 2? Of 20? Of 200?

27. What is the cube root of $\frac{216}{1728}$? Of $\frac{1331}{42875}$?

28. What is the cube root of $11\frac{25}{64}$? Of $37\frac{1}{27}$?

29. A cubical box contains 19683 cubic inches: what is the length of its edge?

30. A block of granite in the form of a cube, contains 41063.625 cubic inches: what is the length of its edge?

31. A cubical bin holds 100 bushels: what is the length of its edge?

32. If 6 ranks of wood, each 128 ft. long, 3 ft. wide, and 6 ft. high, were piled together in the form of a cube, what would be the height of the pile?

## PRINCIPLES AND RULE.

**415.** PRINCIPLES.—1. *The cube root of a number contains as many orders as there are periods of three figures each in the number.*

2. *The left-hand period of a number contains the cube of the first term of its cube root; the two left-hand periods contain the cube of the first two terms of the cube root, etc.*

3. *The cube of a number, composed of tens and units, is equal to the cube of the tens, plus three times the product of the square of the tens by the units, plus three times the product of the tens by the square of the units, plus the cube of the units.*

**416.** RULE.—To extract the cube root of a number,

1. *Begin at the units' order and separate the number into periods of three orders each.*

2. *Find the greatest cube in the left-hand period, and place its cube root at the right for the first term of the root.*

3. *Subtract the cube of the first term of the root from the left-hand period, and to the difference annex the first figure of the next period for a dividend.*

4. *Take three times the square of the first term of the root for a trial divisor, and the quotient for the second term of the root. Cube the root now found, and, if the result is not greater than the two left-hand periods, subtract, and to the difference annex the first figure of the next period for a second dividend. If the cube of the root found is greater than the two left-hand periods, diminish the second term of the root.*

5. *Take three times the square of the two terms of the root found for a second trial divisor, and the quotient for the third term of the root. Cube the three terms of the root found, and subtract the result from the three left-hand periods, and continue the operation in like manner until all the terms of the root are found.*

NOTES.—1. The quotient obtained by dividing the dividend by the trial divisor may be too large, since *three times the square of the next figure of the root* may be a part of the dividend. Usually the term of the root sought is the quotient, or one less than the quotient.

2. When a dividend does not contain the trial divisor, write a cipher for the next term of the root. Take three times the square of the root thus formed for a trial divisor, and to the dividend annex the two remaining figures of the period, and the first figure of the next period for a new dividend.

3. If the number is not a perfect cube, the root may be approximated by annexing periods of decimal ciphers, thus adding decimal terms to the root. Sufficient accuracy is usually secured by continuing the root to two or three decimal places.

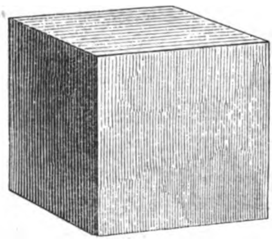
4. When both terms of a common fraction are not perfect cubes, the cube root may be found approximately by multiplying both terms of the fraction by the square of the denominator, and extracting the root of the resulting fraction. The error will be less than one divided by the denominator of the root.

5. The above methods of extracting the square or cube root of numbers, is a general method by which any root may be extracted. The fourth root, for example, is found by dividing the number into periods of *four* figures each, then taking the fourth root of the left-hand period for the first term of the root, four times the cube of this first term for a trial divisor, and the remainder with the first term of the next period annexed, for a dividend, etc.

6. The cube root of a perfect cube may be found by resolving it into its prime factors and taking the product of one of every three of those that are equal.

### Geometrical Explanation of the Process of Extracting the Cube Root.

**417.** The solid contents of a cube are found by cubing the length of its edge, and, conversely, the length of the edge is found by extracting the cube root of the number denoting the solid contents.



Let the annexed cut represent a cube whose solid contents are 15625. Required the length of the edge.

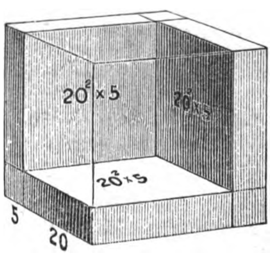
Since the number denoting the solid contents contains two periods, there will be two terms in the cube root, and since the greatest cube in the left-hand period is 8, the tens' term of the root is 2 (Art. 414). Hence, the length of the edge of the cube is 20 plus the units' term of the root.

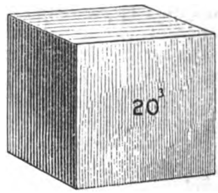
PROCESS.

$$\begin{array}{rr|l}
 & 15625 & 25 \\
2^3 = & \underline{8} & 6 \\
20^2 \times 3 = 1200\,) & \underline{7625} & \\
20^2 \times 5 \times 3 = & 6000 & \\
20 \times 5^2 \times 3 = & 1500 & \\
5^3 = & \underline{125} & \\
 & 7625 &
\end{array}$$

What is the units' term?

Taking from the given cube a cube whose edge is 20 and whose capacity is 8000, there remains a solid whose capacity



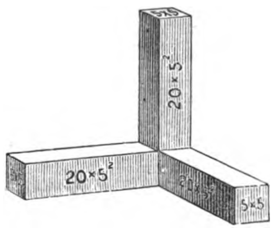

is 15625 — 8000, which is 7625. An inspection of the annexed cut shows that this solid contains *three* equal rectan-

gular solids, whose inner face ($20^2$) is equal to the face of the removed cube and whose thickness equals the units' term of the root. What is the thickness of each of these rectangular solids?

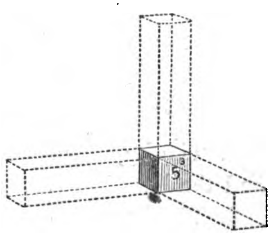
Since they compose only a part of the solid whose solid contents are 7625, their thickness can not be greater than the quotient obtained by dividing 7625 by the area of their joint inner faces, which is $20^2 \times 3$, or 1200. The quotient is 6, which is at least one greater than the thickness of each of the three rectangular solids, since $26^3$ is greater than 15625, the solid contents of the given cube. Try 5 for the thickness. $25^3 = 15625$, and hence 5 is the required thickness, and the length of the edge of the given cube is $20 + 5$, or 25.

The correctness of this result may also be shown by finding the solid contents of the several parts of the given cube. The solidity of the cube removed is, as shown above, $20^3 = 8000$. The joint solidity of the three adjacent rectangular solids is $20^2 \times 5 \times 3 = 6000$.

Removing these three rectangular solids, there remain three other rectangular solids, whose solidity is $20 \times 5^2$, or 500 each, and whose combined solidity is $500 \times 3$, or 1500.




Removing these three rectangular solids, there remains the small cube, whose solidity is $5^3 = 125$.




Adding the solidity of the several parts, we have $8000 + 6000 + 1500 + 125 = 15625$, which is the solidity of the given cube.

It is seen that the cube whose edge is 20, represents *the cube of the tens of the root;* the three ad-

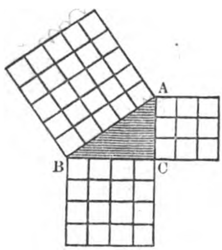
jacent rectangular solids represent *three times the product of the square of tens by the units;* the three smaller rectangular solids, *three times the product of the tens by the square of the units;* and the smaller cube, *the cube of the units.*

## APPLICATIONS OF INVOLUTION AND EVOLUTION.

### I. TO THE RIGHT-ANGLED TRIANGLE.

**418.** The ***Hypotenuse*** of a right-angled triangle is the side opposite the right angle. The other two sides are called the *Base* and the *Perpendicular.* (Art. 155.)

**419.** PRINCIPLES.—1. *The square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.*




This principle, which may be proven by geometry, is illustrated by the annexed diagram.

2. *The square of the base or the perpendicular of a right-angled triangle is equal to the square of the hypotenuse less the square of the other side.*

### PROBLEMS.

1. The base of a right-angled triangle is 8, and the perpendicular 6: what is the length of the hypotenuse?

SOLUTION.—Since the square of the hypotenuse equals the square of the base plus the square of the perpendicular, the hypotenuse equals $\sqrt{8^2 + 6^2} = \sqrt{100} = 10$.

2. The hypotenuse of a right-angled triangle is 20 inches and the base is 16 inches: what is the perpendicular?

3. The hypotenuse of a right-angled triangle is 45 feet and the perpendicular is 27 feet: what is the base?

4. A rectangular field is 192 yards long and 144 yards wide: what is the length of the diagonal?

5. The foot of a ladder is 18 feet from the base of a building, and the top reaches a window 24 feet from the base: what is the length of the ladder?

6. Two boys start from the same point, and one walks 96 rods due north, and the other 72 rods due east: how far are they apart?

7. A flag pole 180 feet high casts a shadow 135 feet in length: what is the distance from the top of the pole to the end of the shadow?

8. A boy in flying his kite let out 240 feet of string, and the distance from where he stood to a point directly under the kite was 208 feet: how high was the kite, supposing the string to be straight?

9. A rectangular field is 84 rods long and 63 rods wide: what is the side of a square field of the same area?

10. A farm is 125 rods square, and a rectangular farm, containing the same number of acres, is 150 rods in length: what is its width?

**420.** RULES.—1. To find the hypotenuse of a right-angled triangle, *Extract the square root of the* SUM *of the squares of the other two sides.*

2. To find the base or the perpendicular of a right-angled triangle, *Extract the square root of the* DIFFERENCE *between the squares of the hypotenuse and the other side.*

### II. TO THE CIRCLE.

**421.** PRINCIPLES.—1. *The area of a circle is equal to the square of its diameter multiplied by* .7854.

2. *The areas of two circles are to each other as the squares of their diameters.*

NOTE.—The above propositions can be proven by geometry. The area of a circle also equals its circumference multiplied by one fourth of its diameter. (Art. 161.)

### PROBLEMS.

11. The diameter of a circle is 15 inches: what is its area?

12. A circular pond is 100 feet in diameter: how many square yards does it contain?

13. A circular room has an area of 78.54 square yards: what is its diameter?

14. How many circles, each 3 inches in diameter, will equal in area a circle whose diameter is 2 feet?

15. How many circles, each 15 inches in diameter, will equal in area a circle whose diameter is 5 feet?

16. A horse, tied to a stake by a rope, can graze to the distance of 40 feet from the stake: on how much surface can it graze?

17. A horse, tied to a stake, can graze on $218\frac{1}{6}$ square yards of surface: to what distance from the stake can it graze?

18. How many circles, each 3 inches in diameter, contain the same area as a surface 2.5 feet square?

### III. TO THE SPHERE.

**422.** PRINCIPLES.—1. *The surface of a sphere is equal to the square of the diameter multiplied by* 3.1416.

2. *The solidity of a sphere is equal to the cube of the diameter multiplied by* .5236.

3. *Two spheres are to each other as the cubes of their diameters.*

NOTE.—The surface of a sphere may also be found *by multiplying the circumference by the diameter;* and the solidity *by multiplying the surface by one sixth of the diameter.* (Arts. 474, 475.)

### PROBLEMS.

19. What is the surface of a sphere whose diameter is 10 inches?

20. How many square miles on the surface of the earth, its mean diameter being 7912 miles?

21. How many cubic miles in the solidity of the earth?

22. How many cubic inches in a cannon ball whose diameter is 7 inches?

23. How many balls 2 inches in diameter, equal in solidity a ball whose diameter is 8 inches?

24. The diameter of the earth is about 4 times the diameter of the moon: how many times larger than the moon is the earth?

25. The diameter of Jupiter, the largest planet, is about 85000 miles, and the diameter of the sun is about 850000 miles: how many times larger than Jupiter is the sun?

26. The surface of the planet Mercury contains about 28274400 square miles: what is its diameter?

27. The planet Uranus contains about 18816613200000 cubic miles: what is its diameter?

SUGGESTION.—Divide the solidity by .5236, and extract the cube root of the quotient.

28. A brass ball contains 904.7808 cubic inches: what is its diameter?

29. A square and a triangle contain an equivalent area, and the base of the triangle is 36.1 inches, and its altitude is 5 inches: what is the side of the square?

30. One of the mammoth pines of California is 110 feet in circumference: what is its diameter?

31. How many cubic feet in a portion of the above tree 100 feet in length, supposing its mean circumference to be $94\frac{1}{4}$ feet?

32. The mean distance of the earth from the sun (new value) is about 91400000 miles, and it revolves in its orbit in $365\frac{1}{4}$ days: what is its mean hourly motion?

33. The mean distance of Mercury from the sun (new value) is about 35400000 miles, and it revolves in its orbit in 87.9 days: what is its mean hourly motion?

34. The diameter of the moon is about 2000 miles: how does the extent of the moon's surface compare with that of the earth?

# SECTION XVI.

## GENERAL REVIEW.

NOTE.—The following problems are selected from several sets used in the examination of pupils for promotion to high schools, and in the examination of teachers.

### MENTAL PROBLEMS.

1. If 3 apples are worth 2 oranges, how many oranges are 24 apples worth?

2. How long will it take a man to lay up \$60, if he earn \$15 a week and spend \$9?

3. $\frac{7}{8}$ of $74\frac{2}{3}$ is $\frac{3}{4}$ of what number?

4. $\frac{5}{6}$ of 45 is $\frac{3}{8}$ of how many times 10?

5. A has 20 cents; and $\frac{4}{5}$ of what A has is $\frac{8}{9}$ of what B has: how many has B?

6. If $\frac{7}{8}$ of a yard of cloth cost 63 cents, what will $\frac{5}{9}$ of a yard cost?

7. If 3 yards of muslin cost $13\frac{1}{2}$ cents, what will $\frac{2}{3}$ of a yard cost?

8. The difference between $\frac{6}{7}$ and $\frac{7}{8}$ of a number is 10: what is the number?

9. What fraction is as much greater than $\frac{4}{5}$ as $\frac{2}{3}$ is less?

10. A piece of flannel lost $\frac{2}{5}$ of its length by shrinkage in fulling, and then measured 30 yards: what was its length before fulling?

11. A horse cost \$90, and $\frac{3}{10}$ of the price of the horse equals $\frac{3}{5}$ of 3 times the cost of the saddle: what did the saddle cost?

12. If to my age you add its half, its third, and 28 years, the sum will be three times my age: what is my age?

13. A boy being asked his age, said that $\frac{3}{4}$ of 80 was $\frac{2}{5}$ of 10 times his age: what was his age?

14. A boy gave $\frac{5}{8}$ of his money for a sled, $\frac{1}{3}$ of it for a hat, and then had 7 cents left: how many cents had he at first?

15. $\frac{3}{5}$ of my money is in my purse, $\frac{3}{8}$ in my hand, and the remainder, which is 25 cents, is in my pocket: how much money have I?

16. A boy having $\frac{3}{5}$ of a dollar, gave $\frac{2}{3}$ of his money to John and $\frac{1}{2}$ of the remainder to James: what part of a dollar did James receive?

17. A farmer sold $\frac{2}{3}$ of his sheep and then bought $\frac{3}{5}$ as many as he had left, when he had 40 sheep: how many had he at first?

18. John lost $\frac{2}{5}$ of his money and spent $\frac{1}{3}$ of the remainder, and then had only 10 cents: how much money had he at first?

19. A man sold a horse for \$60, which was $\frac{4}{5}$ of $\frac{3}{4}$ of its cost: how much was lost by the bargain?

20. A man sold a horse for \$130, which was $\frac{5}{8}$ more than it cost him: what was the cost of the horse?

21. A sold B a horse for $\frac{1}{5}$ more than its cost, and B sold it for \$80, losing $\frac{1}{6}$ of its cost: how much did A pay for the horse?

22. At \$$\frac{5}{7}$ a bushel, how many bushels of corn may be bought for \$8?

23. If $\frac{3}{4}$ of a bushel of wheat cost \$$\frac{3}{5}$, what part of a bushel can be bought for \$$\frac{5}{8}$?

24. If \$$18\frac{3}{4}$ will purchase $\frac{3}{8}$ of a load of corn, what part of it will \$$16\frac{2}{3}$ purchase?

25. If $2\frac{1}{2}$ pounds of cheese cost $3\frac{1}{3}$ dimes, what part of a pound can be bought for 1 dime?

26. How many bushels of coal at $12\frac{1}{2}$ cents a bushel can be bought for \$15?

27. What part of 7 bushels is $\frac{5}{8}$ of a peck?

28. What part of a pound of gold is .25 of an ounce?

29. What part of $\frac{2}{3}$ of a gallon is $\frac{3}{4}$ of a pint?

30. From $\frac{3}{5}$ of a day take $\frac{3}{5}$ of an hour.

31. If a staff 5 feet long cast a shadow 2 feet long at 12

o'clock, what is the height of a steeple whose shadow, at the same hour, is 80 feet?

32. If a five cent loaf weigh 10 ounces when flour is $4 a barrel, what ought it to weigh when flour is $5 a barrel?

33. If 20 bushels of oats will feed 40 horses 80 days, how long will 180 bushels feed them?

34. If a horse eat 2 bushels of oats in 6 days, in how many days will 2 horses eat 18 bushels?

35. If 3 men can mow 18 acres of grass in 4 days, how many men can mow 9 acres in 3 days?

36. A garrison of 20 men is supplied with provisions for 12 days: if 12 men leave, how long will the provisions serve the remainder?

37. A man bought a watch and chain for $80, and the chain cost $\frac{1}{3}$ as much as the watch: how much did each cost?

38. A has $1\frac{1}{2}$ times as many cents as B, and they together have 40 cents: how many has each?

39. A pole 120 feet high fell and broke into two parts, and $\frac{3}{5}$ of the longer part was equal to the shorter: how long was each part?

40. A and B together own 824 sheep, and A has $1\frac{2}{3}$ times as many as B: how many has each?

41. A, B, and C rent a pasture for $42; B pays half as much as A, and C half as much as B: what does each pay?

42. A and B own a farm; A owns $\frac{3}{4}$ as much as B, and B owns 40 acres more than A: how many acres does each own?

43. $\frac{3}{4}$ of A's money is $\frac{2}{3}$ of B's, and $\frac{3}{4}$ of B's is $\frac{2}{3}$ of C's, which is $81: how much have A and B each?

44. If a man can reap $\frac{3}{4}$ of an acre of wheat in a day, how much can 6 men reap in 10 days?

45. A makes a shoe in $\frac{2}{3}$ of a day; B makes one in $\frac{2}{5}$ of a day: how many shoes can both make in a day?

46. A can mow an acre of grass in $\frac{3}{4}$ of a day, and B in $\frac{3}{5}$ of a day: how long will it take both together to mow an acre?

47. A can mow a field of grass in 5 days, and B in 4 days: how long will it take both, working together, to mow it?

48. A can build a house in 20 days, but, with the assistance of C, he can do it in 12 days: in what time can C do it alone?

49. A alone can build a certain wall in 6 days, B alone in 10 days, and C alone in 15 days: in how many days can they all together build it?

50. A, B, and C can do a job in 20 days; A and B can do it in 40 days; and A and C in 30 days: in how many days can each do it alone?

51. A broker bought rail-road stock at 80 and sold it at 70: what per cent. did he lose?

52. A broker bought stock at 70 and sold it at 90: what per cent. did he gain?

53. A merchant bought 40 yards of cloth for $90: at how much a yard must he sell it to gain $33\frac{1}{3}$ per cent.?

54. For how much must tea costing 90 cents, be sold to gain $12\frac{1}{2}$ per cent.?

55. A man bought a hat for $5 and sold it for $6: what per cent. did he gain?

56. I sell cloth at $2.50 a yard and gain 25 per cent.: for how much must I sell it to lose 20 per cent.?

57. A man earned a certain sum of money, and, after adding to it $12.50, found that what he then had was $133\frac{1}{3}$ per cent. of what he earned: how much did he earn?

58. A man sold a watch for $90, and gained 50 per cent.: what per cent. would he have gained if he had sold it for $75?

59. $\frac{2}{3}$ of the price received for an article is equal to $\frac{3}{4}$ of its cost: what is the gain per cent.?

60. Two men, A and B, engaged in trade with different capitals; A lost $33\frac{1}{3}$ per cent. of his capital, and B gained 50 per cent. on his, when each had $600: with what capital did each begin trade?

61. How much grain must I take to mill to bring away 2 bushels after the miller has taken 10 per cent. for toll?

62. At what rate per cent., simple interest, will \$1 double itself in 8 years?

63. The interest on a certain sum for 4 years was $\frac{1}{2}$ the sum: what was the rate per cent.?

64. Two men start from two places 495 miles apart, and travel toward each other; one travels 20 miles a day, and the other 25 miles a day: in how many days will they meet?

65. A owes $\frac{2}{3}$ of B's income, but, by saving $\frac{7}{45}$ of B's income annually, he can pay his debt in 5 years, and have \$50 left: what is B's income?

66. C and D are traveling in the same direction; C is 18 miles ahead of D, but D travels 7 miles while C travels 4: how many miles from the place of starting will D have traveled when he overtakes C?

67. If a man traveling 14 hours a day, perform half a journey in 5 days, how long will it take to perform the other half, if he travel 10 hours a day?

68. If a man can do a piece of work in $9\frac{4}{5}$ days, working 8 hours a day, how long will it take, if he work 6 hours a day?

69. A is 20 years old; the sum of the ages of B and C equals 4 times A's age; C's age is $\frac{1}{9}$ of A's and B's together: what is the age of each?

70. A hare is 30 rods before a hound, but the hound runs 7 rods while the hare runs 5: how far must the hound run to catch the hare?

71. A hare starts 50 leaps before a hound, and leaps 4 times while the hound leaps 3 times; but 2 of the hound's leaps equal 4 of the hare's: how many leaps must the hound take to catch the hare?

72. If a steamer sails 9 miles an hour down stream, and 5 miles an hour up stream, how far can it go down stream and back again in 14 hours?

73. A steamer sails a mile down stream in 5 minutes, and a mile up stream in 7 minutes: how far down stream can it go and return again in one hour?

74. A pipe will fill a cistern in 4 hours, and another will empty it in 6 hours: how long will it take to fill it when both pipes run?

75. At what time between six and seven o'clock are the hour and minute hands of a clock together?

### WRITTEN PROBLEMS.

76. The minuend is 1250, and the remainder 592: what is the subtrahend?

77. The quotient is 71, the divisor 42, and the remainder 15: what is the dividend?

78. If a certain number be multiplied by 22, and 64 be subtracted from the product, and the remainder be divided by 4, the quotient will be 50: what is the number?

79. What will be the cost of 3760 lbs. of hay, at $8.50 a ton?

80. At $24.50 per acre, how many acres of land can be bought for $3560.75?

81. Add $\frac{2}{3}$, $\frac{5}{6}$, $\frac{1}{2}$ of $\frac{7}{8}$, and $\frac{2}{5}$ of $2\frac{1}{2}$.

82. From $17\frac{1}{2}$ take $\frac{3}{5}$ of $6\frac{1}{4}$, and multiply the remainder by $\frac{2}{3}$.

83. Multiply $\frac{3}{4}$ of $\frac{2}{3}$ by $\frac{7}{8}$ of $\frac{5}{7}$, and divide the product by $\frac{3}{10}$.

84. Divide $\frac{3}{5}$ of $6\frac{1}{4}$ by $\frac{2}{3}$ of $7\frac{1}{2}$.

85. What number multiplied by $28\frac{5}{7}$ will produce 145?

86. From the sum of $215\frac{2}{3}$ and $125\frac{3}{4}$ take their difference.

87. Multiply $\frac{5}{8} + \frac{2}{3}$ of $\frac{3}{4}$ by $\frac{5}{8} - \frac{2}{3}$ of $\frac{3}{4}$.

88. Divide $2\frac{1}{2}$ by $3\frac{1}{3}$, and multiply the quotient by $3\frac{1}{5}$.

89. What must $8\frac{13}{16}$ be multiplied by that the product may be 3?

90. A man bought $\frac{9}{16}$ of a section of land for $2880, and sold $\frac{2}{3}$ of it at $10 an acre, and the rest at $12 an acre: how much did he gain?

91. A merchant owning $\frac{7}{8}$ of a ship sells $\frac{2}{5}$ of his share for $16800: what is the value of the whole ship, at this rate, and what part of the ship has he left?

92. Add 9 thousandths, 3 hundredths, and 7 units.

93. From 15 ten-thousandths take 27 millionths, and multiply the difference by 20.5.

94. Multiply 160 by .016, and divide the product by .0025.

95. Multiply 15 thousandths by 15 hundredths, and from the product take 15 millionths.

96. Divide 256 thousandths by 16 millionths.

97. Multiply 625 by .003, and divide the result by 25.

98. Change $\frac{3}{400}$ to a decimal, and divide the result by $2\frac{1}{2}$.

99. Change $\frac{5}{125}$ to a decimal, and divide the result by 5000.

100. Reduce .625 of a pound Troy to lower integers.

101. What decimal of a rod is .165 of a foot?

102. What will 63 thousandths of a cord of wood cost, at $2.25 per cord?

103. How many minutes will there be in the month of February, 1880?

104. How many seconds are there in the three summer months?

105. How many steps, 2 ft. 4 in. each, will a person take in walking $2\frac{1}{4}$ miles?

106. How many times will a wheel, 12 ft. 6 in. in circumference, turn round in rolling one mile?

107. How many acres in a street 4 rods wide and $2\frac{1}{4}$ miles long?

108. How many yards of carpeting, $\frac{3}{4}$ of a yard wide, will it take to cover a parlor, $18\frac{1}{2}$ feet long and 15 feet wide?

109. How many grains in 14 ingots of silver, each weighing 27 oz. 10 pwt.?

110. How many square feet of lumber in 40 boards, each 12 feet long and $7\frac{1}{2}$ inches wide?

111. What will a board 20 feet long and 9 inches wide cost, at $30 a thousand?

112. What will it cost to lay a pavement 36 feet long and 9 feet 6 inches wide, at 40 cents a square yard?

113. A pile of wood, containing 10 cords, is 20 feet long and 8 feet wide: how high is it?

114. What is the value of a pile of wood 40 feet long, 8 feet wide, and $5\frac{1}{2}$ feet high, at \$5.30 a cord?

115. How many sacks, holding 2 bu. 3 pk. 2 qt. each, can be filled from a bin containing 366 bu. 3 pk. 4 qt. of wheat?

116. A lady bought 6 silver spoons, each weighing 3 oz. 3 pwt. 8 gr., at \$2.25 an ounce, and a gold chain, weighing 14 pwt., at \$1.25 a pwt.: what was the cost of both spoons and chain?

117. How many bricks will it require to build a wall 2 rods long, 6 feet high, and 18 inches thick, each brick being 8 inches long, 4 inches wide, and $2\frac{1}{2}$ inches thick?

118. Cincinnati is 7° 49′ west of Baltimore: when it is noon at Baltimore, what is the hour at Cincinnati?

119. New York is 75 degrees of longitude west of London: when it is noon at New York what is the hour at London?

120. Boston is 71° 4′ 9″ W. longitude, and Cleveland is 81° 47′ W.: when it is 4 P. M. at Cleveland, what is the hour at Boston?

121. What part of a rod is 2 ft. 9 in.?

122. Reduce 5 fur. 8 rd. to the decimal of a mile.

123. Reduce $\frac{5}{8}$ of a square yard to the fraction of an acre.

124. From $\frac{7}{8}$ of a pound Troy take $\frac{4}{5}$ of an ounce.

125. Reduce $\frac{3}{8}$ of a quart to the fraction of a bushel.

126. A regiment lost 8% of its men in a battle, and 25% of those that remained died from sickness, and it then mustered 621 men: how many men were in the regiment at first?

127. A quantity of sugar was bought for \$150, and sold for \$167.50: what was the gain per cent.?

128. A merchant bought 500 yards of cloth for \$1800: for how much a yard must he sell it to gain 25%?

129. A man sold a piece of cloth for \$24, and thereby

lost 25 %; if he had sold it for $34, would he have gained or lost, and what per cent.?

130. I sold goods at 20 % gain, and, investing the proceeds, sold at 20 % loss: did I gain or lose by the operation, and what per cent.?

131. Sold 2 carriages, at $240 apiece, and gained 20 % on one and lost 20 % on the other: how much did I gain or lose in the transaction?

132. A man bought a horse for $72, and sold it for 25 % more than cost, and 10 % less than he asked for it: what did he ask for it?

133. A merchant marked a lot of goods, costing $5800, at 30 % above cost, but sold them at 10 % less than the marked price: how much and what per cent. did he gain?

134. What must I ask for cloth, costing $4 a yard, that I may deduct 20 % from my asking price and still make 20 %?

135. A man bought stock at 25 % below par and sold it at 20 % above par: what per cent. did he make?

136. A fruit dealer lost $33\frac{1}{3}$ per cent. of a lot of apples, and sold the remainder at a gain of 50 per cent.: required the per cent. of gain or loss.

137. I bought 63 kegs of nails, each keg containing 100 lbs., at $4\frac{1}{2}$ cents a pound, and sold $\frac{2}{3}$ of them for what $\frac{1}{2}$ of them cost: what per cent. did I lose on the part sold?

138. I bought $128.25 worth of goods; kept them on hand 6 months when money was worth 8 % interest, and then sold them at a net gain of 6 %: for how much were they sold?

139. When money was worth 9 % interest, I bought $800 worth of goods, kept them 4 months and then sold them for $959.10: what per cent. on the cost did I gain?

140. A house valued at $3240 is insured for $\frac{2}{3}$ of its value, at $\frac{3}{4}$ %: what is the premium?

141. I pay $19.20 premium for insuring my house for $\frac{5}{8}$ of its value, at $1\frac{1}{2}$ %: what is the value of my house?

142. A capitalist sent a broker $25000 to invest in cotton, after deducting his commission of $2\frac{1}{2}\%$: how much cotton, at 5 cents a pound, did the broker purchase?

143. An agent received $502.50 to purchase cloth, after deducting $\frac{1}{2}\%$ commission: how many yards did he buy at $1.25 a yard?

144. What is the interest of $125.50 for 7 months and 10 days, at 7%?

145. What is the interest of $50000 for one day, at 8%?

146. What is the interest of $75.50 from June 12, 1869, to Aug. 6, 1870, at $7\frac{1}{2}\%$?

147. A man loaned $800 for 2 years and 6 months, and received $90 interest: what was the rate per cent.?

148. At what rate per cent. will $311.50 amount to $337.40 in 1 yr. 4 mo.?

149. What sum of money will yield as much interest in 3 years, at $4\frac{1}{2}$ per cent., as $540 yields in 1 yr. 8 mo., at 7%?

150. The amount of a certain principal for 3 years, at a certain rate per cent., is $750, and the interest is $\frac{1}{4}$ of the principal: what is the principal, and what is the rate per cent.?

151. A note for $500, dated Oct. 8, 1864, and bearing interest at 9%, is indorsed as follows: Nov. 4, 1865, $30; Jan. 30, 1866, $250. What was due July 1, 1866?

152. What is the present worth of a note of $1320, due in 3 years and 4 months, without interest, money being worth 6%? What is the discount?

153. What is the true discount of $236, due in 3 years, at 6%?

154. What is the bank discount on $125, payable in 90 days, at 8%?

155. What is the difference between the true discount and the bank discount of $359.50, for 90 days, without grace, at 12%?

156. For what sum must I give my note at a bank, payable in 4 months, at 10%, to get $300?

157. I borrow of A $150 for 6 months, and afterward I

lend him \$100: how long may he keep it to balance the use of the sum he lent me?

158. A owes B \$300, of which \$50 is due in 2 months, \$100 in 5 months, and the remainder in 8 months: what is the equated time for the whole sum?

159. A man owes \$300 due in 5 months, and \$700 due in 3 months, and \$200 due in 8 months: if he pays $\frac{1}{2}$ of the whole in 2 months, when ought the other half to be paid?

160. I have sold 50 bushels of wheat for A, and 60 bushels for B, receiving \$150 for both lots: if A's wheat is worth 20% more than B's, how much ought I to pay each?

161. Two men divided a lot of wood, which they purchased together for \$27; one took $5\frac{1}{2}$ cords, the other 8: what ought each to pay?

162. If 8 men cut 84 cords of wood in 12 days, working 7 hours a day, how many men will cut 150 cords in 10 days, working 5 hours a day?

163. If 16 horses consume 84 bushels of grain in 24 days, how many bushels of grain will supply 36 horses 16 days?

164. If the wages of 24 men for 4 days are \$192, what will be the wages of 36 men for 3 days?

165. If 4 men in $7\frac{2}{3}$ days earn $\$53\frac{2}{3}$, how much will 7 men earn in $\frac{4}{49}$ of a day?

166. A and B traded in company and gained \$750, of which B's share was \$600; A's stock was \$1200: what was B's stock?

167. A and B formed a partnership for 1 year, and A put in \$2000 and B \$800: how much more must B put in at the close of 6 months to receive one-half of the profits?

168. A and B engage in trade; A puts in \$200 for 5 months, B \$300 for 2 months; they draw out capital and profits to the amount of \$1389: what was each man's share?

169. What is the square root of 41616? Of 420.25?

170. What is the cube root of 46656? Of 42.875?

171. A certain window is 30 feet from the ground: how far from the base of the building must the foot of a ladder 50 feet long be placed to reach the window?

172. Two men start from the same point; one travels 52 miles north and the other 39 miles west: how far are they apart?

173. A house is 40 feet high from the ground to the eaves, and it is required to find the length of a ladder which will reach the eaves, supposing the foot of the ladder can not be placed nearer to the house than 30 feet.

174. How many rods of fence will inclose 10 acres in the form of a square?

175. A floor is 24 feet long and 15 feet wide: what is the distance between two opposite corners?

176. A room is 20 feet long, 16 feet wide, and 12 feet high: what is the distance from one of the lower corners to the upper opposite corner?

177. How many cubic feet in a stone 8 feet long, $5\frac{1}{2}$ feet wide, and $3\frac{1}{2}$ feet thick?

178. How many square feet on the surface of a stone 6 feet long, 4 feet wide, and $1\frac{1}{2}$ feet thick?

179. There is a circular field 40 rods in diameter: what is its circumference? How many acres does it contain?

180. The area of a circle is $470.8\frac{3}{4}$ square inches: what is the length of its diameter?

181. How many iron balls 2 inches in diameter, will weigh as much as an iron ball 8 inches in diameter?

182. How many cubical blocks, each edge of which is $\frac{1}{3}$ of a foot, are equivalent to a block of wood 8 feet long, 4 feet wide, and 2 feet thick?

183. How many bushels of wheat will fill a bin 8 feet long, 5 feet wide, and 4 feet deep?

184. How many gallons of water will a cistern contain which is 7 feet long, 6 feet wide, and 11 feet deep?

185. How many gallons of water will fill a circular cistern 6 feet deep and 4 feet in diameter?

186. Divide $1000 among A, B, and C, and give A $120 more than C, and C $95 more than B.

187. A can mow 2 acres in 3 days, and B 5 acres in 6 days: in how many days can they together mow 9 acres?

188. A sold cloth to B and gained 10 per cent.; B sold it to C and gained 10 per cent.; C sold it to D for $726 and gained 10 per cent.: how much did it cost A?

189. A man steps 2 feet 8 inches, and a boy 1 foot 10 inches; but the boy takes 8 steps while the man takes 5: how far will the boy walk while the man walks $3\frac{3}{4}$ miles?

190. A father bequeathed $\frac{5}{14}$ of his estate to his eldest son, $\frac{2}{3}$ of the remainder to his second son, and the rest to his youngest son; by this arrangement the eldest's share was $1300 more than the youngest's: what was the share of each son?

191. If 7 bushels of wheat are worth 10 bushels of rye, and 5 bushels of rye are worth 14 bushels of oats, and 6 bushels of oats are worth $3.13, how many bushels of wheat will $50 buy?

192. In a company of 90 persons there are 4 more men than women and 10 more children than men and women together: how many of each in the company?

193. Divide $630 among 3 persons so that the second shall have $\frac{3}{4}$ as much as the first, and the third $\frac{1}{2}$ as much as the other two together.

194. A and B can do a piece of work in 12 days, B and C in 9 days, and A and C in 6 days: how long will it take each alone to do it?

195. A and B perform together $\frac{9}{10}$ of a piece of work in 2 days, when, B leaving, A completes it in $\frac{1}{2}$ a day: in what time can each do it alone?

196. C and D engage in trade with different sums of money; C loses 40 per cent. of his capital, and D gains 50 per cent. on his, when their capitals are equal: how much greater was C's capital than D's when they began business?

197. A man walks 100 miles in 2 days, and $\frac{1}{3}$ of the distance walked the first day, added to $\frac{1}{4}$ the distance walked the second day, equals half the distance walked the first day: how far did he walk each day?

198. How far from the end of a stick of timber 30 feet long, of equal size from end to end, must a handspike be

placed so that 3 men, 2 at the handspike and 1 at the end of the stick, may each carry $\frac{1}{3}$ of its weight?

199. Two trees stand on opposite sides of a stream 40 feet wide; the height of one tree is to the width of the stream as 8 is to 4, and the width of the stream is to the height of the other as 4 is to 5: what is the distance between their tops?

200. A cistern is filled by two pipes, one of which will fill it in two hours, and the other in 3 hours; it is emptied by three pipes, the first of which will empty it in 5 hours, the second in 6 hours, and the third in $7\frac{1}{2}$ hours: if all the pipes be left open, in what time will it be filled?

## TEST QUESTIONS.

1. What is a number? In how many ways may numbers be represented? Name them.

2. What is the difference between numeration and notation? Between the Arabic notation and the Roman? Between the English numeration and the French?

3. What is the simple value of the figure 5 in 452? What is its local value? How is the local value of a figure affected by its removal one order to the left? One order to the right? How is the value of a number affected by annexing a cipher? Why?

4. How many units are there in the sum of two or more integers? Why in addition are like orders of figures written in the same column? Why in adding numbers do we begin at the right hand?

5. Why are the minuend, subtrahend, and difference like numbers? Show that the adding of 10 to a term of the minuend, and 1 to the next higher term of the subtrahend, increases the minuend and subtrahend equally.

6. Why must the multiplier be an abstract number? When the multiplicand is concrete, what is true of the product? Why? How is the product affected by dividing either the multiplicand or the multiplier? Show that annexing two ciphers to an integer, and dividing the result by 4, gives the product of the number multiplied by 25.

7. What kind of number is the quotient when both divisor and dividend are like numbers? When the divisor is abstract and the dividend concrete? What is the difference between short division and long division? Of what order is the first left-hand figure of the quotient?

8. How is the quotient affected by multiplying or dividing both dividend and divisor by the same number? By multiplying the dividend by any number greater than 1? On what principle may the four fundamental rules be reduced to two?

9. Name all the prime numbers from 1 to 20 inclusive. Show that two composite numbers may be prime with respect to each other. What is meant by the factors of a number? The prime factors? Show that the common factor of two or more numbers is a factor of their sum.

10. Why is the factor of a number its divisor? How is a number affected by the canceling of a factor? On what principle may the common factors of a dividend and a divisor be canceled?

11. When is a divisor a *common* divisor? What is the greatest common divisor of two or more numbers? Show that the common divisor of two numbers is a divisor of their sum and difference. In how many ways may the greatest common divisor of two or more numbers be found?

12. How many multiples has every number? What is a *common* multiple? What is the least common multiple of two or more numbers? In how many ways may the least common multiple of two or more numbers be found?

13. What is the difference between a divisor and a multiple of a number? Between the terms factor, divisor, and measure? Is $2\frac{1}{2}$ a divisor of 5? Is $12\frac{1}{2}$ a multiple of 5? Is $12\frac{1}{2}$ a multiple of $6\frac{1}{4}$?

14. What is a fraction? In what two ways may a fraction be expressed? When a fraction is expressed by words, which word or words denote the denominator?

15. What is the difference between *the unit of a fraction* and *a fractional unit?* Which term of a fraction denotes the size of the fractional unit? When is the value of a fraction equal to 1? Greater than 1? Less than 1?

16. Show that the division or multiplication of both terms of a fraction by the same number, does not change its value. How is the value of a *proper* fraction affected by adding the same number to both of its terms? By subtracting the same number from both of its terms?

17. On what principle is a fraction reduced to lower terms? To higher terms? On what principle are two or more fractions reduced to a common denominator?

18. In what two ways may a fraction be multiplied by an integer? Why? In what two ways may a fraction be divided by an integer? In what three ways may a fraction be divided by a fraction? Why must fractions have a common denominator before they can be added or subtracted?

19. What is a decimal fraction? Is the fraction *fifteen-hundredths* a decimal fraction? In what two ways may it be expressed by figures? Which is called the *decimal* form? What is the denominator of a decimal fraction?

20. What is meant by decimal places? What is the name of the third decimal order from units? The sixth? The ninth? How is a decimal read?

21. How is the local value of a decimal figure affected by its removal one order to the right? One order to the left? How is the value of a decimal affected by annexing decimal ciphers? Why? By prefixing decimal ciphers? Why?

22. How is a decimal reduced to a common fraction? A common fraction to a decimal? Why can decimals be added and subtracted like integers?

23. Why does the product contain as many decimal places as both multiplicand and multiplier? Why does the dividend contain as many decimal places as both divisor and quotient?

24. How is a decimal divided by 10, 100, etc.? How is a decimal multiplied by 10, 100, etc.? Why are numbers denoting sums of money added and subtracted like decimals?

25. What is a rectangle? How is its area found? What is a circle? How is its area found?

26. What is a right-angled triangle? How is its area found?

27. What is a rectangular solid? What is the difference between an edge and a face of a solid?

28. Show that the product of the three dimensions of a rectangular solid represents its volume or solid contents. How are the contents of a cylinder found?

29. Is every concrete number denominate? Give examples. What is the difference between a simple denominate number and a compound number? Give examples.

30. What do denominate numbers express? What is the difference between reduction descending and reduction ascending?

31. How are denominate fractions reduced from a higher to a lower denomination? From a lower to a higher? How is a denominate number reduced to the fraction of a higher denomination? Give an example.

32. What is the Metric System? What is the primary unit of the system? What is its length in inches? What is a liter? What is a gram.

33. How are the multiples of the meter, liter, and gram named? How are the subdivisions named?

34. What is the difference between simple addition and compound addition? In what respect are the processes alike?

35. When are compound numbers of the same kind? Give examples. How is a compound number divided by another of the same kind?

36. What part of the equator passes beneath the vertical rays of the sun every hour? What part of the tropic of Cancer? What part of any parallel situated between the polar circles?

37. Why is the time of day earlier at New York than at St. Louis? When the difference in longitude between two places is given, how is the difference in time found?

38. What is meant by 5 per cent. of a number? What is the difference between the terms *rate per cent.* and *rate?* Give examples.

39. What four numbers are considered in percentage? Define each. Give the four cases of percentage and the formula for each.

40. What is the difference between the cost and the selling price of an article? Give the four formulas in profit and loss.

41. What is meant by commission? What is the difference between a factor and a broker? Give the four formulas in commission and brokerage.

42. What is the difference between the market value and the par value of capital? When is capital at a premium? When is it at a discount?

43. What is the difference between a dividend and an assessment? How is the rate of dividend found?

44. What is insurance? What is fire insurance? What is the premium? Give the formulas covering the four cases in insurance.

45. What is life insurance? How is the premium computed? What is a *mutual* insurance company?

46. What is the difference between a poll tax and a property tax? How is a property tax assessed? How is the rate of tax determined?

47. What is an income tax? An excise tax? From what kind of taxes is the internal revenue of the United States derived?

48. What are customs or duties? What is the difference between *specific* duties and *ad valorem* duties? What is a tariff?

49. What is interest? What is the rate of interest?

50. How is the interest of any principal for one year, at any rate per cent., found? Give the formula for the general method of computing interest. Give the formula for the six per cent. method.

51. How many methods are there of finding the time between two dates? Which is called the method by days?

52. On what principle is the United States Rule for partial payments based? What rule is used when a note runs less than a year?

53. What quantities are considered in interest? State the five problems in interest, and give the formula for each.

54. What is discount? What is the difference between true discount and interest? Between true discount and bank discount? Between bank discount and interest?

55. What is meant by days of grace? When does a note with grace become due? How is a note not drawing interest discounted by a bank? How is a note drawing interest discounted?

56. What is a promissory note? What is its face? Who is an indorser? When is a note negotiable? When is a note not negotiable?

57. What is a draft? What are the names of the three parties named in a draft? What is meant by the acceptance of a draft? By its protest?

58. What is a bond? What is a coupon? When bonds are quoted at 108, what are they worth? Name the three principal classes of United States Bonds.

59. What is annual interest? When annual interest is not paid when due, what kind of interest does it draw until paid?

60. What is compound interest? In what respect does compound interest differ from annual interest?

61. On what principle is the common method of finding the equated time of several debts or payments based? What is meant by the equation of accounts?

62. Define ratio. In how many and what ways may the ratio of two numbers be expressed? What are the two terms of a ratio called? Which is the dividend? When is the value of a ratio less than one? When is it greater than one?

63. Why must the two terms of a ratio be like numbers? Why is the value of a ratio not changed by multiplying or dividing both of its terms by the same numbers?

64. What is a compound ratio? How is a compound ratio reduced to a simple ratio?

65. What is a proportion? How many ratios in a simple proportion? When is a proportion called simple? When is it compound? How many terms in a simple proportion?

66. Which terms are called the extremes, and which the means? To what is the product of the extremes equal?

67. How can a missing mean be found? Why? A missing extreme? Why? If the second term of a proportion is greater than the first term, how will the fourth term compare with the third?

68. In stating a problem in proportion, which number is made the third term? Why? What is the relation between the ratio of like causes and the ratio of their effects?

69. How may a compound proportion be reduced to a simple proportion? How may the fourth term of a compound proportion be found?

70. What is the difference between a simple partnership and a compound partnership? On what does the partnership value of capital depend?

71. What is the difference between the power of a number and its root? Give examples. What is the difference between involution and evolution?

72. What is the difference between a perfect power and an imperfect power? Give examples. When is a root called a *surd?*

73. To what is the square of a number composed of tens and units equal? To what is the cube of a number composed of tens and units equal?

74. How many orders in the square of any number? How many orders in the square root of any number? How many orders in the cube of any number? How many orders in the cube root of any number?

75. How is the first term of the square root of any number found? The second term? How is the first term of the cube root of any number found? The second term?

76. To what is the square of the hypotenuse of a right-angled triangle equal? The square of the base or perpendicular?

77. How may the area of a circle be found? When the area is given, how may the diameter be found? What is the relation between the areas of two circles?

78. How is the surface of a sphere found? Its solidity? What is the relation between the solid contents of two spheres?

# APPENDIX.

## NOTATION.

**423.** In the decimal system of notation, with *ten* for its base, *ten* figures are used; in a system with *twenty* for its base, *twenty* figures would be needed; in a system with *five* for its base, only *five* figures (1, 2, 3, 4, 0) would be needed; and, generally, *a system of notation requires as many different figures as there are units in its base.*

**424.** In a system with five for its base, 24 would express *fourteen;* 124 would express *thirty-nine;* 1120 would express *one hundred and sixty.*

### EXERCISES.

1. What number is expressed by 200 on a scale of five?
2. What number is expressed by 1240 on a scale of five?
3. Express forty on a scale of five.
4. Express one hundred on a scale of five.
5. Express two hundred on a scale of five.

## PROOF OF THE SIMPLE RULES BY "CASTING OUT THE 9's."

**425.** The method of proving the elementary operations of arithmetic by "casting out the 9's" is based on the principle, that *the excess of 9's in any number is equal to the excess of 9's in the sum of its digits.*

Take, for example, 2345. Dividing it by 9, we have the remainder 5, for the excess of 9's; and adding the digits ($2 + 3 + 4 + 5 = 14$), and dividing the sum by 9, we have the same remainder.

**426.** This principle may be thus explained:

$$2345 \begin{cases} 2000 = 222 \times 9 + 2 \\ 300 = 33 \times 9 + 3 \\ 40 = 4 \times 9 + 4 \\ 5 = 5 \end{cases}$$

It is seen that 2000 is 222 times 9, with a remainder 2; 300 is 33 times 9, with a remainder 3; 40 is 4 times 9, with a remainder 4. Hence, the remainders obtained by dividing the several parts of a number, denoted by the local value of its digits, by 9, are respectively the digits of the number; and the remainder obtained by dividing the number itself by 9, equals the remainder obtained by dividing the sum of its digits by 9. Hence,

*The excess of 9's in any number is found by adding its digits and finding the excess of 9's in their sum.*

### 427. Proof of Addition.

Process.

| | | |
|---|---|---|
| 325 | *Excess* | 1 |
| 256 | " | 4 |
| 358 | " | 7 |
| 939 | " | 3 |

The excess of 9's in the first number, found by adding its digits, is 1; in the second number, 4; in the third, 7. The excess of 9's in the sum of these excesses is 3, which equals the excess of 9's in 939, the amount. Hence,

*The excess of 9's in the sum of several numbers is equal to the excess of 9's in the sum of their excesses.*

1. Add and prove 2346, 5084, 6784, 8653, and 9045.
2. Add and prove 30483, 50678, 346864, and 706037.
3. Add and prove 530902, 672084, 567084, and 1345602.

### 428. Proof of Subtraction.

Process.

| | | |
|---|---|---|
| 3676 | *Excess* | 4 |
| 1508 | " | 5 |
| 2168 | " | 8 |
| | " | 4 |

Since the minuend is equal to the sum of the subtrahend and remainder, *the excess of 9's in the minuend equals the excess of 9's in the* SUM *of the excesses in the subtrahend and remainder.*

1. From 40603 take 27475, and prove the result.
2. From 607853 take 492097, and prove the result.

### 429. Proof of Multiplication.

PROCESS.

| | | |
|---|---|---|
| 347 | *Excess* | 5 |
| 53 | " | 8 |
| 1041 | | 40 |
| 1735 | | |
| 18391 | *Excess* | 4 |

Since 347 contains a certain number of 9's with an excess of 5, and 53 contains a certain number of 9's with an excess of 8, the product of 347 and 53 consists of the product of the number of 9's in them, plus the product of 5 and 8, the excesses of 9's. Hence,

*The excess of 9's in the product of two numbers is equal to the excess of 9's in the product of the excesses in these numbers.*

1. Multiply 45603 by 708, and prove the result.
2. Multiply 60875 by 690, and prove the result.

### 430. Proof of Division.

PROCESS.

```
347 ) 18496 ( 53
      1735
      ----
       1146
       1041
       ----
        105
```

| | | |
|---|---|---|
| 18496 | *Excess* | 1 |
| 347 | " | 5 |
| 53 | " | 8 |
| 105 | " | 6 |
| $5 \times 8 + 6 = 46$ | " | 1 |

Since the dividend equals the product of divisor and quotient, plus the remainder, the excess of 9's in the dividend is equal to the excess of 9's in the product of divisor and quotient, plus the excess in the remainder. Hence,

*The excess of 9's in the dividend is equal to the excess of 9's in the product of the excesses in divisor and quotient, plus the excess in the remainder.*

1. Divide 6480 by 47, and prove the result.
2. Divide 15685 by 625, and prove the result.

## CIRCULATING DECIMALS.

**431.** A ***Circulating Decimal*** is an interminate decimal, containing the same figure or set of figures, repeated in the same order indefinitely. (Art. 121.)

**432.** The figure or set of figures repeated is called a *Repetend.*

A repetend is denoted by a dot placed over the first and last of its figures; as, $.\dot{5}$ $.\dot{1}\dot{6}$ $.\dot{3}2\dot{5}$.

**433.** When a circulating decimal has no figure but the repetend, it is called a *Pure Circulate;* as, $.\dot{3}2\dot{5}$.

When a circulating decimal has one or more figures before the repetend, it is called a *Mixed Circulate;* as, $.45\dot{2}\dot{6}$.

**434.** A pure circulate is reduced to a common fraction *by taking the repetend for the numerator, and as many 9's for the denominator as there are figures in the repetend.*

PROOF.

Let $.\dot{6}\dot{3}$ be a pure circulate.

| | | | |
|---|---|---|---|
| Then, | $63.\dot{6}\dot{3}$ | $=$ | 100 *times the pure circulate.* |
| | $.\dot{6}\dot{3}$ | $=$ | 1 *time* " " " |
| Subtracting, | $63.$ | $=$ | 99 *times* " " " |
| Hence, | $\frac{63}{99}$ | $=$ | *the value of* " " |

**435.** A mixed circulate is reduced to a common fraction *by subtracting the terms which precede the repetend from the whole repetend, and taking the difference for the numerator; and, for the denominator, taking as many 9's as there are figures in the repetend, with as many ciphers annexed as there are decimal figures before the repetend.*

PROOF.

Let $.45\dot{1}2\dot{4}$ be a mixed circulate.

| | | | |
|---|---|---|---|
| Then, | $45124.\dot{1}2\dot{4}$ | $=$ | 100000 *times the mixed circulate.* |
| And, | $45.\dot{1}2\dot{4}$ | $=$ | 100 " " " " |
| Subtracting, | $45079$ | $=$ | 99900 " " " " |
| Hence, | $\frac{45079}{99900}$ | $=$ | *the value of* " " " |

**436.** Pure or mixed circulates may be added, subtracted, multiplied, or divided *by first reducing them to common fractions.*

NOTE.—Circulates may be added, subtracted, multiplied, or divided without first reducing them to common fractions; but the processes are not of sufficient practical importance to justify their explanation in a school arithmetic. In all computations, circulates are carried to enough places to avoid any appreciable error in the result, and then are treated as other decimals.

# 437. TABLES OF DENOMINATE NUMBERS.

## I. CURRENCIES.

### 1. United States Money.

The denominations are *mills*, *cents*, *dimes*, *dollars*, and *eagles*.

TABLE.

| | | |
|---|---|---|
| 10 *m.* | = | 1 *ct.* |
| 10 *ct.* | = | 1 *d.* |
| 10 *d.* | = | $1 |
| $10 | = | 1 *E.* |

### 2. English Money.

The denominations are *farthings* (q.), *pence* (d.), *shillings* (s.), and *pounds* (£).

TABLE.

| | | |
|---|---|---|
| 4 *q.* | = | 1 *d.* |
| 12 *d.* | = | 1 *s.* |
| 20 *s.* | = | 1 *£.* |

1 £ = $4.84.

## II. MEASURES OF EXTENSION AND TIME.

### 1. MEASURES OF LINES AND ARCS.

#### Long Measure.

The denominations are *inches*, *feet*, *yards*, *rods*, *furlongs*, and *miles*.

TABLE.

| | | |
|---|---|---|
| 12 *in.* | = | 1 *ft.* |
| 3 *ft.* | = | 1 *yd.* |
| 5½ *yd.* | = | 1 *rd.* |
| 40 *rd.* | = | 1 *fur.* |
| 8 *fur.* | = | 1 *m.* |

*Also:*

| | | |
|---|---|---|
| 3 *barleycorns* | = | 1 *inch.* |
| 4 *inches* | = | 1 *hand.* |
| 3 *feet* | = | 1 *pace.* |
| 6 *feet* | = | 1 *fathom.* |
| 3 *miles* (*geog.*) | = | 1 *league.* |

60 *geographic miles* / 69½ *statute miles* (*nearly*) } = 1 *degree at the equator.*

#### Circular Measure.

The denominations are *seconds*, *minutes*, *degrees*, *signs*, and *circumferences*.

TABLE.

| | | |
|---|---|---|
| 60″ | = | 1′ |
| 60′ | = | 1° |
| 30° | = | 1 *s.* |
| 12 *s.* / 360° | = | 1 *C.* |

#### Cloth Measure.

(*Little used.*)

| | | |
|---|---|---|
| 2¼ *in.* | = | 1 *nail.* |
| 4 *n.* | = | 1 *quarter.* |
| 4 *qr.* | = | 1 *yard.* |
| 5 *qr.* | = | 1 *Ell Eng.* |

## 2. MEASURES OF SURFACES OR AREAS.

### Square Measure.

The denominations are *square inches*, *square feet*, *square yards*, *square rods* (or *perches*), *roods*, *acres*, and *square miles*.

Table.

| | | |
|---|---|---|
| 144 *sq. in.* | = | 1 *sq. ft.* |
| 9 *sq. ft.* | = | 1 *sq. yd.* |
| $30\frac{1}{4}$ *sq. yd.* | = | 1 *P.* |
| 40 *P.* | = | 1 *R.* |
| 4 *R.* | = | 1 *A.* |
| 640 *A.* | = | 1 *sq. mi.* |

### Surveyor's Measure.

Table.

| | | |
|---|---|---|
| 7.92 *in.* | = | 1 *link* (*l.*). |
| 25 *l.* | = | 1 *rod.* |
| 4 *rd.* | = | 1 *chain* (*ch.*) |
| 80 *ch.* | = | 1 *mile.* |

*Also:*

| | | |
|---|---|---|
| 625 *sq. l.* | = | 1 *P.* |
| 16 *P.* | = | 1 *sq. ch.* |
| 10 *sq. ch.* | = | 1 *A.* |
| 640 *A.* | = | 1 *sq. mi.* |
| 1 *sq. mi.* | = | 1 *section.* |
| 36 *sect.* | = | 1 *township.* |

## 3. MEASURES OF SOLID CONTENTS OR CAPACITY.

### Cubic Measure.

The denominations are *cubic inches*, *cubic feet*, and *cubic yards*.

Table.

| | | |
|---|---|---|
| 1728 *cu. in.* | = | 1 *cu. ft.* |
| 27 *cu. ft.* | = | 1 *cu. yd.* |

### Wood Measure.

Table.

| | | |
|---|---|---|
| 16 *cu. ft.* | = | 1 *cord ft.* |
| 8 *cd. ft.*, or 128 *cu. ft.* | = | 1 *cord.* |

$24\frac{3}{4}$ *cu. ft.* = 1 *perch of stone.*
40 *cu. ft. round timber* = 1 *ton.*
50 *cu. ft. hewn timber* = 1 *ton.*

### Dry Measure.

The denominations are *pints*, *quarts*, *pecks*, and *bushels*.

Table.

| | | |
|---|---|---|
| 2 *pt.* | = | 1 *qt.* |
| 8 *qt.* | = | 1 *pk.* |
| 4 *pk.* | = | 1 *bu.* |

### Liquid Measure.

Table.

| | | |
|---|---|---|
| 4 *gills* | = | 1 *pt.* |
| 2 *pt.* | = | 1 *qt.* |
| 4 *qt.* | = | 1 *gal.* |
| $31\frac{1}{2}$ *gal.* | = | 1 *bbl.* |
| 63 *gal.* | = | 1 *hhd.* |
| 42 *gal.* | = | 1 *tierce.* |

Note.—The standard bushel contains $2150\frac{2}{5}$ cu. in.; the liquid gallon, 231 cu. in.; and the beer gallon (little used), 282 cu. in.

### 4. MEASURES OF DURATION OR TIME.

#### Time Measure.

The denominations are *seconds*, *minutes*, *hours*, *days*, *years*, and *centuries*.

Table.

| | | |
|---|---|---|
| 60 *sec.* | = | 1 *min.* |
| 60 *min.* | = | 1 *h.* |
| 24 *h.* | = | 1 *da.* |
| 365 *da.* | = | 1 *common yr.* |
| 366 *da.* | = | 1 *leap yr.* |
| $365\frac{1}{4}$ *da.* | = | 1 *solar yr.* |
| 100 *s. yr.* | = | 1 *century.* |

*Also:*

| | | |
|---|---|---|
| 7 *da.* | = | 1 *week.* |
| 4 *w.* | = | 1 *lunar mo.* |

Calendar Months.

| | | | | |
|---|---|---|---|---|
| January, | 1st | mo., | 31 | days. |
| February, | 2d | " | 28 | or 29. |
| March, | 3d | " | 31 | days. |
| April, | 4th | " | 30 | " |
| May, | 5th | " | 31 | " |
| June, | 6th | " | 30 | " |
| July, | 7th | " | 31 | " |
| August, | 8th | " | 31 | " |
| September, | 9th | " | 30 | " |
| October, | 10th | " | 31 | " |
| November, | 11th | " | 30 | " |
| December, | 12th | " | 31 | " |

*Also:*

A Julian year contains 13 lunar mo. 1 da. 6 h.
A civil year contains 12 calendar months.
A solar year contains 365 da. 5 h. 48 min. 48 sec.

## III. WEIGHTS.

#### Avoirdupois Weight.

The denominations are *drams*, *ounces*, *pounds*, *hundred-weights*, and *tons*.

Table.

| | | |
|---|---|---|
| 16 *dr.* | = | 1 *oz.* |
| 16 *oz.* | = | 1 *lb.* |
| 100 *lb.* | = | 1 *cwt.* |
| 20 *cwt.* | = | 1 *T.* |

*Also:*

| | | |
|---|---|---|
| 196 *lb. flour* | = | 1 *barrel.* |
| 200 *lb. beef or pork* | = | 1 " |

| | | |
|---|---|---|
| 100 *lb. fish* | = | 1 *quintal.* |
| 56 *lb. corn or rye*<br>60 *lb. wheat*<br>32 *lb. oats* | = | 1 *bushel.* |
| 14 *lb. iron or lead* | = | 1 *stone.* |
| $21\frac{1}{2}$ *stones* | = | 1 *pig.* |
| 8 *pigs* | = | 1 *fother.* |

### Troy Weight.

The denominations are *grains*, *pennyweights*, *ounces*, and *pounds*.

Table.

24 *gr.* = 1 *pwt.*
20 *pwt.* = 1 *oz.*
12 *oz.* = 1 *lb.*

4 *gr.* = 1 *carat.*

### Apothecaries Weight.

The denominations are *grains*, *scruples*, *drams*, *ounces*, and *pounds*.

Table.

20 *gr.* = 1 ℈
3 ℈ = 1 ʒ
8 ʒ = 1 ℥
12 ℥ = 1 ℔

### COMPARISON OF WEIGHTS.

1 *lb. Avoir.* = $1\frac{31}{144}$ *lb. Troy* = $1\frac{31}{144}$ ℔ *Apoth.*
1 *oz.* " = $\frac{175}{192}$ *oz.* " = $\frac{175}{192}$ ʒ "

## IV. MISCELLANEOUS TABLE.

| | | |
|---|---|---|
| 12 *things* | are | 1 *dozen.* |
| 12 *dozen* | " | 1 *gross.* |
| 12 *gross* | " | 1 *great gross.* |
| 20 *things* | " | 1 *score.* |
| 18 *inches* | " | 1 *cubit.* |
| 22 *inches* (*nearly*) | " | 1 *sacred cubit.* |

### Paper.

| | | |
|---|---|---|
| 24 *sheets* | are | 1 *quire.* |
| 20 *quires* | " | 1 *ream.* |
| 2 *reams* | " | 1 *bundle.* |
| 5 *bundles* | " | 1 *bale.* |

### Books.

| | | | | | |
|---|---|---|---|---|---|
| *A sheet* | *folded in* | 2 | *leaves* | *is called* | *a folio.* |
| " | " | 4 | | " | *a quarto*, or 4*to.* |
| " | " | 8 | | " | *an octavo*, or 8*vo.* |
| " | " | 12 | | " | *a duodecimo*, or 12*mo.* |
| " | " | 16 | | " | *a* 16*mo.* |
| " | " | 24 | | " | *a* 24*mo.* |

Note.—In estimating the size of the leaves, as above, the double medium sheet (23 by 26 inches) is taken as a standard.

## LEGAL RATES OF INTEREST IN THE SEVERAL STATES.

**438.** When no rate is mentioned, the legal rate in

| | |
|---|---|
| Louisiana, except on bank interest (6 %), is | 5 % |
| New York, New Jersey, Michigan, Wisconsin, Minnesota, South Carolina, and Georgia | 7 % |
| Alabama and Texas | 8 % |
| California, Oregon, Kansas, Nebraska, Nevada, and Colorado | 10 % |
| All the other States and District of Columbia | 6 % |

When stipulated in the contract, the legal rate in

| | |
|---|---|
| Ohio, Florida, and Louisiana is as high as | 8 % |
| Illinois, Iowa, Michigan, Arkansas, Mississippi, Missouri, and Tennessee | 10 % |
| Minnesota, Texas, Wisconsin | 12 % |
| Nebraska | 15 % |
| Kansas | 20 % |
| Massachusetts, Rhode Island, California, Nevada, and Colorado, any per cent. agreed upon. | |
| The legal rate in England and France is | 5 % |
| Canada, Nova Scotia, and Ireland | 6 % |

NOTE.—Since the rate of interest is often changed by legislation, the above rates may not in all cases be strictly accurate.

## LIFE INSURANCE.

**439.** The rate of premium in life insurance is based on the applicant's *expectation of life,* as shown by life statistics or bills of mortality.

The annual premium must be such a sum as, when put at interest, will amount to the sum insured at the close of average extension of life beyond the applicant's age.

**440.** There are two tables showing the Expectation of Life, called the *Carlisle Table* and the *Wigglesworth Table.* The former is based on bills of mortality prepared in England, and the latter is based on the mortality in the United States. Both tables are in use in this country.

**441.** The Expectation of Life, as shown by the two tables, is as follows:

| AGE. | EXPECTATION BY C. TABLE. | EXPECTATION BY W. TABLE. | AGE. | EXPECTATION BY C. TABLE. | EXPECTATION BY W. TABLE. | AGE. | EXPECTATION BY C. TABLE. | EXPECTATION BY W. TABLE. | AGE. | EXPECTATION BY C. TABLE. | EXPECTATION BY W. TABLE. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 38.72 | 28.15 | 24 | 38.59 | 32.70 | 48 | 22.80 | 22.27 | 72 | 8.16 | 9.14 |
| 1 | 44.68 | 36.78 | 25 | 37.86 | 32.33 | 49 | 21.81 | 21.72 | 73 | 7.72 | 8.69 |
| 2 | 47.55 | 38.74 | 26 | 37.14 | 31.93 | 50 | 21.11 | 21.17 | 74 | 7.33 | 8.25 |
| 3 | 49.82 | 40.01 | 27 | 36.41 | 31.50 | 51 | 20.39 | 20.61 | 75 | 7.61 | 7.83 |
| 4 | 50.76 | 40.73 | 28 | 35.69 | 31.08 | 52 | 19.68 | 20.05 | 76 | 6.49 | 7.40 |
| 5 | 51.25 | 40.88 | 29 | 35.00 | 30.66 | 53 | 18.97 | 19.49 | 77 | 6.10 | 6.99 |
| 6 | 51.17 | 40.69 | 30 | 34.34 | 30.25 | 54 | 18.28 | 18.92 | 78 | 6.02 | 6.59 |
| 7 | 50.80 | 40.47 | 31 | 33.68 | 29.83 | 55 | 17.58 | 18.35 | 79 | 5.80 | 6.21 |
| 8 | 50.24 | 40.14 | 32 | 33.03 | 29.43 | 56 | 16.89 | 17.78 | 80 | 5.51 | 5.85 |
| 9 | 49.57 | 39.72 | 33 | 32.36 | 29.02 | 57 | 16.21 | 17.20 | 81 | 5.21 | 5.50 |
| 10 | 48.82 | 39.23 | 34 | 31.68 | 28.62 | 58 | 15.55 | 16.63 | 82 | 4.93 | 5.16 |
| 11 | 48.04 | 38.64 | 35 | 31.00 | 28.22 | 59 | 14.92 | 19.04 | 83 | 4.65 | 4.87 |
| 12 | 47.27 | 38.02 | 36 | 30.32 | 27.78 | 60 | 14.34 | 15.45 | 84 | 4.39 | 4.66 |
| 13 | 46.51 | 37.41 | 37 | 29.64 | 27.34 | 61 | 13.82 | 14.86 | 85 | 4.12 | 4.57 |
| 14 | 45.75 | 36.79 | 38 | 28.96 | 26.91 | 62 | 13.31 | 14.26 | 86 | 3.90 | 4.21 |
| 15 | 45.00 | 36.17 | 39 | 28.28 | 26.47 | 63 | 12.81 | 13.66 | 87 | 3.71 | 3.90 |
| 16 | 44.27 | 35.76 | 40 | 27.61 | 26.04 | 64 | 12.30 | 13.05 | 88 | 3.59 | 3.67 |
| 17 | 43.57 | 35.37 | 41 | 26.97 | 25.61 | 65 | 11.79 | 12.43 | 89 | 3.47 | 3.56 |
| 18 | 42.87 | 34.98 | 42 | 26.34 | 25.19 | 66 | 11.27 | 11.96 | 90 | 3.28 | 3.73 |
| 19 | 42.17 | 34 59 | 43 | 25.71 | 24.77 | 67 | 10.75 | 11.48 | 91 | 3 26 | 3.32 |
| 20 | 41.46 | 34.22 | 44 | 25.09 | 24.35 | 68 | 10.23 | 11.01 | 92 | 3.37 | 3.12 |
| 21 | 40.75 | 33.84 | 45 | 24.46 | 23.92 | 69 | 9.70 | 10.50 | 93 | 3.48 | 2.40 |
| 22 | 40.04 | 33.46 | 46 | 23.82 | 23.37 | 70 | 9.18 | 10.06 | 94 | 3.53 | 1.98 |
| 23 | 39.31 | 33.08 | 47 | 23.17 | 22.83 | 71 | 8.65 | 9.60 | 95 | 3.53 | 1.62 |

NOTE.—A comparison shows that the Wigglesworth table has a less expectation of life than the Carlisle table for all ages below 50 years; and that the latter table has a less expectation than the former for all ages from 50 to 90 years inclusive.

## EQUATION OF PAYMENTS.

**442.** In 1860, the author published a demonstration of the correctness of the common Mercantile Rule for finding the equated time for the payment of several debts, due at different times without interest. The inaccuracy of the rule by *present worths*, commended by several authors as "*the only accurate rule*", was thus pointed out:

"The equated time for the payment of \$200, of which \$100 is now due, and the other \$100 is due in two years, as found by this rule, is 11.32 months. Now, the amount of \$100 for 11.32 months, at 6 per cent., is \$105.66; the present worth of the other \$100, due in 12.679 months, is \$94.038, and \$105.66 + \$94.038 = \$199.698, whereas it ought to be \$200.

"It is also evident that the equated time, as found by this 'accurate' rule, will not be the same for all rates of interest. At 50 per cent. the equated time of the above example is 8 months, and the error, by the above test, \$8.33⅓; at 100 per cent. it is 6 months, with an error of \$10.

"This supposed accurate rule is based upon the principle that the amount to be paid on a debt due at a future date, without interest, *at any time previous to this date*, is the *present worth* of the debt at any prior date, plus the interest of the present worth up to date of payment. The incorrectness of this principle is easily shown. Suppose I owe a man \$100, due in two years, without interest; how much ought I to pay in one year?

"The present worth of \$100, due in two years (at 6 per cent.), is \$89.2857, and the interest on this sum for one year is \$5.3571; hence, the sum to be paid is \$89.2857 + \$5.3571 = \$94.6428. The true amount to be paid, however, is the present worth of \$100, due in one year, which is \$94.339."

NOTE.—The accuracy of the Mercantile Rule and the inaccuracy of the rule by Present Worths were rigidly demonstrated by Prof. A. Schuyler, in an article published in the *Ohio Educational Monthly*, for 1862, p. 116.

## ARITHMETICAL PROGRESSION.

**443.** An ***Arithmetical Progression*** is a series of numbers which so increases or decreases that the difference between the consecutive numbers is constant.

**444.** The numbers which form the series are called *Terms*, the first and last terms being the *Extremes*, and the intervening terms the *Means*.

The difference between the consecutive terms is called the *Common Difference*.

**445.** An *Ascending Series* is one in which the terms increase; as, 2, 5, 8, 11, 14, etc.

A *Descending Series* is one in which the terms decrease; as 20, 17, 14, 11, 8, etc.

**446.** In an arithmetical progression five quantities are

considered; and such is the relation between them, that, if any three are given, the other two may be found.

These quantities are:

1. *The first term.*
2. *The last term.*
3. *The common difference.*
4. *The number of terms.*
5. *The sum of all the terms.*

**447.** The ascending series, 2, 5, 8, 11, 14, having 5 terms, may be expressed in three forms, as follows:

| | | | | | |
|---|---|---|---|---|---|
| (1) | 2 | 5 | 8 | 11 | 14 |
| (2) | 2 | $2+3$ | $2+(3+3)$ | $2+(3+3+3)$ | $2+(3+3+3+3)$ |
| (3) | 2 | $2+3$ | $2+3\times2$ | $2+3\times3$ | $2+3\times4$ |

A comparison of these three forms of the same series shows, that each term is composed of two parts, viz.: (1) the first term; (2) the common difference taken as many times as there are *preceding* terms. Hence,

1. *The last term of an ascending series is equal to the first term, plus the common difference taken as many times as there are terms in the series less one.* Conversely,

2. *The first term of an ascending series is equal to the last term, minus the common difference taken as many times as there are terms in the series less one.*

3. *The common difference is equal to the difference between the first and last terms, divided by the number of terms less one.*

4. *The number of terms less one is equal to the difference between the first and last terms, divided by the common difference.*

**448.** Let

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 3 | 5 | 7 | 9 | 11 | 13 | be an arithmetical series, |
| and, | 13 | 11 | 9 | 7 | 5 | 3 | be the series reversed. |

Then, $16 + 16 + 16 + 16 + 16 + 16 =$ *twice* the sum of the terms.
and $\quad 8 + 8 + 8 + 8 + 8 + 8 =$ the sum of the terms.

An inspection of the above shows that the sum of the first and last terms of an arithmetical series, multiplied by the number of terms, is equal to *twice* the sum of all the terms. Hence, *The sum of all the terms of an arithmetical series is equal to the product of one half the sum of the first and last terms, multiplied by the number of terms.*

NOTE.—One half of the sum of the first and last terms is equal to the *average* of the several terms of the series.

**449.** From the above principles may be deduced the following

### FORMULAS.

1. *Last term* $=$ *first term* $\pm$ (*com. difference* $\times$ *number of terms less one*).

2. *First term* $=$ *last term* $\mp$ (*com. difference* $\times$ *number of terms less one*).

3. *Common difference* $= \left\{ \begin{array}{l} \textit{last term} - \textit{first term} \\ \textit{first term} - \textit{last term} \end{array} \right\} \div$ *number of terms less one.*

4. *Number of terms less one* $= \left\{ \begin{array}{l} \textit{last term} - \textit{first term} \\ \textit{first term} - \textit{last term} \end{array} \right\}$ $\div$ *common difference.*

5. *Sum of terms* $= \frac{1}{2}$ (*first term* $+$ *last term*) $\times$ *number of terms.*

NOTE.—The first term of an ascending series corresponds to the last term of a like descending series, and the last term of a descending series corresponds to the first term of a like ascending series.

### PROBLEMS.

1. What is the tenth term of the series 5, 7, 9, 11, etc.?

2. The first term of an ascending series is 4, the common difference 3, and the number of terms 8: what is the last term?

3. The last term of a descending series is 1, the common difference 4, and the number of terms 12: what is the first term?

4. The extremes of an arithmetical series are 47 and 3, and the number of terms 12: what is the common difference?

5. The 1st term is 7 and the 21st term 57: what is the common difference?

6. The 4th term of a series is 21 and the 9th term is 41: what are the four mean terms?

7. The two extremes of a series are 12 and 177, and the common difference 5: what is the number of terms?

8. The two extremes of a series are 20 and 152, and the number of terms 45: what is the sum of all the terms?

9. What is the sum of all the terms of the series described in the 6th problem above? In the 7th problem?

10. How many strokes does the hammer of a clock make in 24 hours?

11. A man agreed to dig a trench 50 yards long for 2 cents for the first yard, 5 cents for the second yard, 8 cents for the third, and so on, the price of each yard being 3 cents more than that of the preceding yard: what did he receive for digging the last yard? For digging the trench?

## GEOMETRICAL PROGRESSION.

**450.** A ***Geometrical Progression*** is a series of numbers which so increases or decreases that the ratio between the consecutive terms is constant.

The first and last terms are called the *Extremes*, and the intervening terms are called the *Means.*

**451.** A geometrical progression is *ascending* or *descending* according as the series increases or decreases from left to right.

**452.** In a geometrical progression five quantities are considered, and these (as in arithmetical progression) are so related to each other that, any three being given, the other two may be found.

These five quantities are

1. *The first term.*
2. *The last term.*
3. *The common ratio.*
4. *The number of terms.*
5. *The sum of all the terms.*

**453.** The ascending series, 2, 6, 18, 54, 162, 486, has 6 terms, and the first term is 2, and the common ratio or multiplier is 3. This series may be expressed in three forms, as follows:

| | | | | | | |
|---|---|---|---|---|---|---|
| (1) | 2 | 6 | 18 | 54 | 162 | 486 |
| (2) | 2 | $2\times3$ | $2\times3\times3$ | $2\times3\times3\times3$ | $2\times3\times3\times3\times3$ | $2\times3\times3\times3\times3\times3$ |
| (3) | 2 | $2\times3$ | $2\times3^2$ | $2\times3^3$ | $2\times3^4$ | $2\times3^5$ |

A comparison of the corresponding terms of the three forms, shows that each term of the series is composed of two factors, viz.: (1) the first term, and (2) the common ratio raised to a power whose exponent or degree is equal to the number of *preceding* terms. Hence,

1. *The last term of a geometrical series is equal to the first term, multiplied by the common ratio, raised to a power whose degree is one less than the number of terms.* Conversely,

2. *The first term is equal to the last term divided by the common ratio, raised to a power whose degree is one less than the number of terms.*

3. *The common ratio is equal to the root whose index is one less than the number of terms, of the quotient of the last term divided by the first term.*

**454.** By an algebraic process it may be shown that

4. *The sum of a geometrical series is equal to the product of the last term and the common ratio, less the first term, divided by the common ratio less one.*

**455.** When the number of terms in a descending geometrical series is infinite, the last term is 0, and *the sum of the series is equal to the first term divided by the ratio less one.*

**456.** From the above principles may be deduced the following

FORMULAS.

1. *Last term = first term* $\times$ *ratio* $^{n-1}$.
2. *First term = last term* $\div$ *ratio* $^{n-1}$.
3. *Ratio* $= \sqrt[n-1]{\textit{last term} \div \textit{first term}}$.
4. *Sum of series* $= \dfrac{(\textit{last term} \times \textit{ratio}) - \textit{first term}}{\textit{ratio} - 1}$
5. *Sum of infinite descending series = first term* $\div$ (*ratio* — 1).

NOTES.—1. By "ratio $^{n-1}$," in 1st and 2d formulas, is meant the ratio raised to a power whose degree is the *number of terms less* 1. The index of the root, in the 3d formula ($n$—1), is the number of terms less 1.

2. In an ascending series the ratio is greater than 1, and in a descending series the ratio is less than 1.

PROBLEMS.

1. What is the 6th term of the series 5, 10, 20? etc.

2. The first term of a geometrical series is 5, the ratio is 3, and the number of terms 7: what is the last term?

3. The first term of a series is 1220, the ratio $\frac{1}{3}$, and the number of terms 6: what is the last term?

4. The last term of a series is 64, the ratio 2, and the number of terms 10: what is the first term?

5. What is the sum of the series described in the 4th problem? In the 3d problem?

6. The first term of a series is 5, and the sixth term is 1215: what is the ratio?

7. The first term of a series is 10, the sixth term 2430, and the ratio 3: what is the sum of the six terms?

8. A father gave his son 50 cents on his 12th birthday, and agreed to double the amount on each succeeding birthday to and including the 21st: how much did the son receive on his 21st birthday? How much in all?

9. A man worked 15 days on condition that he should receive 1 cent the first day, 5 cents the second day, and so

on, the wages of each day being 5 times the wages of the previous day: how much did he receive?

## ALLIGATION.

**457.** ***Alligation*** is the process of finding the average value or quality of a mixture composed of articles of different values or qualities.

It is also the process of compounding several articles of different values or qualities to form a mixture of an average value or quality.

The first process is called *Alligation Medial*, and the second *Alligation Alternate*.

NOTE.—The term Alligation is derived from the Latin *alligare*, to *bind* or *link*. The term is applied to this process because some of the problems may be solved by joining or linking the numbers.

### Case I.

**458.** *Several ingredients of a mixture, and their respective values given, to find their average value.*

#### PROBLEMS.

1. A farmer mixed 25 bushels of oats, at 50 cents a bushel; 15 bushels of rye, at 80 cents a bushel; and 30 bushels of corn, at 70 cents a bushel: what was the value of a bushel of the mixture?

PROCESS.

$$\begin{array}{rcrcr}
\textit{cts.} & & & & \textit{cts.} \\
50 & \times & 25 & = & 1250 \\
80 & \times & 15 & = & 1200 \\
70 & \times & \underline{30} & = & \underline{2100} \\
 & & 70 & ) & \underline{4550} \\
 & & & & 65\ \textit{cts., Ans.}
\end{array}$$

Since the total value of the 70 bushels of grain mixed together was 4550 cents, the value of 1 bushel was $\frac{1}{70}$ of 4550 cents. which is 65 cents.

2. A grocer mixed 20 pounds of coffee worth 28 cents, 30 pounds worth 35 cents, and 50 pounds worth 33 cents: what is a pound of the mixture worth?

## Case II.

**459.** *The values of several articles given, to find in what proportion they must be compounded to make a mixture of a given value.*

3. A grocer has sugars worth 16, 18, and 24 cents a pound: in what proportion must they be taken to make a mixture worth 20 cents a pound?

### I. Solution by Analysis.

On each pound of sugar worth 16 cents taken, there is a gain of 4 cents, and on each pound at 24 cents, there is a loss of 4 cents. Hence, these two kinds of sugar may be taken in equal quantities, or 1 pound of each. On each pound worth 18 cents there is a gain of 2 cents, and hence 2 pounds of it must be taken to offset a loss of 4 cents on 1 pound at 24 cents. Hence, the simplest proportionals are 1 lb. at 16 cts., 2 lb. at 18 cts., and 2 lb. at 24 cts.

### II. Another Solution.

$$\left.\begin{array}{llllll} 1 \text{ lb. at} & 16 \text{ cts. selling for} & 20 \text{ cts.} & \text{gains} & 4 \text{ cts.} \\ 1 \quad " & 18 \quad " \quad " & 20 \quad " & \quad " & 2 \text{ cts.} \end{array}\right\} \text{ 6 cts. gain.}$$

$$\begin{array}{llllll} 1 \quad " & 24 \quad " \quad " & 20 \quad " & \text{loses} & 4 \text{ cts.} \ldots 4 \text{ cts. loss.} \end{array}$$

Taking two pounds each of the first two kinds, the loss will be 12 cents, and by taking 3 pounds of the third kind, the loss will be 12 cents. Hence, the proportionals 2, 2, 3 make *the gains and losses equal.*

### III. Solution by Linking.

$$20 \left| \begin{array}{l} 16 \\ 18 \\ 24 \end{array} \right. \quad \left.\begin{array}{r} \ldots\ldots 4 \\ \ldots\ldots 4 \\ .\ 4+2=6 \end{array}\right\} \div 2 = \left\{\begin{array}{l} 2 \\ 2 \\ 3 \end{array}\right.$$

Note.—When only two articles of different values are given, they can be compounded in but one way; but when more than two articles are given, they may be compounded *in an infinite number of ways.* They may be combined two and two in such proportions as to make, in each case, a mixture of the required value, and then these compounds may be united in any proportions whatever.

4. A merchant has teas worth \$1.25, \$1.40, \$1.60, and \$1.75: how much of each kind must be taken to make a mixture worth \$1.50?

### Case III.

**460.** *The values of the several ingredients of a mixture, their average value, and the quantity of one or more of the ingredients given, to find the respective quantities of the other ingredients.*

5. A grocer wishes to mix 100 pounds of coffee at 25 cts. with coffees at 22, 28, and 30 cts., making a mixture worth 27 cts.: how much of each kind must he take?

SUGGESTION.—Find the proportionals of the ingredients by Case II, and then multiply each proportional by the quotient of 100 lbs. divided by the proportional for the coffee worth 25 cts.

6. A farmer wishes to mix 60 bushels of corn at 60 cts., with rye at 75 cts., barley at 50 cts., and oats at 40 cts., to make a mixture worth 65 cts.: how many bushels each of rye, barley, and oats must he take?

### Case IV.

**461.** *The values of the ingredients, and the quantity and value of the mixture given, to find the quantity of each ingredient.*

7. How much gold 16 carats fine, 18 carats fine, and 22 carats fine, must be taken to make 12 rings 20 carats fine, and weighing $4\frac{1}{2}$ pwt. each?

SUGGESTION.—Find the proportionals by Case II, and then divide the whole quantity into parts proportional to these proportionals.

8. How much sugar worth 15 cts., 17 cts., and 20 cts. must be taken to make a mixture of 200 pounds, worth 18 cts.?

9. How much water must be mixed with vinegar, worth 60 cts. a gallon, to make 90 gallons, worth 50 cts. a gallon?

# DUODECIMALS.

**462.** A ***Duodecimal*** is a denominate number in which *twelve* units of any denomination make a unit of the next higher denomination.

A duodecimal may be regarded as a fraction whose denominator is a power of 12; or a number whose scale is 12. The term is derived from the Latin *duodecim, twelve.*

**463.** Duodecimals are used by artificers in measuring surfaces and solids.

The foot is divided into *primes,* marked $'$; the primes into *seconds* ($''$); the seconds into *thirds* ($'''$), etc., as is shown in the following

TABLE.

| | | |
|---|---|---|
| 12 fourths ($''''$) | are | $1'''$ |
| 12 thirds | " | $1''$ |
| 12 seconds | " | $1'$ |
| 12 primes | " | 1 ft. |

The accents used to mark the different denominations, are called *Indices.*

**464.** The prime denotes *the twelfth of a foot;* the second, *the twelfth of the twelfth of a foot,* etc.

When a duodecimal denotes the area of a surface, the foot is a *square foot;* the prime, *the twelfth of a square foot;* the second, *the twelfth of a twelfth of a square foot,* etc.

When a duodecimal denotes the contents of a solid, the foot is a *cubic foot;* the prime, *the twelfth of a cubic foot,* etc.

## 465. ADDITION AND SUBTRACTION.

**PROBLEMS.**

1. Add 12 ft. $8'$ $11''$, 16 ft. $10'$ $9''$, and 24 ft. $6''$.

$$\text{PROCESS: } \left\{\begin{array}{lll} 12 \text{ ft.} & 8' & 11'' \\ 16 \text{ ft.} & 10' & 9'' \\ 24 \text{ ft.} & 0' & 6'' \\ \hline 53 \text{ ft.} & 8' & 2'' \end{array}\right. \quad \textit{Ans.}$$

2. Add 12 ft. 9′ 11″ 4‴, 23 ft. 7″ 10‴, and 10′ 6″ 9‴.

3. From 21 ft. 7′ 10″ take 15 ft. 9′ 4″.

| | | | | |
|---|---|---|---|---|
| PROCESS: | 21 ft. | 7′ | 10″ | |
| | 15 ft. | 9′ | 4″ | |
| | 5 ft. | 10′ | 6″ | *Ans.* |

4. From the sum of 30 ft. 8″ 4‴ and 14 ft. 7′ 10‴, take their difference.

## 466. MULTIPLICATION OF DUODECIMALS.

5. Multiply 13 ft. 7′ 8″ long and 6 ft. 5′ wide?

PROCESS.

| | | | | |
|---|---|---|---|---|
| 13 ft. | 7′ | 8″ | | |
| | 6 ft. | 5′ | | |
| 5 ft. | 8′ | 2″ | 4‴ | |
| 81 ft. | 10′ | 0″ | | |
| 87 ft. | 6′ | 2″ | 4‴, | *Ans.* |

Multiply first by 5′ and then by 6 ft., and add the partial products.

Since $1 \times \frac{1}{12} = \frac{1}{12}$, $\frac{1}{12} \times \frac{1}{12} = \frac{1}{144}$, $\frac{1}{144} \times \frac{1}{12} = \frac{1}{1728}$, etc., feet × primes (or twelfths) must produce *primes;* primes by primes, *seconds;* seconds by primes, *thirds;* and, generally, the denomination of the product of any two denominations is denoted by *the sum of their indices.*

6. What are the superficial contents of a board 9 ft. 7′ 4″ long and 10′ 6″ wide?

7. What are the solid contents of a block of marble 7 ft. 6′ long, 2 ft. 8′ wide, and 1 ft. 4′ thick?

NOTE.—The answers to the 5th and 6th problems are in *square feet* and duodecimal parts of a square foot, and the answer to the 7th problem is in *cubic feet* and duodecimal parts of a cubic foot (Art. 464).

## 467. DIVISION OF DUODECIMALS.

8. Divide 87 ft. 6′ 2″ 4‴ by 13 ft. 7′ 8″.

PROCESS.

| *Dividend.* | | | | *Divisor.* |
|---|---|---|---|---|
| 87 ft. | 6′ | 2″ | 4‴ | ) 13 ft. 7′ 8″ |
| 81 ft. | 10′ | | | 6 ft. 5′, *Q'nt.* |
| 5 ft. | 8′ | 2″ | 4‴ | |
| 5 ft. | 8′ | 2″ | 4‴ | |

The process is the reverse of that in multiplication. For convenience in multiplying, place the divisor at the right of the dividend, and the terms of the quotient below those of the divisor.

9. Divide 62 ft. 11″ 3‴ by 8 ft. 6′ 9″.

10. Multiply 10 ft. 5′ 8″ by 3 ft. 10′, and divide the product by 5 ft. 2′ 10″.

## PERMUTATIONS.

**468.** Permutations are the changes of order, which a number of objects may undergo, and each object enter once and but once in each result.

**469.** The diagram at the right shows the number of permutations of 1, 2, and 3 letters.

$$a\begin{cases} ba\begin{cases} cba\\ bca\\ bac\end{cases}\\ ab\begin{cases} cab\\ acb\\ abc\end{cases}\end{cases}$$

The letter *a* permits no change of order. The letter *b* may be placed *before* and *after* the letter *a*, giving *two* $(1\times2)$ permutations of two letters—*ba*, *ab*. The letter *c* may be placed *before*, *between*, and *after* the two letters *a b*; and the same for *b a*, giving *six* $(1 \times 2 \times 3)$ permutations of three letters.

A fourth letter, as *d*, may evidently occupy *four* different positions in each of the six combinations of these letters, giving *twenty-four* $(1\times2\times3\times4)$ permutations of four letters.

In like manner it may be shown that *the number of permutations of any number of objects is equal to the continued product of all the integers from* 1 *to the given number of objects inclusive.*

### PROBLEMS.

1. In how many different orders may 6 boys sit on a bench?

2. In how many different orders may all the letters in the word *permutation* be written?

3. How many permutations may be made of the nine digits?

4. How many different combinations of eight notes each may be made of the octave?

## ANNUITIES.

**470.** An ***Annuity*** is a sum of money, payable annually, for a given number of years, for life, or forever. The term is also applied to sums of money payable at any regular intervals of time.

**471.** A *Certain Annuity* is an annuity that is payable for a given number of years.

A *Contingent Annuity* is an annuity payable for an uncertain period, as during the life of a person.

A *Perpetual Annuity* is one that continues forever.

**472.** An *Immediate Annuity* is an annuity whose payment begins at once.

A *Deferred Annuity* is an annuity whose payment begins at a future time.

**473.** The *Forborne* or *Final Value* of an annuity is the sum of the compound amounts of all its payments, from the time each is due to the end of the annuity.

The *Present Value* of an annuity is the present worth of the forborne or final value.

NOTE.—The principal applications of the subject of annuities are in leases, life estates, rents, dowers, life insurance, etc.; and the problems arising are readily solved by means of tables which give the present and final values of $1 at the usual rates of interest. A full discussion of the principles involved in the construction of these tables, can not well be presented in a school arithmetic.

## RULES OF MENSURATION.

### 474. SURFACES AND LINES.

1. To find the area of a rectangle, *Multiply the length by the width.*

2. To find either side of a rectangle, *Divide the area by the other side.*

3. To find the area of a triangle, *Multiply the base by one half of the altitude.*

4. To find the area of any quadrilateral having two sides parallel, *Multiply one half of the sum of the two parallel sides by the perpendicular distance between them.*

5. To find the circumference of a circle,

1. *Multiply the diameter by* 3.1416. Or,
2. *Divide the area by one fourth of the diameter.*

6. To find the area of a circle,

1. *Multiply the square of the diameter by* .7854. Or,
2. *Multiply the square of the radius by* 3.1416. Or,
3. *Multiply the circumference by one half of the radius.*

7. To find the diameter of a circle, whose area is given, *Divide the area by* .7854, *and extract the square root of the quotient.*

8. To find the side of the largest square that can be inscribed in a circle, *Multiply the radius by the square root of* 2.

9. To find the side of the largest equilateral triangle that can be inscribed in a circle, *Multiply the radius by the square root of* 3.

10. To find the area of an ellipse, the two diameters being given, *Multiply the product of the two diameters by* .7854.

11. To find the surface of a sphere,

1. *Multiply the circumference by the diameter.* Or,
2. *Multiply the square of the diameter by* 3.1416.

12. To find the entire surface of a right prism or right cylinder, *Multiply the perimeter or circumference of the base by the height, and, to the product, add the surface of the two bases.*

13. To find the convex surface of a pyramid or cone, *Multiply the perimeter or circumference of the base by one half the slant height.*

14. To find the hypotenuse of a right-angled triangle,

*Extract the square root of the sum of the squares of the other two sides.*

15. To find the base or the perpendicular of a right-angled triangle, *Extract the square root of the difference between the square of the hypotenuse and the square of the other side.*

### 475. Contents of Solids.

1. To find the solid contents of a rectangular solid, *Multiply the length, width, and thickness together.*

2. To find either dimension of a rectangular solid, *Divide the solid contents by the product of the other two dimensions.*

3. To find the solid contents of a cylinder, *Multiply the area of the base by the altitude.*

4. To find the solid contents of a sphere,
   1. *Multiply the cube of the diameter by* .5236. Or,
   2. *Multiply the surface by one third of the radius.*

5. To find the solid contents of a cone or pyramid, *Multiply the area of the base by one third of the altitude.*

6. To find the solid contents of the frustum of a cone or pyramid, *To the sum of the areas of the two bases, add the square root of their product, and multiply the result by one third of the altitude.*

7. To measure timber, as planks, joists, etc., by board measure, *Find the number of square feet in one surface, and multiply the result by the thickness in inches.*

## HENKLE'S METHOD OF WRITING DECIMALS.

**476.** It is seen from the decimal scale, that the *tens* of any number of *tenths*, the *hundreds* of any number of *hundredths*, the *thousands* of any number of *thousandths*, etc., each falls in *the order of units* when the decimal is expressed decimally. Thus, 42 *tenths*, written decimally, is 4.2, the

4 (tens) falling in units' order; 1265 *hundredths*, written decimally, is 12.65, the 2 (hundreds) falling in units' order; and 425 *thousandths*, written decimally, is .425 or 0.425, the 0 (thousands) falling in units' order. Hence, the following

RULE.—To write a decimal, *Begin at the left and write the term corresponding to the name of the decimal, in the order of units.*

## SCHUYLER'S CONTRACTED METHOD OF MULTIPLYING DECIMALS.

**477.** RULE.—*Write the multiplier so that its units' place shall fall under that term of the multiplicand whose order is one lower than the lowest required in the product. Multiply first by the units, if any, carrying to the nearest unit from the part rejected. In multiplying by a term on either side of the unit, commence with that term of the multiplicand as far on the other side of the term of the multiplicand over the units' place of the multiplier, carrying to the nearest unit. Add the partial products, reject the sum of the last column, after carrying to the nearest unit. Point off the required number of decimal places in the sum.*

PROCESS.

```
  3.45641
    12.345
  -------
  6913
 34564
  1037
   138
    17
 ------
 42.669
```

NOTE.—Before commencing to multiply by any term of the multiplier, it is convenient to mark that term, also the term with which we commence to multiply in the multiplicand.

# ANSWERS

TO

# THE WRITTEN PROBLEMS.

---

N. B.—The *last* answer is given when a problem has several answers, and also when several problems are united.

## NOTATION.

**Page 11.**

**9.** 40,605.
**10.** 700,007.
**11.** 5,005,500.
**12.** 60,060,060.
**13.** 700,700,700.
**14.** 560,068,000.
**15.** 4,014,045,000.
**16.** 65,000,006,050.
**17.** 350,049,000,000.
**18.** 17,070,000,700,400.
**19.** 56,000,016,000,090.
**20.** 7,000,085,000,000,204.

## ADDITION.

**Page 14.**

**13.** 108,657.
**14.** 442,555.
**15.** 63,077,833.
**16.** 74,467,648.
**17.** 12,369 bush.

**Page 15.**

**18.** 443,275 sq. m.
**19.** 8,984,342.
**20.** 3,578,392 sq. m.
**21.** 258.

**Page 16.**

**22.** 383.
**23.** 512.
**24.** 462.
**25.** 649.
**26.** 598.
**27.** 732.
**28.** 803.
**29.** 631.
**30.** 865.
**31.** 636.
**32.** 633.

## SUBTRACTION.

**Page 18.**

**16.** $4,075.
**17.** 49,894,136 m.
**18.** 429,559.
**19.** 35,965.
**20.** 1,116,942 sq. m.
**21.** 914,054 sq. m.
**22.** 56,077,528 bush.
**23.** $467.
**24.** $2,330.
**25.** $1,032.

**Page 19.**

**26.** $800.
**27.** 6,890 bush.
**28.** All, 1802 A.
**29.** 26,956.

## MULTIPLICATION.

**Page 21.**

**11.** 2,499,120.
**12.** 230,668,800.
**13.** 503,232,000.
**14.** 364,800,000,000.
**15.** 17,424,000 ft.
**16.** 1,572,480 m.
**17.** 25,600,000 A.
**18.** 28,500.
**19.** Gained $798.

**Page 23.**

**7.** 4,560,000.
**8.** 305,000,000.
**9.** 347,000,000,000.
**10.** 88,900,000.
**16.** 15,300.
**17.** 16,200.
**18.** 84,400.

**Page 24.**

**19.** $13{,}549{,}333\frac{1}{3}$.
**20.** 867,000.
**21.** 1,362,000.
**22.** $691{,}066\frac{2}{3}$.
**23.** 45,766.
**24.** 5,666,328.
**25.** 40,740,411.
**26.** 86,772,642.
**27.** 61,870,306,330.
**28.** 322,096.

**Page 25.**

**29.** 652,919.
**30.** 94,240.
**31.** 25,629,438.
**32.** 4,432,246.
**33.** 613,566.

## DIVISION.

**Page 26.**

**15.** 54.
**16.** 233, with 20 R.
**17.** 7, with 600 R.
**18.** 1, with 109,304 R.
**19.** 3,464.
**20.** 8,743.
**21.** 4,567.
**22.** 41 cars.
**23.** 5m. 2,700 ft.

**Page 27.**

**24.** 205 h.
**25.** 548, with 128 R.

**Page 29.**

**8.** 356.
**9.** 46, with 35 R.
**10.** 38, with 4,602 R.
**11.** 95.

**Page 30.**

**18.** 9, with 200 R.
**19.** 24, with 800 R.
**20.** 9, with 10,800 R.
**21.** 2, with 1,600 R.
**28.** 125.

**Page 31.**

**29.** 26, with 190 R.
**30.** 45, with 900 R.
**32.** 761, with 39 R.
**33.** 190, with 28 R.
**34.** 387, with 13 R.
**35.** 57, with 39 R.

**Page 32.**

**36.** 480 with 7 R.
**37.** 1,487, with 26 R.
**38.** 5,203 with 40 R.
**39.** 3,604, with 16 R.
**40.** 433, with 3 R.

## PROPERTIES OF NUMBERS.

**Page 33.**

**15.** 2, 2, 2, 2, 2, 5.
**16.** 5, 5, 7.
**17.** 2, 2, 2, 2, 2, 2, 2, 2.
**18.** 5, 5, 13.
**19.** 2, 3, 5, 11.
**20.** 2, 2, 3, 5, 7.
**21.** 2, 3, 7, 11.
**22.** 2, 5, 7, 7.
**23.** 2, 3, 3, 3, 11.
**24.** 2, 2, 11, 17.
**25.** 3, 3, 7, 11.
**26.** 3, 5, 7, 11.

**Page 34.**

**27.** 3, 3.
**28.** 5, 2.
**29.** 2, 2, 2, 2, 2.
**30.** 2, 5, 5.
**31.** 5, 5.
**32.** 2, 2, 3, 3.

**Page 35.**

**34.** 7.
**35.** 12.
**36.** 55.

**Page 36.**

**38.** $\frac{1}{6}$.
**39.** 8.
**40.** $\frac{5}{21}$.
**41.** 54 cts.
* **42.** 9 men.

* Revised Edition.

## GREATEST COMMON DIVISOR.

**Page 38.**

**12.** 12.
**13.** 63.
**14.** 48.
**15.** 28.
**16.** 42.
**17.** 128.
**18.** 48.
**19.** 4.
**20.** 48.
**21.** 37.
**22.** 1.
**23.** 252.
**24.** 14.
**25.** 192.
**26.** 57.
**27.** $52.
**28.** $165.
**29.** 39 lbs.
**30.** 25.
**31.** 120.
**32.** 72.
**33.** 7.
**34.** 8.
**35.** 5.
**36.** 13.
**37.** 1.

## LEAST COMMON MULTIPLE.

**Page 41.**

**12.** 120.
**13.** 126.
**14.** 480.
**15.** 108.
**16.** 144.
**17.** 210.
**18.** 300.
**19.** $720.
**20.** 420.
**21.** 180.
**22.** 280.
**23.** 180.
**24.** 600.
**25.** 756.
**26.** 720.
**27.** 1,200.
**28.** 1,890.
**29.** 3,360.
**30.** 2,520.

## FRACTIONS.

**Page 46.**

**8.** $\frac{2923}{9}$.
**9.** $\frac{988}{15}$.
**10.** $\frac{4061}{30}$.
**11.** $\frac{763}{12}$.
**12.** $\frac{1121}{15}$.
**13.** $\frac{6187}{30}$.

**Page 47.**

**14.** $\frac{5803}{40}$.
**15.** $\frac{3433}{25}$.
**16.** $\frac{20409}{50}$.
**17.** $\frac{24613}{41}$.
**18.** $\frac{8767}{24}$.
**19.** $\frac{7291}{35}$.
**20.** $\frac{7289}{12}$.
**28.** $16\frac{7}{16}$.
**29.** 27.
**30.** $17\frac{2}{3}$.
**31.** $39\frac{1}{6}$.
**32.** $19\frac{7}{8}$.
**33.** 9.
**34.** $17\frac{5}{8}$.
**35.** $38\frac{2}{7}$.
**36.** $46\frac{2}{25}$.
**37.** $109\frac{3}{13}$.
**38.** $12\frac{4}{333}$.
**39.** $53\frac{1}{15}$.
**40.** $2,016\frac{5}{7}$.

**Page 48.**

**51.** $\frac{3}{7}$.
**52.** $\frac{13}{14}$.
**53.** $\frac{12}{23}$.
**54.** $\frac{11}{20}$.
**55.** $\frac{5}{8}$.
**56.** $\frac{21}{29}$.
**57.** $\frac{7}{8}$.
**58.** $\frac{16}{25}$.
**59.** $\frac{1}{6}$.
**60.** $\frac{5}{13}$.
**61.** $\frac{3}{5}$.
**62.** $\frac{5}{7}$.
**63.** $\frac{13}{7}$.
**64.** $\frac{5}{4}$.

**Page 49.**

**65.** $\frac{18}{11}$.

**Page 50.**

**87.** $\frac{32}{72}$, $\frac{66}{72}$, $\frac{34}{72}$, $\frac{69}{72}$.
**88.** $\frac{80}{120}$, $\frac{72}{120}$, $\frac{100}{120}$, $\frac{105}{120}$.
**89.** $\frac{42}{60}$, $\frac{32}{60}$, $\frac{33}{60}$, $\frac{26}{60}$.
**90.** $\frac{49}{84}$, $\frac{32}{84}$, $\frac{33}{84}$, $\frac{84}{84}$.
**91.** $\frac{24}{60}$, $\frac{55}{60}$, $\frac{33}{60}$, $\frac{58}{60}$, $\frac{31}{60}$.
**92.** $\frac{4025}{7245}$, $\frac{2898}{7245}$, $\frac{3542}{7245}$, $\frac{1380}{7245}$, $\frac{6363}{7245}$.

**Page 52.**

**107.** $\frac{2}{5}$.
**108.** $\frac{3}{7}$.
**109.** $\frac{5}{4}$.

**110.** $\frac{8}{5}$.
**111.** $\frac{20}{21}$.
**112.** $\frac{4}{5}$.
**113.** $\frac{3}{2}$.
**114.** $\frac{49}{40}$.
**115.** $\frac{128}{8}$.
**116.** $\frac{5}{3}$.
**117.** $\frac{500}{15}$ gal.
**118.** $12\frac{2}{3}$.
**119.** $\$63\frac{7}{11}$.
**120.** $\frac{12}{30}$, $\frac{21}{30}$, $\frac{22}{30}$.
**121.** $\frac{150}{12}$, $\frac{225}{12}$, $\frac{400}{12}$.
**122.** $\frac{36}{65}$.
**123.** $\frac{15}{24}$, $\frac{20}{24}$, $\frac{14}{24}$.
**124.** $\frac{5}{6}$, $\frac{33}{6}$, $\frac{2}{6}$.
**125.** $\frac{28}{63}$, $\frac{36}{63}$, $\frac{386}{63}$.
**126.** $\frac{15}{12}$, $\frac{30}{12}$, $\frac{25}{12}$.
**127.** $\frac{14}{24}$, $\frac{48}{24}$, $\frac{185}{24}$.
**128.** $\frac{105}{120}$, $\frac{450}{120}$, $\frac{1000}{120}$, $\frac{276}{120}$.

**Page 53.**

**12.** $2\frac{2}{3}$.
**13.** $2\frac{14}{45}$.
**14.** $2\frac{71}{160}$.
**15.** $1\frac{43}{48}$.

**Page 54.**

**16.** $1\frac{41}{70}$.
**17.** $1\frac{25}{48}$.
**18.** $2\frac{17}{36}$.
**19.** $2\frac{1}{6}$.
**20.** $2\frac{1}{3}$.
**21.** $2\frac{4}{5}$.
**22.** $2\frac{31}{45}$.
**23.** $2\frac{91}{162}$.
**24.** $2\frac{5}{36}$.
**25.** $\frac{7}{30}$.
**26.** $1\frac{31}{40}$.
**27.** $1\frac{5}{8}$.
**28.** $3\frac{5}{6}$.
**29.** $1\frac{17}{42}$.
**30.** $4\frac{3}{10}$.
**31.** $193\frac{1}{4}$.
**32.** $288\frac{2}{3}$.
**33.** $274\frac{11}{12}$.
**34.** 174.
**35.** $65\frac{14}{15}$.
**36.** $\$20.37\frac{1}{2}$.
**37.** $68\frac{71}{84}$.
**38.** $3\frac{17}{40}$.

**Page 55.**

**13.** $\frac{3}{11}$.
**14.** $\frac{43}{90}$.

**Page 56.**

**15.** $\frac{17}{45}$.
**16.** $\frac{5}{36}$.
**17.** $\frac{17}{54}$.
**18.** $\frac{11}{100}$.
**19.** $\frac{5}{42}$.
**20.** $\frac{19}{60}$.
**21.** $\frac{13}{132}$.
**22.** $\frac{35}{108}$.
**23.** $\frac{7}{36}$.
**24.** $\frac{7}{30}$.
**25.** $\frac{7}{36}$.
**26.** $2\frac{39}{40}$.
* **27.** $\frac{5}{72}$.
**28.** $\frac{12}{77}$.
**29.** $92\frac{13}{20}$.
**30.** $46\frac{7}{8}$.
**31.** $19\frac{1}{10}$.
**32.** $17\frac{13}{21}$.
**33.** $88\frac{11}{24}$.
**34.** $\$.70\frac{5}{6}$.
**35.** $\$.63\frac{3}{4}$.
**36.** $\frac{13}{24}$.
**37.** $9\frac{11}{12}$.
**38.** $1\frac{1}{2}$.
**39.** $\frac{119}{120}$.
**40.** $1\frac{7}{15}$.
**41.** $1\frac{13}{36}$.
**42.** $7\frac{3}{8}$.
**43.** $\frac{17}{60}$.
**44.** $\frac{2}{5}$.

**Page 57.**

**45.** $\frac{31}{63}$.
**46.** $\frac{3}{20}$.

**Page 58.**

**11.** $4\frac{4}{5}$.
**12.** $5\frac{5}{12}$.
**13.** $5\frac{5}{7}$.
**14.** $7\frac{6}{7}$.
**15.** $2\frac{1}{12}$.
**16.** $2\frac{15}{16}$.
**17.** $5\frac{1}{6}$.
**18.** $4\frac{11}{25}$.
**19.** 2,250.
**20.** 3,648.
**21.** \$166.
**22.** $\$3,466\frac{2}{3}$.

**Page 59.**

**34.** $51\frac{1}{3}$.
**35.** $50\frac{4}{15}$.
**36.** $29\frac{3}{11}$.
**37.** $254\frac{4}{5}$.
**38.** $623\frac{1}{13}$.
**39.** $465\frac{3}{10}$.
**40.** $432\frac{4}{7}$.
**41.** $1,702\frac{1}{2}$.
**42.** $522\frac{2}{3}$.
**43.** $13,736\frac{1}{3}$.
**44.** 572.
**45.** 693.
**46.** 808.
**47.** $8,223\frac{3}{4}$.
**48.** 8,649.
**49.** $13,533\frac{1}{3}$.
**50.** $45,196\frac{2}{3}$.
**51.** $17,427\frac{2}{3}$.
**52.** $42,745\frac{1}{2}$.

* Revised Edition.

**Page 60.**

**62.** $\frac{11}{16}$.
**63.** $\frac{20}{27}$.
**64.** $\frac{3}{10}$.
**65.** $\frac{3}{4}$.
**66.** $\frac{2}{7}$.
**67.** $\frac{1}{20}$.
**68.** $\frac{35}{64}$.
**69.** $\frac{1}{2}$.
**70.** $8\frac{1}{3}$.
**71.** $24\frac{3}{4}$.
**72.** $22\frac{11}{12}$.
**73.** 28.
**74.** $\$\frac{49}{96}$.
**75.** $34\frac{3}{8}$ cts.
**76.** $\$3\frac{1}{3}$.
**77.** $\$253\frac{3}{4}$.

**Page 61.**

**78.** $\$7,729\frac{1}{6}$.
**79.** $1\frac{13}{36}$.
**80.** $\$2.81\frac{1}{4}$.

**Page 62.**

**11.** $\frac{2}{21}$.
**12.** $\frac{2}{23}$.
**13.** $\frac{3}{40}$.
**14.** $\frac{3}{47}$.
**15.** $\frac{3}{61}$.
**16.** $\frac{3}{77}$.
**17.** $\frac{7}{24}$.
**18.** $\frac{11}{24}$.
**19.** $\frac{2}{3}$.

**Page 63.**

**28.** 36.
**29.** 45.
**30.** 75.
**31.** $123\frac{1}{3}$.
**32.** $169\frac{7}{17}$.
**33.** $297\frac{13}{21}$.
**34.** 12.
**35.** 36.
**36.** $40\frac{10}{11}$.

**Page 64.**

**45.** $\frac{3}{4}$.
**46.** $1\frac{7}{26}$.
**47.** $\frac{5}{8}$.
**48.** $\frac{4}{5}$.
**49.** $1\frac{1}{2}$.
**50.** $1\frac{9}{13}$.
**51.** $\frac{8}{15}$.
**52.** 5.
**53.** $\frac{7}{9}$.
**54.** $\frac{1}{2}$.
**55.** $4\frac{2}{3}$.
**56.** $2\frac{10}{13}$.

**Page 65.**

**57.** $5\frac{1}{4}$ months.
**58.** $15\frac{3}{5}$ bu.
**59.** $46\frac{7}{8}$ yds.
**60.** $6\frac{3}{22}$ h.
**61.** $25\frac{1}{4}$ A.
**62.** $44\frac{4}{9}$.
**63.** $25\frac{55}{81}$.

**Page 66.**

**65.** $\frac{5}{12}$.
**66.** $\frac{1}{28}$.
**67.** 27.
**68.** $\frac{2}{3}$.
**69.** $\frac{2}{3}$.
**70.** $1\frac{1}{2}$.
**71.** 20.
**72.** $\frac{1}{20}$.
**73.** $2\frac{6}{7}$.
**74.** $\frac{4}{33}$.
**75.** $\frac{2}{5}$.
**76.** $2\frac{1}{4}$.
**77.** 9.
**78.** $\frac{96}{413}$.
**79.** $3\frac{8}{9}$.
**80.** $\frac{160}{729}$.

**Page 71.**

**51.** 5.
**52.** $\frac{1}{12}$.
**53.** $41\frac{1}{3}$.
**54.** $8\frac{1}{6}$.
**55.** $1\frac{13}{32}$.
**56.** $1\frac{27}{28}$.
**57.** $504\frac{1}{6}$.
**58.** $1\frac{1}{6}$.
**59.** $188\frac{5}{8}$ A.
**60.** 26 sq. rd.
**61.** $13\frac{11}{69}$ h.
**62.** $12\frac{1}{2}$ T.
**63.** $\$3,132\frac{6}{7}$.
**64.** $14\frac{1}{3}$ yds.
**65.** 105.
**66.** $115\frac{1}{5}$ m.
**67.** $1,375.
**68.** $14,175.

**Page 72.**

**69.** 390.
**70.** $14,616.
**71.** $\$2,555\frac{5}{9}$.
**72.** $\frac{49}{160}$.
**73.** $\frac{4}{35}$.
**74.** A, 220 A; B, 176 A.
**75.** A's $2,310; B's $2,800; C's $1,050.
**76.** $35,200.
**77.** $4,875.
**78.** 8 of each.
**79.** $9,000.

## DECIMAL FRACTIONS

**Page 78.**

**58.** .0205
**59.** .040034
**60.** .02004
**61.** .0000615
**62.** 600.0015

**63.** 15.015
**64.** .00300303
**65.** .5000085
**66.** .00012
**67.** 400.000465
**68.** 25.025
**69.** 5000.005
**70.** 375.000000375

**Page 79.**

**71.** .30046
**72.** .001000045
**73.** 80040.0306
**74.** 15000.0015
**75.** 75.005043
**76.** 1000000.000001
**7.** .0674000
**8.** .07500
**9.** 62.700
**10.** 5.3300
**11.** 3.00
**12.** 45.0000
**13.** .045
**14.** 5.24

**Page 80.**

**19.** $\frac{1}{8}$
**20.** $\frac{3}{4}$
**21.** $\frac{3}{40}$
**22.** $\frac{1}{16}$
**23.** $\frac{13}{80}$
**24.** $\frac{9}{40}$
**25.** $\frac{1}{250}$
**26.** $\frac{9}{16}$
**27.** $\frac{1}{80}$
**28.** $\frac{141}{400}$
**29.** $3\frac{21}{40}$
**30.** $37\frac{3}{4}$
**31.** $62\frac{1}{10}$
**32.** $37\frac{5}{8}$
**33.** $56\frac{3}{8}$
**34.** $247\frac{1}{3}$
**35.** $16\frac{2}{3}$
**36.** $214\frac{1}{400}$

**Page 81.**

**42.** .625
**43.** .5625
**44.** .04
**45.** .78125
**46.** .512
**47.** .64
**48.** 1.28
**49.** 3 625
**50.** .096
**51.** .075
**52.** .0875
**53.** .095
**54.** .325
**55.** .0175
**56.** .092
**57.** .0032
**58.** $.0013\frac{1}{3}$
**59.** .04375
**60.** 12.15
**61.** 25.032
**62.** 37.1625
**63.** $.0083\frac{1}{3}$
**64.** $.076\frac{2}{3}$
**65.** $.126\frac{14}{111}$

**Page 82.**

**2.** 210.08595
**3.** 111.0188
**4.** $\$267.322\frac{2}{3}$
**5.** $120.0905\frac{1}{3}$
**6.** .2806484
**7.** .0252077
**8.** $148.58\frac{1}{3}$ rd.
**9.** $5.00\frac{1}{3}$ lb.
**10.** 22.84 in.

**Page 83.**

**3.** 2.0425
**4.** .61625
**5.** 11.9995
**6.** .594
**7.** .043956
**8.** .026095
**9.** .005005193
**10.** 4.95 miles.
**11.** 2.55 in.
**12.** .22°

**Page 84.**

**9.** 4.875
**10.** .2795
**11.** .23328
**12.** 1.152
**13.** .3136
**14.** 1.1772
**15.** 2.048
**16.** .5454
**17.** 1.344
**18.** 4.
**19.** 2.55
**20.** 640.
**21.** 1.08
**22.** 49.45
**23.** .75375
**24.** 256.
**25.** .00000943
**26.** 625.

**Page 85.**

**27.** 3406.
**28.** 48.
**29.** 25.6

**Page 87.**

**12.** 18.
**13.** 4.
**14.** 8.
**15.** 30.
**16.** 2500.
**17.** 20.
**18.** 150.
**19.** 24.
**20.** .05
**21.** .08
**22.** 2.07
**23.** .27
**24.** .0066
**25.** 790.

**26.** 900.
**27.** .009
**28.** .009
**29.** $3413\frac{1}{3}$.
**30.** .001024
**31.** .00005
**32.** 20000.
**33.** .00001
**34.** 100000.
**35.** .00005
**36.** 36.
**37.** 192.
**38.** 1500.
**39.** $2294.11\frac{13}{17}$
**40.** .00025
**42.** .48375
**43.** .00545
**44.** .00005

**Page 88.**

**1.** .024
**2.** .0028
**3.** $\frac{13}{40}$
**4.** $\frac{9}{2000}$
**5.** 1.08
**6.** 63.5475
**7.** 39.056875
**8.** 320.
**9.** 399.514
**10.** $\$177.66\frac{2}{3}$
**11.** .264
**12.** 105.
**13.** .000064
**14.** 50000000000.

## UNITED STATES MONEY.

**Page 90.**

**5.** $10.50
**6.** $40.605
**7.** $100.374
**8.** $25.005
**9.** $.065
**10.** $.104
**11.** 35000 cts.
**12.** 165000 m.
**13.** 17 m.
**14.** 4008 cts.
**15.** 10000 m.
**16.** $15.
**17.** $15.
**18.** 45 cts.
**19.** 25080 m.
**20.** 100010 m.

**Page 91.**

**7.** $271.64
**8.** $37.775
**9.** $617.20
**10.** $3361.25
**11.** $1053.10
**12.** $225.
**13.** $378.60

**Page 92.**

**14.** $83.50
**8.** $1010.25
**9.** $9.22+
**10.** $\$8666\frac{2}{3}$
**11.** $40.75
**12.** 60 carriages.

**Page 93.**

**13.** 94 tons.
**14.** $68.55
**15.** $527.05
**16.** $163.20

**Page 94.**

**1.** $236.35
**2.** $389.19
**3.** $4569.02
**4.** $53463.64

**Page 96.**

**9.** $565.50
**10.** $1000.
**11.** $4004.
**12.** $2343.75
**13.** $480.
**14.** 242 dozens.
**15.** 360 yards.
**16.** $1.969
**17.** $1.9625
**18.** $31.825

**Page 97.**

**19.** $35.4375
**20.** $2.245

**Page 98.**

**2.** $469.125
**3.** $40.946
**4.** $160.758

**Page 99.**

**5.** $22.017
**6.** $\$39.68\frac{1}{3}$

## MENSURATION.

**Page 103.**

**11.** $862\frac{1}{2}$ sq. ft.
**12.** 404.625 sq. yd.
**13.** $\begin{cases} 897\frac{3}{4} \text{ sq. ft.} \\ 450 \text{ sq. ft.} \end{cases}$
**14.** 7035 sq. ch.
**15.** 13000 sq. ft.
**16.** 66 yards.

**Page 104.**

**17.** 24 rods.
**18.** 165 yards.
**19.** 904.7 sq. yd.
**20.** 15 inches.
**21.** 113.0976 sq. in.
**22.** 314.16 sq. ft.

**Page 106.**

9. $1102\frac{1}{2}$ cu. ft.
10. 166.375 cu. yd.
11. 1953.125 cu. ft.
12. 5232 cu. ft.
13. 8 ft.
14. 76545 bricks.
15. 225 cans.
16. 2827.44 cu. in.

## DENOMINATE NUMBERS.

**Page 109.**

32. 9911 dr.
33. 778 pt.
34. 23983 yd.
35. 1902 P.
36. 7 bu. 3 pk. 3qt. 1 pt.
37. 10 gal. 1 pt.
38. 1 mi. 7 fur. 22 rd. 5 yd. 2 ft.
*39. 3° 27′ 40″.
40. 8 h. 31 min. 24 sec.
41. 57040 d.
42. 51320 P.
43. 2 cwt. 55 lb. 5 oz.
44. 2 mi. 3 fur. 18 rd. 1 yd.
45. 867240 in.
46. 1 mi. 6 fur. 12 rd. 4 yd. 1 ft. 8 in.
47. 26 cd. 7 cd. ft. 10 cu. ft.
48. 160 bu. 4 qt.
49. 140160 h.
50. 31622400 sec.
51. 2167200 min.
52. 236.8 pt.
53. 74.25 ft.
54. 43.8 oz.
55. 56436″.
56. 163° 28′ 7″.
57. 64944 ft.
58. 52 w. $1\frac{1}{4}$ da.
59. 4 A. 1 R. 26.35 P.
60. 8064 A.

**Page 111.**

26. $1\frac{1}{25}$ dr.
27. $19\frac{1}{2}$ in.
28. 220 pwt.
29. 1.2 d.
30. 1.092 h.
31. .04 lb.
32. .02 rd.
33. .09625 mi.
34. 6.144 pt.
35. $\frac{1}{1584}$ l.

**Page 112.**

36. $\frac{1}{672}$ cir.
37. $979\frac{1}{5}$ min.
38. 128 h.
39. $\frac{1}{1620}$ da.
40. .07 A.
41. 108.96 cd. ft.
42. 9.472 dr.
43. 326160 gr.
44. $\frac{11}{15}$ pt.
45. 12830.4 ft.
46. 3.24 £.
47. $.00026\frac{2}{3}$ ~~min.~~ miles
48. $\frac{7}{360}$ da.
49. 21.12 yd.
50. $.00045\frac{5}{11}$ mi.

**Page 113.**

14. 6 fur. 8 rd. 4 yd. 2 ft. 8 in.
15. 2 da. 22 h.
16. 5 oz. 12 pwt.
17. 4 yd. 1 ft. $4\frac{7}{8}$ in.
18. 3 R. $26\frac{2}{3}$ P.
19. 112 cu. ft.
20. 13 oz. 9.6 dr.
21. 6 cwt. 50 lb.
22. 3 in.
23. 3 qt. 1 pt. 2 gi.
24. 56 lb. 4 oz.
25. 1.728 cu. ft.

* Revised Edition.

**Page 114.**

12. $\frac{1}{3}$ rd.
13. $\frac{53}{160}$ bu.
14. $\frac{4}{7}$ lb.
15. $.001\frac{7}{73}$ c. yr.
16. .6125 bu.
17. $.66\frac{2}{3}$ £.

**Page 115.**

18. .3375 A.
19. .7 lb.
20. $.18\frac{2}{11}$ mi.
21. $\frac{23}{28}$.
22. $\frac{2}{15}$.
23. $\frac{1}{8}$.
24. $\frac{3}{16}$.

**Page 117.**

11. $85\frac{1}{3}$ yd.
12. $22\frac{1}{2}$ A.
13. $241920.
14. $\begin{cases} 1340\frac{5}{8} \text{ boards.} \\ 1345, \text{ in practice.} \end{cases}$
15. $95.185+.

**Page 118.**

16. 7722 bricks.
17. 56 rings, with.8 pwt. R.
*19. 110+ P.
18. 75 reams.
20. $7111\frac{1}{9}$ P.
21. $222.962+.
22. 90 bu.
23. 4.417875 A.
24. $174.5\frac{1}{3}$ cu. ft.
25. 930.24 gal.
26. 640 A.
27. 5760 A.
28. $3640.
29. $20.50.

**Page 124.**

17. 5765.06 grams.
18. 643.5 liters.
19. 6070.5 meters.
20. 6040.08 grams.
21. 23456 grams.
22. 3.458 grams.
23. 45060 liters.
24. 3540 liters.
25. 8450 sq. meters.
26. 1.324 meters.
27. 24 meters.
28. 4.345 liters.
29. 322500 grams.
30. 7.4635 kilograms.
31. 240.59 yards.
32. 27.6523 miles.
33. 9.4488 inches.
34. 709.375 bushels.
35. 9.9+ gallons.
36. 330.69 pounds.
37. 991.872 liters.
38. 72.49 steres.
39. 30.348 ares.
40. 536.448 meters.

## COMPOUND NUMBERS.

**Page 125.**

1. 3 T. 10 cwt. 15 lb. 1 oz. $11\frac{5}{16}$ dr.
2. 58 mi. 3 fur. 9 rd. 2 ft. $3\frac{7}{12}$ in.
3. 36 w. 22 h. 11 min. 47 sec.
4. 101 lb. 7 oz. 12 pwt. 12 gr.
5. 131 bu. 1 pk. 1. qt. 1 pt.
6. 1 C. 11 S. 15° 54′ 24″.
7. 18 cd. 54 cu. ft.

**Page 126.**

9. 5 mi. 5 fur. 17 rd. 4 yd.
10. 16 rd. 4 yd. 9 in.
11. 20° 27′ 44″.
12. 51° 22′ 6″.
13. 375 A. 3 R. 28 P.
14. 5 yr. 3 mo. 23 da.
15. 27 yr. 6 mo. 24 da.

* Revised Edition.

16. 93°.
17. 15° 31′ N.

**Page 128.**

2. 2 fur. 35 rd. 2 yd. 1 ft. 6 in.
3. 2248 bu. 1 pk. 2 qt.
4. 60 T. 16 cwt. 28 lb.
5. 2 lb. 8 oz. 3 pwt.
6. 2 mi. 6 fur. 14 rd. 1 yd. 2 ft. $8\frac{1}{7}$ in.
7. 2 mi. 2 fur. 12 rd. 2 ft. 5 in.
8. 1° 52′ $1\frac{19}{21}$″.
9. 1 lb. 7 oz. 15 pwt. $17\frac{9}{15}$ gr.
10. 35 bu. 3 pk. 4 qt.

**Page 129.**

11. 3 da. 2 h. 51 min. 40 sec.
12. 21 rd. 4 yd. 1 ft. $3\frac{6}{11}$ in.
13. 10 lb. 10 oz. $10\frac{4}{9}$ dr.
14. 95 rings, with 15 gr. R.
15. 12 kegs.
16. 264 rotations.
17. 120 lengths.
18. 48 barrels.
19. 627 axes, with 1 lb. 7 oz. R.
20. 1188 steps.
21. 213 da. 1 h. 23 min. 28 sec.

**Page 133.**

16. 22 min. 4 sec.
17. 27 min. 59 sec. past 9 o'clock A. M.
18. 9 min. 5 sec. past 3 P. M.
19. 34 min. $31\frac{3}{5}$ sec. past 3 P. M.
20. 37 min. $52\frac{2}{15}$ sec. past 5 P. M.
21. 23° 48′.
22. 33° 47′ 30″.
23. 40° 15′ W.
24. 97° 2′ 30″ W.
25. $51\frac{7}{13}$ min. past 10 A. M.
26. $52\frac{4}{53}$ min. past 9 A. M.

**Page 134.**

27. 1 h. 42 min.
*28. 31 min. 16 sec.
29. 1 h. 6 min. $3\frac{2}{5}$ sec.
30. 2 h. 32 min. 1 sec.
31. 3 h. 13 min. 45 sec.
32. 5 h. 37 min. $52\frac{2}{15}$ sec.

## PERCENTAGE.

**Page 136.**

12. .09
13. .45
14. 2.20
15. $.24\frac{2}{5}$, or .244
16. $.30\frac{1}{4}$, or .3025
17. $.00\frac{7}{15}$
18. $.20\frac{1}{4}$, or .2025

**Page 138.**

6. 12.25
7. 32.4
8. 180.
9. 9.375
10. $49.50
11. $3.
12. $5.4075
13. 14 lb.
14. 1.52 lb.
15. 70 days.
16. $2.70
17. $4.
18. 93 ft.
19. $.36375
20. $321\frac{2}{3}$ days.
21. $.09
22. .135
23. $1\frac{7}{9}$.
24. .054
25. .046875
26. 36 miles.
27. 2618 lb.
28. $28\frac{3}{4}$ tons.
29. $206.25
30. By R. R., 17062.5 bushels.

**Page 139.**

7. 6%
8. 20%
9. $33\frac{1}{3}$%
10. 20%
11. $16\frac{2}{3}$%
12. $2\frac{1}{2}$%
13. $\frac{1}{2}$%
14. $7\frac{1}{2}$%
15. 22%
16. 6%
17. $13\frac{1}{3}$%
18. 20%

* Revised Edition.

**19.** 75%
**20.** 40%
**21.** $83\frac{1}{3}$%
**22.** 30%
**23.** 15%
**24.** $91\frac{2}{3}$%
**25.** $55\frac{5}{9}$%

**Page 140.**

**26.** $78\frac{2}{3}$%
**27.** 45%
**28.** $20\frac{5}{6}$%

**Page 141.**

**7.** $15200.
**8.** $731\frac{3}{7}$.
**9.** 800.
**10.** $716\frac{2}{3}$.
***11.** 1500 sheep.
**12.** 7920 pounds.
**13.** $182.50
**14.** $80.60
**15.** 525 pupils.
**16.** 14500.
**17.** $50.
**18.** $14400.
**19.** 375 barrels.
**20.** $40000.
**21.** 150000000 sq. mi.
**22.** 80000.

**Page 143.**

**12.** $\$581.81\frac{9}{11}$
**13.** 240.
**14.** 125.
**15.** 2430.
**16.** 450 acres.
**17.** $120.
**18.** $\$.66\frac{2}{3}$
**19.** 600 pupils.
**20.** 800.
***21.** 145400.

**Page 144.**

***16.** 74422.

**Page 145.**

**17.** { 144 acres. / 122.4 " / 80 " / 133.6 "
**18.** 60%
**19.** $36\frac{6}{19}$.
**20.** $700.
**21.** $6480.
**22.** $9500.
**23.** $14175.
**24.** { Apple, 540. / Peach, 264. / Cherry, 150. / Pear, 246.
**25.** C's share, $7175.
**26.** $24570.
**27.** 37760.
***28.** 60000.

**Page 148.**

**19.** $7593.75
**20.** $29.70
**21.** 20%
**22.** 30%
**23.** $14\frac{2}{7}$%
**24.** $12\frac{59}{68}$%
**25.** $16\frac{2}{3}$%
**26.** $65280.
***27.** $2.50
**28.** $216.
**29.** $6750.
***30.** $36375.
**31.** $360.
**32.** $14\frac{2}{5}$%
**33.** $4.735

**Page 149.**

**34.** Sold at cost.

**Page 151.**

**13.** $103.275
**14.** $\$199.06\frac{1}{4}$
**15.** $88.136
**16.** { $38.59+ / $578.91—
**17.** $76312.50
**18.** $1950.

**Page 152.**

**19.** 12%
**20.** $6\frac{2}{3}$%
**21.** $30000.
**22.** $4170.
**23.** $5948.25
***24.** $3460.
**25.** $11600. C'n $174.
**26.** $4.80
**27.** $39.
**28.** $113.01+
***29.** $1357.886
**30.** 4434.59 bu.

**Page 153.**

**31.** 25% of sales.
**32.** $12857.14+
**33.** $2077.82+
**34.** { 25256.9+ lb. / $403.06+

**Page 156.**

**11.** $6937.50
**12.** $300.
**13.** $507.50

**Page 157.**

**14.** $456.
**15.** 40 shares.
**16.** $270.
**17.** $600.
**18.** $22000.
**19.** $8500.
**20.** 8.48+%
**21.** { Rate, $10\frac{37}{86}$% / Div., $469.36+
**22.** $5\frac{3}{10}$%
**23.** 80 shares.
**24.** 100 shares.
**25.** $7200 worth.

* Revised Edition.

**Page 158.**

**26.** $252.
**27.** 50 shares.
**28.** $3\frac{1}{3}$%

**Page 160.**

**8.** $625.
**9.** $120.875

**Page 161.**

**10.** $36.94—
**11.** $2\frac{1}{5}$.
**12.** $\frac{4}{5}$.
**13.** $15600.
**14.** $35000.
**15.** $495.
**16.** $840.
**17.** 1st Co., $20000.; 2d Co., $24000.; 3d Co., $16000.
**18.** $3290.
**19.** $20000.
**20.** $32000.

**Page 162.**

**21.** $22500.
**22.** $12040.
**23.** $12000.
**24.** $2600.
**25.** $34650.
**26.** $48636.

**Page 163.**

**27.** $111.90
**28.** $141.60
**29.** $2225.
**30.** $1250.

**Page 164.**

**31.** $6958.40

**Page 165.**

**2.** $1\frac{1}{5}$%, or 12 mills.
**3.** $1\frac{3}{10}$%, or 13 mills.
**5.** $12845.487, or $12861.368
**6.** $130280.51+

**Page 166.**

**7.** $478747.83
**8.** $791131.56+
**9.** A's $203.40; B's $147.759; C's $750.90
**10.** 5%, $69.80; 3%, $41.88
**11.** $96.
**12.** $258.16+

**Page 167.**

**14.** $10.62
**15.** $14.40
**16.** $1.287
**17.** $51.075
**18.** $12.9075
**19.** $81.12
**20.** $1284.
**21.** $1363.62
**22.** $2255.10
**23.** $3380.25

**Page 169.**

**1.** $328.90
**2.** $33480.
**3.** $4037.50
**4.** $7660.80
**5.** $3990.87+
**6.** $22680.
**7.** $3589.525
**8.** $2.20; $3.696

**Page 170.**

**1.** 62%
**2.** Last, $1385.70
**3.** 55%; $1375.

**Page 171.**

**4.** $64\frac{3}{5}$ cents.

## INTEREST.

**Page 173.**

20. $225.596
21. $6.53
22. $314.951
23. $79.08$\frac{1}{3}$
24. $4.462

**Page 174.**

25. $13.144
26. $16.78
27. $80.336
28. $136.81
29. $18.48
30. $173.919
31. $126.939
32. $807.202
33. $2.237
34. $166.31
35. $93.634
37. $23.869
38. $6516.66$\frac{2}{3}$
39. $90.782
40. $155.564
41. $238.565
42. $187.716
43. $297.498
44. $242.804

**Page 175.**

45. $66.54
46. $4.385
47. $814.123

**Page 178.**

29. $117.072
30. $38.88
31. $41.483
32. $13.105
33. $1120.759
34. $77.54+
35. $891.377
36. $119.523
37. $2238.944
38. $8.206
39. $19.188
40. $58.49+
41. $286.888
42. $31.711
43. $53.946
44. $410.84
45. $169.928
46. $1440.555
47. $101.767
48. $76.611

**Page 180.**

2. $3.075
3. $25.744
4. $67.225
5. $77.643
6. $123.33
7. $427.653
8. $153.208
9. $82.809
10. $152.40
11. $507.097

**Page 183.**

2. $508.717
3. $170.151
4. $283.103
5. $293.147

**Page 184.**

6. $463.761
7. $526.335
8. $210.806

**Page 185.**

9. $436.923
10. $125.496
1. $3.906

* Revised Edition.

**Page 186.**

2. $32.682
3. Am't, $56.729
4. $86.458
5. $229.858
6. $2.354 (by days.)
7. $952.66 "
8. $7.791 "
9. 7%
10. 5%
11. 10%

**Page 187.**

12. 8%
13. 8%
14. 8%
15. 8%
16. 7%
18. 2 yr. 6mo.
19. 3 yr. 11 mo.
20. 2 yr. 7 mo. 15 da.

**Page 188.**

21. 1 yr. 10 mo.
22. 3 yr. 3 mo. 9 da.
23. 10 yr.
24. 8⅓ yr.
25. 33⅓ yr.

**Page 189.**

27. $317.50
28. $3600.
29. $9000.
30. $5400.
31. $20000.
32. $7500.
33. $2540.

**Page 190.**

35. $64.244
36. $540.
37. $360.
38. $2400.

**Page 191.**

1. $35.636
2. $178.88
3. 7% (nearly.)
*4. 8% "
5. 2 yr. 9 mo. 23 da.
6. 1 mo. 8 da.
*7. 6 mo. 24 da.
8. $408.
9. $35.60
10. $630.
1. $192.78
12. $600.
13. $265.
14. $383.20 (by days.)
15. 10 years.
16 $445.783
17 $14285.714

**Page 192.**

1. $12 (dis.)
2. $10.

**Page 193.**

3. $20.905
4. $7.722
5. $19.736
6. $94.821
7. $25.937
8. $20.203
9. $59.125
10. $4.764
11. $7.927
12. $20.216
13. $1.62+
14. $4.233
15. $7.863
16. $.61+
17. $400
18. $5.

**Page 195**

2. $244.833 (proceeds.)
3. $143.223
4. $79.906
5. $980.625

* Revised Edition.

6. $741.
7. $1228.54+
8. $54.852
9. $117.973
10. $491.50
11. $8.50

**Page 196.**

12. $391.944
13. $.29+

NOTE.—The 90 days include days of grace.

14. $.43+
15. $.25+
16. July 12.
17. April 9.
18. December 12.
19. $461.317
20. $124.242
21. $21.488
22. $90.20+

**Page 197.**

*23. $379.489

NOTE.—The amount due April 4, 1870, is found by the "Merchant's Rule" (p. 184).

24. $500.
25. $2000
26. $836.729
*27. $2000.
28. $725.
29. $319 60 (no grace.)
30. $978.50
31. $878.75
32. { $87.83, with grace. / $90.77, without grace. }

**Page 202.**

2. $1243.75
3. $1052.625
4. $2537.50

**Page 203.**

5. $502.25
6. $644.15
7. $314.613
8. $256.
9. $980.
10. $1250.
11. $500.
12. $800.
13. $360.

**Page 204.**

1. $1642.50
2. $1425.
3. $4500.

**Page 205.**

4. $616.197
5. $9775.
6. $1250.
7. $456.524
8. $958.33$\frac{1}{3}$
9. $490.909
10. $.888
11. $273.125
12. 7$\frac{1}{5}$%
13. (1.) 6$\frac{42}{53}$, (2.) 6$\frac{12}{53}$, (3.) 5$\frac{35}{53}$.

**Page 206.**

2. $966.72
3. $1298.80

**Page 207.**

*4. $4415.60
5. $770.12 (no grace.)
*6. $807.066
*7. $544.956
8. $316.425

**Page 209.**

2. $129.303
3. $389.568
4. $900.407
5. $141.191

**Page 211.**

6. $1797.418
7. $800.516
8. $2074.296

* Revised Edition.

**Page 212.**

2. 7 months.
3. 8 months.
4. 4 months.
5. 7 months.
6. $5\frac{1}{11}$ months.
7. 4 months.
8. 49 days (48.9)
9. 46 days.

**Page 213.**

10. 68 days.
11. 46 days.
12. July 4, 1870.
13. July 29, 1870.
14. July 6, 1870.

**Page 214.**

15. 6 months after maturity.
16. 6 months after maturity.
17. 3 " " "

**Page 215.**

18. $400.
19. $10\frac{1}{3}$ months after maturity.
20. Nov. 1, 1870.
1. July 15, 1870.

**Page 216.**

2. Oct. 14, 1869.
3. Oct. 9, 1870.
4. Aug. 30, 1868. $949.49 (at 6%).

**Page 219.**

8. March 6, 1870.
9. Nov. 6, 1870.
10. Oct. 6, 1870.

## RATIO AND PROPORTION.

**Page 221.**

13. 35 : 17.
14. 3.4 : .62.
15. $2\frac{1}{3}$ : $\frac{3}{5}$
16. $3\frac{1}{5}$.
17. $2\frac{2}{3}$.
18. $\frac{3}{5}$.
19. 2.2
20. .45
21. $\frac{2}{5}$.
22. 9.
23. $\frac{50}{121}$.
24. 6.
25. $\frac{1}{4}$.
26. $\frac{1}{3}$.
27. $\frac{1}{5}$.
28. $\frac{3}{10}$.
29. 2 : 5.
30. 5 : 12.
31. 7 : 12.
32. 11 : 20.
33. 3 : 4.
34. 3 : 7.
35. 7 : 13.
36. 13 : 5.
37. 16 : 7.
38. 15 : 8.
39. 10 : 9.
40. 8 : 15.
41. 10 : 11.
42. 14 : 15.
43. 14 : 15.
44. 33 : 28.
45. 52 : 51.
46. 7 : 80.
47. 14 : 5.
48. 56 : 21.
49. 4 : 9, or $\frac{4}{9}$.

**Page 222.**

50. 36 : 55, or $\frac{36}{55}$.
51. 4 : 7, or $\frac{4}{7}$.
52. 8 : 35, or $\frac{8}{35}$.

**Page 226.**

14. 12.

15. 48.
16. 6.
17. 30.
18. 36.
19. .13
20. 14.4
21. .75
22. $\frac{15}{16}$.
23. $\frac{63}{80}$.
24. $1\frac{13}{36}$.
25. $\frac{5}{18}$.
26. 54 lb.
27. 14 oz.
28. 90 days.
29. 1.

**Page 227.**

31. $1757.777
32. $56.25
33. $8571.43—
34. 180 acres.
35. $6.90
36. 36000.
37. 66600.
38. 34° 40′.
39. 208 bbl. (52 w.)
40. 54 bbl.
41. 200 ft.
42. 150 ft.

**Page 228.**

43. $6075.
44. $39.60
45. $18\frac{1}{2}$ tons.
46. 96 apples.
47. $41\frac{2}{3}$ acres.
49. 75 days.
50. 36 men.
51. 40 and 50.
52. 288 and 352.

**Page 229.**

53. $4500; $5100.
54. 60 miles; 90 miles.

**Page 230.**

2. 2 : 3.
3. 25 : 27.
4. 10 : 9.
5. 16 : 45.
6. 16 : 15 :: 16 : 15.

**Page 231.**

7. 7 : 5 :: 42 : 30.
8. 3 : 14 :: 12 : 56.
*9. $108 : $216 :: 1 : 2.
10. 10 : 9 :: 10 : 9.
11. 39.
12. 10.
13. $33\frac{1}{3}$.
14. 81.
15. 49.

**Page 232.**

16. 3 men.
17. 18 men.
18. $60.
19. 960 cu. ft.

**Page 233.**

20. 63 bushels.
21. 256 miles.
22. 125 slabs.
23. $22.75
24. $99.
25. $675.
26. 8 men.
27. 10 days.
*28. 6 men.

**Page 236.**

2. { A's $2500. B's $2000.
3. { A's $1600. B's $ 900. C's $ 700.
4. { A's $2800. B's $2100. C's $1400.
5. $4400 and $3300.

* Revised Edition.

6. Proceeds { A's \$500. B's \$800. C's \$700.

**Page 238.**

*8. { A's \$3175. B's \$5715.

*9. { A's \$1700. B's \$2300. C's \$1000.

10. { A's \$1350. B's \$1200. C's \$1050.

**Page 243.**

61. { Younger, \$5985. Elder, \$8550.

62. { Younger, \$2896. Elder, \$3258.

63. { \$12040. \$15480.

64. \$3240 and \$4860.

65. A, \$387.50; B, \$310.

66. 7220.

**Page 244.**

67. 19800 steps.

68. \$620.

69. \$212.50

70. \$289.20

71. \$857.50

72. \$3756.

73. \$131.25

74. \$5654.40

75. \$949.218

76. \$137.50

77. 20 days.

78. 180 cords.

**Page 245.**

79. 15 days.

80. { A's \$1250. B's \$2000.

81. { A's \$1250. B's \$ 900.

82. { A's \$2550. B's \$1275. C's \$ 850.

83. { A's \$2502.50 B's \$2957.50

84. { A's $\$1577\frac{1}{7}$. B's $\$\ 822\frac{6}{7}$.

85. { A's \$900. B's \$850.

86. 56 days.

87. { 1st, 50 yd. 2d, 45 yd. 3d, 40 yd.

## INVOLUTION AND EVOLUTION.

**Page 246.**

7. 164836.

8. 74088.

9. 331776.

10. 1048576.

11. 42.25

12. .074088

13. .07776

14. $\frac{5832}{15625}$.

15. $\frac{625}{1024}$.

16. $\frac{14641}{20736}$.

**Page 247.**

17. 388129.

* Revised Edition.

**18.** 1157625.
**19.** 1197.16
**20.** .000004096
**21.** 3.8416
**22.** .002025
**23.** $\frac{2197}{3375}$.
**24.** $\frac{2401}{6561}$.
**25.** $277\frac{7}{9}$.
**26.** $123\frac{37}{81}$.
**27.** $244\frac{9}{64}$.
**28.** $\frac{256}{1089}$.
**29.** .000000000003125
**30.** 8.489664
**31.** $\frac{1}{2401}$.

### Page 248.

**32.** (29). 2809.
**33.** (30). 2025.
**34.** (31). 5625.
**35.** (32). 1444.
**36.** (33). 15625.
**37.** (34). 15376.

### Page 249.

**38.** (35). 148877.
**39.** (36). 91125.
**40.** (37). 32768.
**41.** (38). 79507.
**42.** (39). 157464.
**43.** (40). 42875.
**44.** (41). 140608.
***45.** (42). 166375.
**46.** (43). 512000.

### Page 253.

**13.** 58.
**14.** 65.
**15.** 88.
**16.** 79.
**17.** 87.
**18.** 365.
**19.** 459.
**20.** 648.
**21.** 702.

### Page 254.

**22.** 6400.
**23.** 3.24
**24.** 13.3
**25.** .25
**26.** .094
**27.** 24.221+
**28.** 2.343+
**29.** 116.047+
**30.** $\frac{25}{48}$.
**31.** $32\frac{1}{4}$.
**32.** 2.236

### Page 259.

**11.** 35. (2). 45.
**12.** 49. (2). 57.
**13.** 65. (2). 74.
**14.** 89. (2). 97.
**15.** 364.
**16.** 145. (2). 325.
**17.** 345. (2). 352.
**18.** 301. (2). 802.
**19.** 57. (2). 504.
**20.** 47. (2). 3002.
**21.** 2.5 (2). .422+
**22.** 4.6 (2). .25

### Page 260.

**23.** 3.83+ (2). 2.31+
**24.** .035 (2). 4.08
**25.** 2.08+ (2). 3.141+ (3) 3.683+
**26.** 1.259+ (2). 2.714+ (3). 5.846+

* Revised Edition.

27. $\frac{6}{12}$, or $\frac{1}{2}$. (2). $\frac{11}{35}$.
28. $2\frac{1}{4}$. (2). $3\frac{1}{3}$.
29. 27 inches.
30. 34.5 inches.
31. 59.9+ inches.
32. 24 feet.

**Page 264.**

1. 10.
2. 12 inches.
3. 36 feet.

**Page 265.**

4. 240 yards.
5. 30 feet.
6. 120 rods.
7. 225 feet.
8. 119.73+ feet.
9. 72.74+ rods.
*10. $104\frac{1}{6}$ rods.

**Page 266.**

11. 176.715 sq. in.
12. $872\frac{2}{3}$ sq. yd.
13. 10 yards.
14. 64.
15. 16.
16. 558.5+ sq. yd.
17. 25 feet.
18. 100.
19. 314.16 sq. in.
20. 196663355.75 sq. m.

**Page 267.**

21. 259333411782.86
22. 179.594 cu. in.
23. 64 balls.
24. 64.
25. 1000.
26. 3000 miles.
27. 33000 miles.
28. 12 inches.
29. 9.5 inches.
30. 35.014+ feet.
31. 70687.5 cu. ft.
32. 65512.7+ miles.
33. 105434.8+ miles.
34. Moon's surface about $\frac{1}{13}$ of the earth's surface.

* Revised Edition.

# GENERAL REVIEW PROBLEMS.

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**Page 273.**

**76.** 658.
**77.** 2997.
**78.** 12.
**79.** \$15.98
**80.** $145\frac{23}{98}$ A.
**81.** $2\frac{15}{16}$.
**82.** $9\frac{1}{6}$.
**83.** $1\frac{1}{24}$.
**84.** $\frac{3}{4}$.
**85.** $5\frac{10}{201}$.
**86.** $251\frac{1}{2}$.
**87.** $\frac{9}{64}$.
**88.** $2\frac{2}{5}$.
**89.** $\frac{16}{47}$.
**90.** \$960.
**91.** Value, \$32000; Part left, $\frac{7}{20}$.

**Page 274.**

**92.** 7.039
**93.** .0301965
**94.** 1024.
**95.** .002235
**96.** 16000
**97.** .075
**98.** .003
**99.** .000008
**100.** 7 oz. 10 pwt.
**101.** .01
**102.** .14175
**103.** 41760 min.
**104.** 7948800 sec.
**105.** $5091\frac{3}{7}$ steps.
**106.** $422\frac{2}{5}$.
**107.** 18 A.
**108.** $41\frac{1}{9}$ yd.
**109.** 184800 gr.
**110.** 300 sq. ft.
**111.** 45 cts.
**112.** \$15.20

**Page 275.**

**113.** 8 ft.
**114.** \$72.875
**115.** 130, with 1 bu. 1 pk. R.
**116.** \$60.25
**117.** $6415\frac{1}{5}$.
**118.** 28 min. 44 sec. past 11 A. M.
**119.** 5 P. M.
**120.** 42 min. $51\frac{2}{5}$ sec. past 4 P. M.
**121.** $\frac{1}{6}$.
**122.** .65
**123.** $\frac{1}{7744}$.
**124.** $\frac{97}{120}$ lb.
**125.** $\frac{3}{256}$.
**126.** 900 men.
**127.** $11\frac{2}{3}\%$
**128.** \$4.50
**129.** Gained $6\frac{1}{4}\%$

**Page 276.**

**130.** 4% loss.
**131.** Lost, $20.
**132.** $100.
**133.** { $986. / Gained, 17%.
**134.** $6 per yard.
**135.** 60%
**136.** 0%
**137.** 25%
**138.** $141.382
**139.** $16\frac{7}{8}\frac{1}{0}$%
**140.** $16.20
**141.** $2048.

**Page 277.**

**142.** 487804.8 lb.
**143.** 400 yd.
**144.** $5.368
**145.** $11.111
**146.** $6.511
**147.** $4\frac{1}{2}$%
**148.** $6\frac{2}{8}\frac{1}{9}$.
**149.** \$$466\frac{2}{3}$.
**150.** { $600. / $8\frac{1}{3}$%
**151.** $289.532
**152.** Discount, $220.
**153.** $36.
**154.** $2.583
**155.** $.314
**156.** $310.61
**157.** 9 months.

**Page 278.**

**158.** 6 months.
**159.** In $6\frac{2}{3}$ months.
**160.** $75.
**161.** $11 and $16.
**162.** 24 men.
**163.** 126 bushels.
**164.** $216.
**165.** $1.
**166.** $4800.
**167.** $2400.
**168.** Profits, { A, \$$555\frac{5}{9}$. / B, \$$333\frac{3}{9}$.
**169.** 20.5
**170.** 3.5
**171.** 40 feet.

**Page 279.**

**172.** 65 miles.
**173.** 50 feet.
**174.** 160 rods.
**175.** 28.3+ feet.
**176.** 28.28+ feet.
**177.** 154 cu. ft.
**178.** 78 sq. ft.
**179.** Area, 7.854 A.
**180.** 24.4+ in.
**181.** 64 balls.
**182.** 1728 blocks.
**183.** 128.57+ bu.
**184.** 3456 gal.
**185.** 564.019 gal.
**186.** { A, $445. / B, $230. / C $325.
**187.** 6 days.

**Page 280.**

**188.** $545.454
**189.** $4\frac{1}{8}$ miles.
**190.** { 1st, $3250. / 2d, $3900. / 3d, $1950.
**191.** $23\frac{3}{3}\frac{0}{1}\frac{1}{3}$ bu.

192. W., 18.; M., 22.; Ch., 50.

193. 1st, \$240.; 2d, \$180.; 3d, \$210.

194. A, $14\frac{2}{5}$ days.; B, 72 "; C, $10\frac{2}{7}$ "

195. A in 5 days.; B in 4 days.

*196. $2\frac{1}{2}$ times.

197. 1st, 60 miles.; 2d, 40 miles.

198. $7\frac{1}{2}$ feet.

**Page 281.**

199. 50 feet.

200. 3 hours.

## APPENDIX.

**Page 287.**

1. Fifty.
2. One hundred and ninety-five.
3. 130.
4. 400.
5. 1300.

**Page 299.**

1. 23.
2. 25.
3. 45.

**Page 300.**

4. 4.
5. $2\frac{1}{2}$.
6. 25, 29, 33, 37.
7. 34.
8. 3870.
9. 6th, 225.; 7th, 3213.
10. 156.
11. Last yard, \$1.49; Trench, \$37.75

**Page 302.**

1. 160.
2. 3645.
3. $5\frac{5}{243}$.
4. $\frac{1}{8}$.
5. 4th, $127\frac{7}{8}$.; 3d, $1827\frac{119}{243}$.
6. 3.
7. 3640.
8. \$256.; \$511.50
9. \$76293945.31

**Page 303.**

2. \$.326

**Page 305.**

4. 1 lb. of each. (1 ans.)
5. 300 lb. at 22 cents.; 200 lb. at 28 "; 500 lb. at 30 "
6. Rye, 270 bu.; Barley, 60 bu.; Oats, 60 bu.

7. $\begin{cases} 10\frac{4}{5} \text{ pwt. } 16 \text{ carat.} \\ 10\frac{4}{5} \text{ “ } 18 \text{ “} \\ 32\frac{2}{5} \text{ “ } 22 \text{ “} \end{cases}$

8. $\begin{cases} 50 \text{ lb. at } 15 \text{ cents.} \\ 50 \text{ “ } 17 \text{ “} \\ 100 \text{ “ } 20 \text{ “} \end{cases}$

9. 15 gal.

**Page 307.**

2. 36 ft. 9′ 1″ 11‴.
4. 29 ft. 2′ 1″ 8‴.
6. 8 sq. ft. 4′ 11″.
7. 26 cu. ft. 8′.

**Page 308.**

9. 7 ft. 3′
10. 7 ft. 8′.
1. 720.
2. 39916800.
3. 362880.
4. 40320.

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